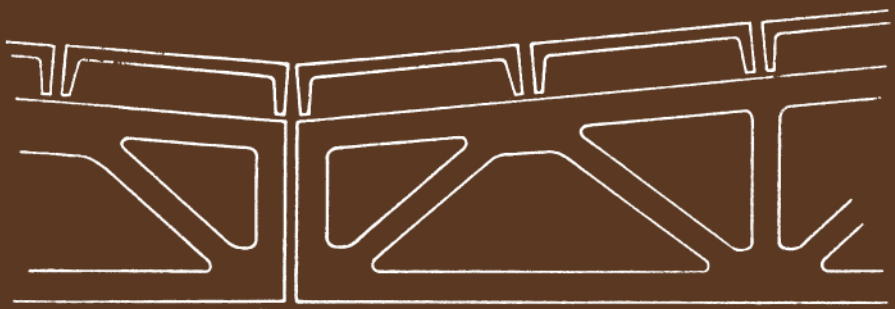
V. BAIKOV, E. SIGALOV

REINFORCED CONCRETE STRUCTURES

VOLUME 2

DESIGN OF BUILDINGS AND STRUCTURES



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The book is in two volumes. The first volume of the book presents a systematic and step-by-step approach to the physical and mechanical properti of concrete, reinforcing steel and reinforced concrete, and to the experimental basis for the strength theory of reinforced concrete. Ample space is devoted to the design principles of reinforced concrete members used in industrial and civil buildings. Attention is also given to the calculation technique for engineering and economic evaluation of structures.

The second volume of the book, set aside for Soviet practice in the design of reinforced concrete buildings and structures, introduces the reader to the design of bearing-wall and skeleton types of construction, floors, frames, foundations and other structures.

В. Н. БАЙКОВ

Э. Е. СИГАЛОВ

ЖЕЛЕЗОБЕТОННЫЕ КОНСТРУКЦИИ

«СТРОЙИЗДАТ», МОСКВА

V. BAIKOV, E. SIGALOV

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VOLUME 2

DESIGN OF BUILDINGS AND STRUCTURES

Translated from the Russian by
Alexander Kuznetsov

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The Russian Alphabet and Transliteration

А а	a	К к	k	Х х	kh
Б б	b	Л л	l	Ц ц	ts
в	v	М м	m	Ч ч	ch
Г г	g	Н н	n	Ш ш	sh
Д д	d	О о	o	Щ щ	shch
Е е	e	П п	p	ъ	''
Ё ё	e	Р р	r	ы	y
Ж ж	zh	С с	s	ь	'
З з	z	Т т	t	Э э	e
И и	i	У у	u	Ю ю	yu
Й й	y	Ф ф	f	Я я	ya

The Greek Alphabet

Α α	Alpha	Ι ι	Iota	Ρ ρ	Rho
Β β	Beta	Κ κ	Kappa	Σ σ	Sigma
Γ γ	Gamma	Λ λ	Lambda	Τ τ	Tau
Δ δ	Delta	Μ μ	Mu	Υ υ	Upsilon
Ε ε	Epsilon	Ν ν	Nu	Φ φ	Phi
Ζ ζ	Zeta	Ξ ξ	Xi	Χ χ	Chi
Η η	Eta	Ο ο	Omicron	Ψ ψ	Psi
Θ θ	Theta	Π π	Pi	Ω ω	Omega

На английском языке

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CHAPTER I

GENERAL PRINCIPLES OF ECONOMICAL DESIGN FOR REINFORCED CONCRETE BUILDINGS

I.1. PRINCIPLES OF LAYOUT FOR REINFORCED CONCRETE STRUCTURES

1. Construction Types

Industrial and civil buildings are made up of separate members combined in a single structure. This structure must stand up well to horizontal deformation induced by service loads and other factors, that is, it must have an adequate spatial rigidity. A load imposed on a building member is shared by the other members, so that the structure behaves as an integrated space system. Since the individual structural members (such as panels, floor beams, columns and walls) are designed to be sufficiently strong, stable, rigid and crack-resistant, and to contribute to the performance of the whole building, proper account of the behaviour of a building as a space system can substantially cut down the cost of the structure.

Reinforced concrete buildings that satisfy the above requirements may be of the skeleton type of construction or of bearing-wall construction; they may also be of multi- or one-storey types. The skeleton of a multi-storey building consists essentially of vertical and horizontal members, namely columns and girders (Fig. I.1). In a skeleton-type building, horizontal loads (such as wind, earthquake shocks and so on) may be taken up either jointly by the skeleton and filler walls combined in a single system by floor slabs, or solely by the skeleton which, in the absence of filler walls, is itself a space frame. In a multi-storey bearing-wall-type building, horizontal loads are jointly carried by the transverse and longitudinal bearing walls likewise combined in a single system by floor slabs. The skeleton of a one-storey building is constituted by columns embedded in a foundation, and hinged or fixed girders.

Whatever construction type, reinforced concrete structures should be designed so as to make the most of machines and mechanisms in the manufacture of members and the erection of buildings, and to

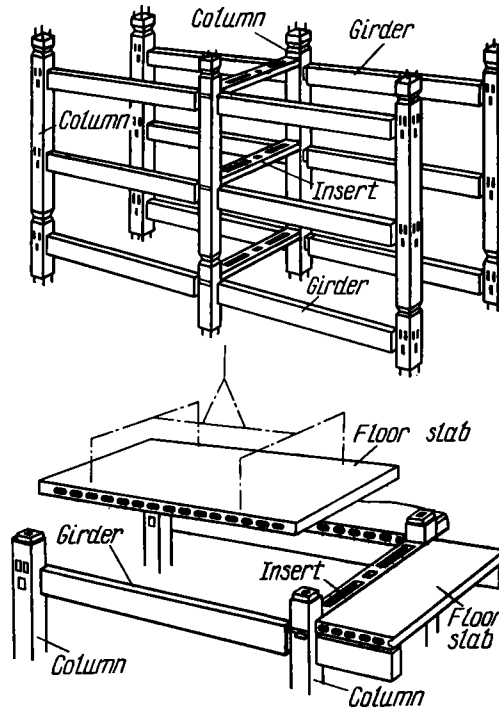

Fig. I.1. Reinforced concrete skeleton of a multi-storey building

keep labour consumption and the cost of construction materials to a minimum. From this point of view, the most suitable are prefabricated reinforced concrete structures.

2. Movement (Functional) Joints

The dimensions of reinforced concrete structures are subject to various factors. Falling or rising ambient temperature results in the contraction or expansion of reinforced concrete members, respectively, whereas shrinkage reduces their length. Nonuniform settlement of foundations gives rise to vertical displacements of structural parts.

Reinforced concrete structures are mostly statically indeterminate systems. So temperature variations, shrinkage and nonuniform settlement of foundations induce additional forces which may cause some part of a structure to crack or fail.

In order to reduce the forces brought about by temperature variations and shrinkage, reinforced concrete structures are divided in the longitudinal and transverse directions into what we shall call

strain blocks by movement or functional joints*. If, at a temperature of more than —40°C, the spacing of joints does not exceed the limits specified by Table I.1, the temperature and shrinkage analysis

TABLE I.1. Maximum Safe Spacing of Joints in Reinforced Concrete Structures

Construction type	Spacing of joints, m	
	heated buildings and buried structures	exterior structures and unheated buildings
Precast skeleton	60	40
Precast bearing-wall	50	30
In-situ and precast-in-situ skeleton	50	30
Same, bearing-wall	40	25

of non-prestressed reinforced concrete structures and prestressed structures subject to Category Three of cracking-resistance requirements may be omitted.

For one-storey skeleton-construction buildings, the spacing between joints may be increased by 20% over the tabulated data. The

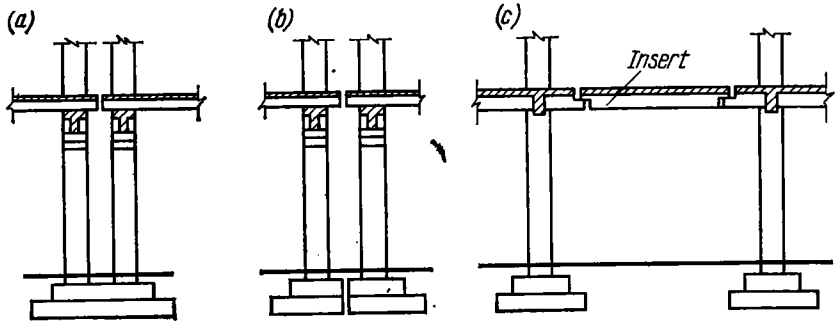

Fig. I.2. Movement (functional) joints
(*a*) expansion joint between twin columns; (*b*) settlement joint between twin columns; (*c*) settlement joint formed by a slab-and-beam insert

joint spacings of Table I.1 are given for skeleton-construction buildings with vertical members in the middle of a strain block. If the vertical members are at the ends of a strain block, the behaviour of the building resembles that of a continuous structure.

Movement joints are made in the overground part of a building (from the roof to the top of the foundation) so that floors and walls

* It is customary to class movement or functional joints in (1) expansion joints, (2) contraction joints, and (3) shrinkage joints.—*Translator's note.*

are separated. The width of joints usually ranges between 2 and 3 cm, according to strain-block length and temperature range. The best results are achieved when a movement joint is installed between twin columns and supporting twin beams (Fig. I.2*a*).

Settlement joints are provided between building parts differing in height or in buildings erected on a site with different soils; such joints are also made in foundations (Fig. I.2*b*). In addition, they may be formed by slab-and-beam inserts (Fig. I.2*c*). Settlement joints simultaneously act as expansion joints.

I.2. PRINCIPLES OF DESIGN OF PRECAST MEMBERS

1. Standardization of Precast Members

Prefabrication of reinforced concrete members is most efficient when a factory produces each type of elements on a repetitive basis. In this case, the manufacturing process can continuously be improved, the labour consumption and the cost of the members cut down, and the quality of the products enhanced. In view of this, it pays to limit the number of member types used in a building, and adapt them to as large a number of building designs as possible.

For this purpose, members are standardized. This refers to the selection of the most rational, field-tested design type which, as compared with other alternatives, requires the least amount of materials, is easier to manufacture and erect, and is less expensive. The member type thus chosen is adopted for quantity production.

Experience shows that for bending members, say, floor panels, varying in length or subjected to different loads, it is advisable to maintain the same cross-sectional dimensions while varying the steel area. For roof beams whose length and loading conditions vary over a wide range, it is recommended to modify both the cross-sectional dimensions and the amount of reinforcement. For the columns of multi-storey civil (and, sometimes, industrial) buildings, the cross-sectional dimensions should be maintained the same for all floors while varying the amount of reinforcement or, if necessary, the concrete brand. In this case, despite an excess of concrete in the upper floor columns, the overall cost of the structure will be reduced owing to the repeated use of the forms and standard reinforcing cages. In addition, standard columns make it possible to use floor beams of the same type in the various floors.

The necessary data on standard precast reinforced concrete members can be found in appropriate catalogues which are intended as guides in designing buildings and structures. With progress in technology and experience, standard elements are continually improved, so new catalogues containing advanced design types are published from time to time.

2. Standardization of Building Dimensions and Construction Types

In order that the same standard members can be used in a variety of buildings, the column and floor spacings in plan (grid plans) are standardized, that is, reduced to a limited range of sizes. For this purpose, the member sizes are set up to form a modular system based on a 100-mm module or a larger module which is a multiple of 100 mm.

In one-storey industrial buildings designed for overhead cranes, the centre-to-centre distance between columns in the longitudinal

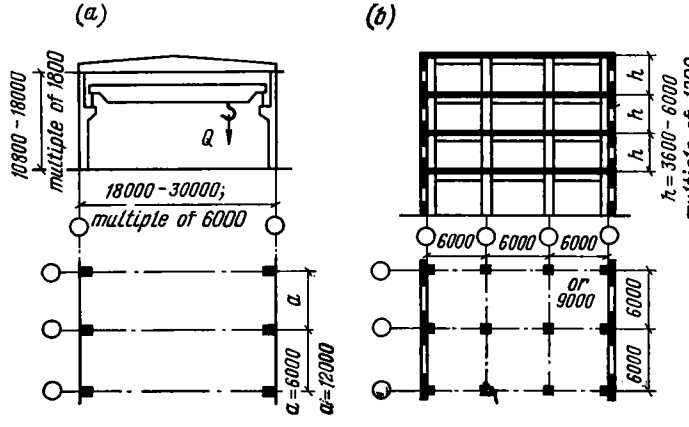

Fig. I.3. Standardized dimensions of industrial buildings

direction (the bay of a building) is assumed as 6 or 12 m, and the centre-to-centre distance in the transverse direction (the span of a building) is taken as a multiple of 6 m, that is, 18 m, 24 m, 30 m and so on (Fig. I.3*a*). The spacing between the floor and the bottom of the main load-bearing structure is adopted as a multiple of 1.8 m, for example, 10.8 m, 12.6 m, etc., up to 18 m.

In multi-storey industrial buildings, a standard grid plan with a grid size of 9 × 6 m is adopted for a basic live load of 5, 10 and 15 kN/m², and 6 × 6 m for a basic live load of 10, 15, 20 and 25 kN/m²; the floor spacing is assumed as a multiple of 1.2 m, for example, 3.6 m, 4.8 m, and 6 m (Fig. I.3*b*)

The module for civil buildings is a multiple of 200 mm. The longitudinal and transverse grid spacings vary between 2 and 6.4 m; in skeleton-construction buildings, they are usually taken as 6 m. The floor spacing in office buildings is a multiple of 300 mm, being 3.3 m, 3.6 m, and so on.

Dimensional standardization has made it possible to reduce building design types to a limited number of type designs, that is, designs using standard skeletons and members.

For the dimensional coordination of standard building members in the USSR, three dimension categories have been set up, namely nominal, structural and actual dimensions. *Nominal dimensions* refer to the centre-to-centre distances in plan. Thus, for a bay of

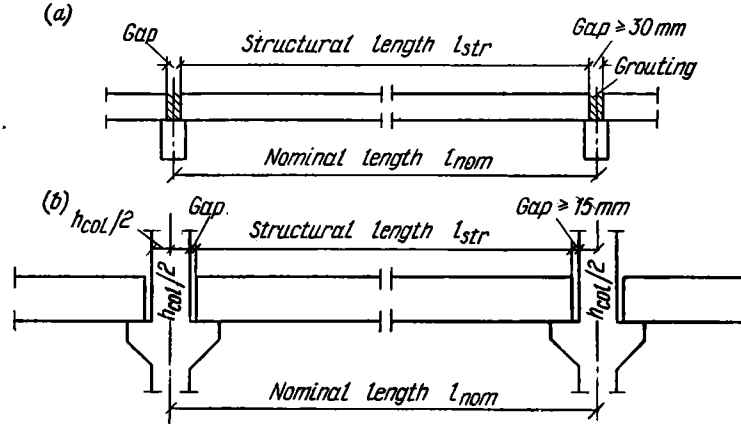

Fig. I.4. Nominal and structural dimensions of (*a*) prefabricated slabs and (*b*) prefabricated girders

6 m, a roof panel will have a nominal length of 6 m. The *structural dimensions* of a member differ from nominal by the width of gaps and joints. For example, a roof panel having a nominal length of 6 000 mm will have a structural length of 5 970 mm, that is the gap is 30 mm wide (Fig. I.4). The gap width depends on the erection conditions and technology, and it should be large enough for the convenient assembly of members and, if necessary, grouting. In the latter case, the gap width should be not less than 30 mm. *Actual dimensions* depend on the accuracy of manufacture and may differ from the structural dimensions by a certain value, known as a tolerance, which may range between 3 and 10 mm. Structural dimensions are assigned with allowance for the necessary gaps in joints, and standard tolerances.

3. Preassembly of Members into Larger Units

When designing buildings, it is necessary to provide for the preassembly of precast reinforced concrete members into larger units. Preassembled units reduce the number of field-assembly operations,

cut down the number of joints to be made in the course of erection, help to industrialize the manufacture of structural elements and, as a consequence, reduce the volume of finishing on the site. For civil buildings it is good practice to use floor panels covering a whole room and wall panels a whole floor high and a whole room wide. For industrial buildings, it is advisable to use large roof slabs which could be placed directly on roof trusses (thus avoiding beams). The size of preassembled units is limited by the weight and overall dimensions of the unit, which may vary according to the lifting power of erection mechanisms, load-carrying capacity of vehicles and methods of transportation.

For better use of erecting cranes, preassembled units should approximately be equal in weight which, in turn, should be as close to the lifting power of cranes as possible.

In the USSR, the maximum length of precast members to be transported by road or by rail is 24 m.

Since the size of preassembled units is sometimes limited by their weight, it pays to use thin-walled and hollow-core members, high-brand concrete and high-strength reinforcing steel. It is also advisable to design structures from lightweight concrete.

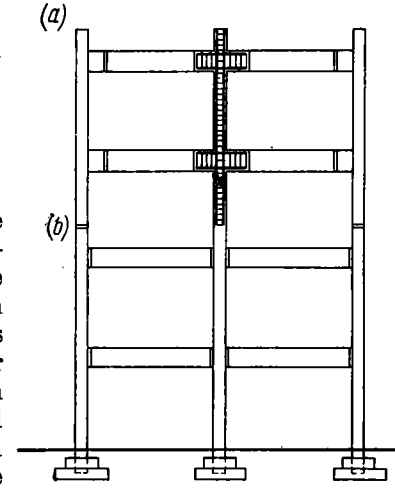

Fig. I.5. Division of a multi-storey frame into members to be prefabricated
(*a*) with cantilevers; (*b*) linear

4. Manufacture and Erection of Precast Members

In the USSR, r.c. members are said to adapt themselves to streamlined manufacture if they can be fabricated in quantity off-site with the help of high-output machines and mechanisms eliminating labour consuming manual work. The design of such members should be tailored to the manufacturing methods and facilities available. For example, the skeleton for a multi-storey building might be divided into separate units by breaking up the girders at points with minimum bending moments (Fig. I.5*a*). However, the overall width of the element includes the cantilevers whose arms are several times the column width. In flow-line and stage-by-stage production, a column with cantilevers would be ill-adapted to streamlined manufacture because a conveyer trolley is narrow and can carry only one column at a time, which markedly reduces output.

If we divide the skeleton for a multi-storey building into linear members, this will adapt them to fabrication by flow-line and stage-by-stage methods (Fig. I.5*b*). Now, although the bending moments and shearing forces at the break points sharply increase and make the erection more critical, this technique markedly raises the productivity; in view of this, it has been adopted as a standard procedure in the USSR. On casting beds or precasting yards, columns with cantilevers are relatively easy to fabricate, so they are adapted to streamlined manufacture in this case.

An appropriate arrangement of reinforcing steel and embedded parts is also important for the streamlined manufacture of reinforced concrete members.

Prefabricated members should also adapt themselves to streamlined erection (that is, they should be easy to install and secure in a design position, and permit removal of the crane hook). In fact, the breakdown of a structure into units is sometimes governed solely by erection requirements. For example, the columns of a multi-storey building skeleton are designed to be joined together at 800 to 1000 mm above floor level.

Joints between prefabricated members should be designed for both strength and ease of fabrication. Field-welded joints should be relatively few and easy to concrete.

Precast reinforced concrete members should contain suitable lifting eyes, lifting holes and so on. The required cross-sectional area for lifting eyes should be found by strength analysis. In the USSR, lifting eyes are only made of class A-II grade 10ГТ and class A-I grade ВСт3сп2 hot-rolled steel with a definite yield.

5. Loading Systems for Precast Members in Transit and Erection

When lifted, moved or placed in permanent position, prefabricated members are subjected to loads induced by their self-weight. In any of these situations, the disposition of loads may considerably differ from that existing in service. In fact, the cross-sectional area designed to carry service loads may sometimes prove insufficient to resist the loads arising in transit and during erection. Because of this, the loading systems for precast members should be designed so as to minimize the forces likely to be induced in transit and during erection. For this purpose, lifting eyes, holes and points of support should properly be arranged and shown in the accompanying drawings.

Precast members should be designed to carry their self-weight multiplied by a dynamic factor which should be 1.8 for transit and 1.5 for lifting and erection. Here, the self-weight overload factor is $n = 1$. Relevant standards allow the dynamic factor to be reduced

and taken as not less than 1.25 if experience with a given type of structure validates such a reduction.

A typical example of a member for which the design loading system differs from those existing in transit and during erection is furnished by a column (Fig. I.6*a*). In transit and during erection, the column is subjected to bending instead of compression, and the position of the compression zone and the compressive and tensile

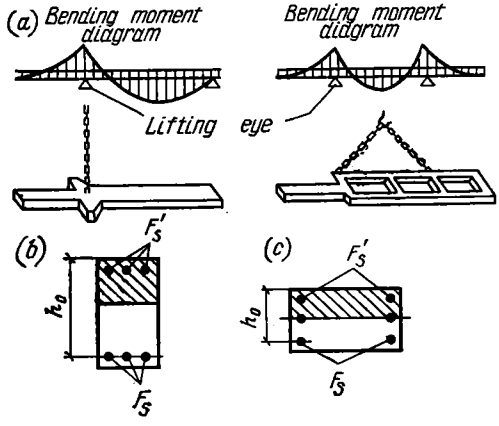

Fig. I.6. Loading systems for a prefabricated column during erection

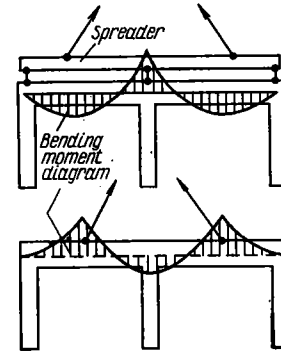

Fig. I.7. Loading systems for a prefabricated frame during erection

steel is other than in service (Fig. I.6*b* and *c*). To improve the disposition of loads during erection, it is advisable to move the lifting eyes from the ends towards the middle of the column; then the column will behave as a single-span beam with cantilevers, and the bending moments arising in erection will be reduced.

To illustrate a rational loading system during erection, consider a two-span frame (Fig. I.7). Using a spreader, the frame can be hooked at its joint ends. Then the bending moments in the girders will have the same sign as in service, and no additional reinforcement will be required to take care of erection stresses. If the frame were hooked up at two points directly (without a spreader), the bending moment distribution diagram would be different: negative bending moments would arise in the midspans of the girders, and additional reinforcement unnecessary in service would be required.

Members relatively deep but narrow in cross section (such as deep beams, trusses and wall panels) are usually transported in their service position (on the edge), because their load-bearing capacity in a horizontal position is small and the above strength-improving measures would be ineffective.

When designing reinforced concrete structures, it is necessary to provide for the stability of the individual members and the building as a whole in the course of erection, and for safety of labour.

When designing prefabricated reinforced concrete structures, it is necessary to specify not only the concrete brand to be used, but also the cube crushing strength at which the product may be released for transit and erection.

6. Joints and Ends of Precast Members

Precast reinforced concrete members making up a building or any other structure act together under load owing to the joints that provide reliable connection between them. Joints in precast structures may be classed according to the function served by the members joined together, and also according to the nature of forces.

Functionally, there may be joints between columns and foundations, between two columns, between girders and columns, between

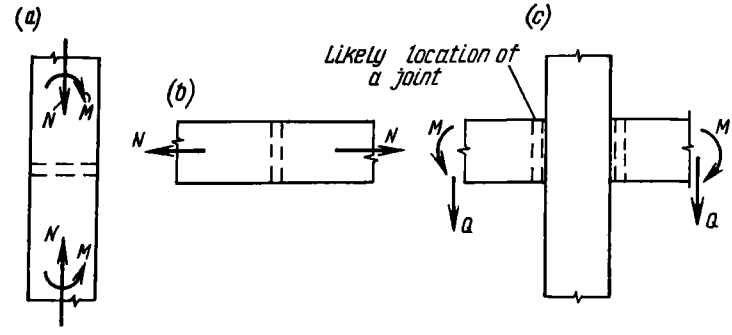

Fig. I.8. Types of joints between precast members and forces arising in them

crane beams and columns, between trusses and columns, between floor beams and columns, between panels and girders, and so on.

As regards the nature of forces, there may be compressive joints, such as joints between two columns (Fig. I.8*a*); tensile joints, for example, joints in the tensile chord of a truss (Fig. I.8*b*); joints subjected to combined bending and shear, such as those between a girder and a column (Fig. I.8*c*); and the like.

At a joint, the forces from one member are transferred to the other through welded load-bearing reinforcement, embedded steel parts and the concrete of the joint. A joint must be designed so as to have adequate strength and rigidity under service conditions, keep the members it connects in an unchanged relative position, and make the members easily adaptable to streamlined manufacture and erection. Joints should be disposed so that the members can be quickly

erected and reliably secured in the service position by simple devices (such as conductors) without special handling facilities. At the same time, their design should be such that all of the forces would be transferred from one member to the other. This primarily refers to joints between columns which, in the course of erection, transfer the loads due to the self-weight of the columns and the structural members lying above.

The gap width between the members is usually assigned so that the reinforcement stick-outs can conveniently be welded and the joint concreted to make up for the manufacturing and erection tolerances, and may range between 50 and 100 mm or even more. If a joint is to be grouted, especially under pressure, its width may be reduced, but it should not be less than 20 mm.

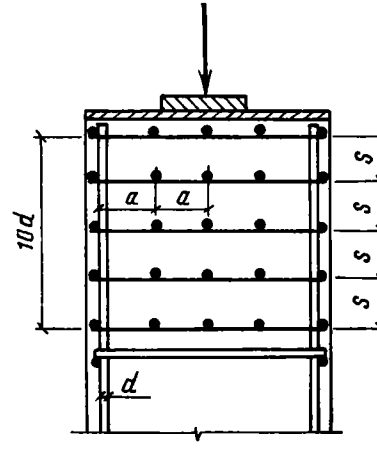

Fig. I.9. Member end additionally reinforced with transverse fabric

To prevent corrosion and ensure the necessary fire-resistance of members, embedded steel parts are given a coat of grout applied over steel fabric. Because of this, they should be buried deep enough so that no local protrusion would be noticeable after the protective coating has been applied. Where this condition is difficult to satisfy, special protective coatings should be used. Embedded steel parts should be as small as possible, but large enough to carry welds of an ample length.

The member ends in compression (for example, the ends of precast columns) are additionally reinforced by transverse steel fabric. If the longitudinal load-bearing reinforcement is cut off at a joint, the amount and distribution of transverse steel fabric should be determined by design. At each end, at least four pieces of steel fabric should be disposed over a distance of not less than $10d$ (where d is the deformed bar diameter), as reckoned from the end face of the member. The fabric spacing, s, should be not less than 60 mm, but not more than 1/3 of the smaller side of the cross section and not more than 150 mm, whichever is the smaller (Fig. I.9). The fabric meshsize should be at least 45 mm square, but not more than 1/4 of the least side of the cross section, nor more than 100 mm square. Local compression analysis should be carried out by an empirical formula, and the following condition (see Sec. IV.8, Volume 1) should be satisfied

$$N \leqslant R^*_{pr} F_{cr} \qquad (I.1)$$

where

$$R_{pr}^{*} = R_{pr}\gamma_c + k\mu_{ind}^{f} R_s^{f}\gamma_{ind} \tag{I.2}$$

$$\gamma_c = \sqrt[3]{F/F_{cr}} \leqslant 3.5$$

$$\gamma_{ind} = (4.5 \text{ to } 3.5)\, F_{cr}/F_k \tag{I.3}$$

$$k = (5 + \alpha)/(1 + 4.5\alpha) \tag{I.4}$$

is the efficiency of indirect reinforcement

$$\alpha = \mu_{ind}^{f} R_s^{f}/R_{pr} \tag{I.5}$$

μ_{ind}^{f} is the fabric reinforcement ratio defined as

$$\mu_{ind}^{f} = 2f/as \tag{I.6}$$

where a is the square-mesh size, f is the cross-sectional area of one fabric wire, and s is the spacing between the fabric pieces; R_s^{f} is the design strength of the fabric steel; F_{cr} is the area of crushing;

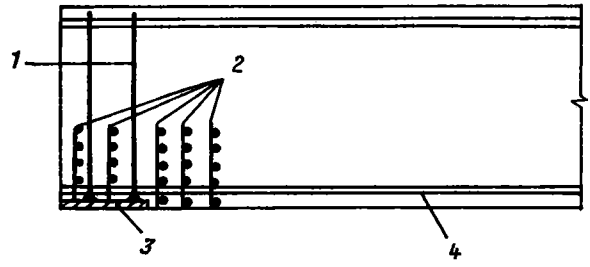

Fig. I.10. Additional reinforcement at the ends of a prestressed member
1—additional transverse bars; *2*—indirect fabric reinforcement; *3*—embedded steel part; *4*—prestressed longitudinal reinforcement

F is the design cross-sectional area of a member; and F_k is the cross-sectional area of the concrete within the outer wires (kern area).

The ends of prefabricated prestressed members should be locally reinforced against longitudinal splitting cracks which may form when the prestress is transferred from the steel to the concrete (Fig. I.10). This purpose can be served by additional transverse prestressed or nonprestressed reinforcing steel of cross-sectional area

$$F_s = kN_0/R_s \tag{I.7}$$

where $k = 0.15$ for prestressed steel; $k = 0.2$ for nonprestressed steel; $k = 0.3$ for nonprestressed steel in structures subject to fatigue analysis; N_0 is the prestressing force with allowance for the early losses; and R_s is the design strength of the additional steel.

Additional transverse nonprestressed reinforcement should extend the entire depth of a member and be welded to the embedded part at the support.

Also, the ends of prestressed members should additionally be congested with indirect reinforcement having μ_{ind} equal to at least 2%. The indirect reinforcement should extend for not less than $0.6\,l_{trans}$ and not less than 20 cm (if the longitudinal reinforcement has no anchors).

Embedded steel parts at the joints between precast reinforced concrete members are often made as plates and T-welded anchors provided to carry the forces, M, N and Q (Fig. I.11). In the design of the anchors, the bending moment is replaced by a couple with an arm, z, and the forces are multiplied by empirical coefficients. The cross-sectional area of the anchors in the most stressed row is given by

$$F_s = 1.1\sqrt{N_s^2 + (Q_s/kk_1)^2}/R_s \qquad \text{(I.8)}$$

Here, the maximum tensile force per anchor row (for n rows) is

$$N_s = N/n + M/z \qquad \text{(I.9)}$$

the maximum compressive force per anchor row is

$$N_{com} = -N/n + M/z \qquad \text{I.10)}$$

the shearing force per anchor row with allowance for friction is

$$Q_s = (Q - 0.3N_{com})/n \qquad \text{(I.11)}$$

k is the coefficient which, for anchors 8 to 25 mm in diameter and of cross-sectional area f_s, and for M-150 to M-600 heavy concrete, is defined as

$$k = 7\sqrt[3]{R_{pr}}/(1 + 0.15f_s)\sqrt{R_s} \leqslant 0.7 \qquad \text{(I.12}a\text{)}$$

and

$$k_1 = 1/\sqrt{1 + \omega N_s/Q_s} \qquad \text{(I.12}b\text{)}$$

For $N_{com} \geqslant 0$, $\omega = 0.3$; for $N_{com} < 0$, $\omega = 0.6$. The anchorage length l_{an} is calculated as prescribed in Chapter I of Volume 1. The centre-to-centre spacing between the anchors is given in Fig. I.11.

In tensile members, the joints are made by welding together stick-outs or embedded parts and in prestressed members, by passing the wire strands, cables or prestressed bars through the ducts or slots and then tensioning them. The welded joints between tensile members should be designed so as to prevent embedded parts and fishplates from unbending and the concrete from spalling.

In order to transfer shearing forces from one member to the other, notches are made on the surface of the members being joined, which, after the joint has been concreted, form concrete keys. Concrete keys can be used to advantage in the joints between girders and columns without cantilevers where they are arranged so that the keys are subjected to diagonal compression. They are also used in the joints

between slabs, to enhance the rigidity of panel structures in the plane of joints, etc. (Fig. I.12).

The dimensions of concrete keys are determined by the following formulas

$$\delta_{key} \geqslant Q/R_{pr} l_{key} n_{key} \tag{I.13}$$

$$h_{key} \geqslant Q/2R_{pr} l_{key} n_{key} \tag{I.14}$$

where Q is the shearing force; δ_{key}, h_{key} and l_{key} are the depth, height and length of a key, respectively; and n_{key} is the number of

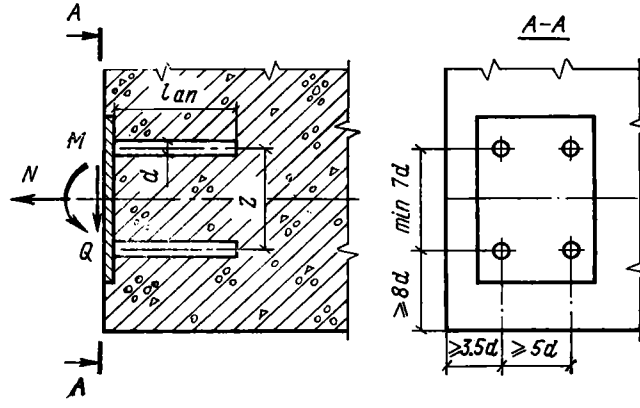

Fig. I.11. Embedded steel parts at the joints between r.c. members

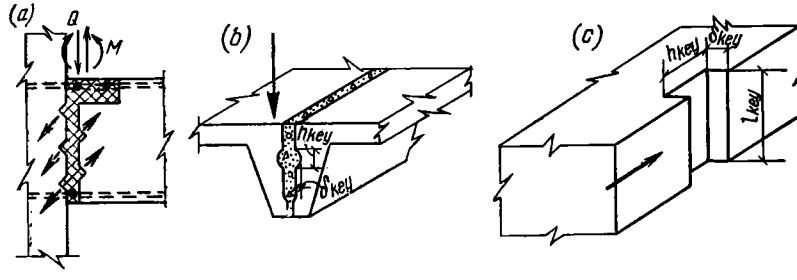

Fig. I.12. Concrete keys at the joints between r.c. members (*a*) between a girder and a column; (*b*) between slabs

keys used in design calculations (in shear-strength analysis, it should not exceed three).

When a key is subjected to a constant compressive force, N, the key height is determined with allowance for the relieving effect of friction, by the following formula

$$h_{key} = (Q - 0.7N)/2R_{pr} l_{key} n_{key} \tag{I.15}$$

Experience shows that the concrete field-placed at joints develops a good bond with the concrete of precast members, provided the surface of the precast members is properly cleaned, moistened, and so on. Joints should be concreted in reusable formwork, the concrete mix should be applied under pressure and electrically heated to speed up hardening, which is advisable even in the warm season.

Main welded joints are made in the downhand and vertical position. In the course of welding, the reinforcement, the embedded parts and, as a consequence, the concrete are locally heated. Experiments have shown that, when heated, concrete somewhat loses its mechanical strength. This, however, occurs only directly at the weld and does not affect the joint as a whole. If stick-outs are welded properly, the initial welding stresses (tensile in the reinforcing steel and compressive in the concrete) do not affect the load-bearing capacity of the joint, either.

7. Technical and Economic Evaluation of Reinforced Concrete Structures

For the purpose of evaluation at the design stage the following technical and economic parameters of separate members and entire structures are used: the amount of reinforcement in tonnes; the volume of concrete in m^3; man-days expended in manufacture and erection; and the cost. Ordinarily, these quantities are referred to a product or structure, although it may sometimes be necessary to know the above parameters as determined per m^3, m^2 or m of a structure or product. The main economic parameter of reinforced concrete structures is the cost which is defined as the sum of the costs of the construction materials, manufacture, erection, energy, process fuel and materials, investment, and maintenance (see Chapter IX of Volume 1).

In choosing a design for a reinforced concrete building and structure, it is usual to weigh several relevant alternatives by comparing them for the main technical and economic parameters. The parameters are determined on the basis of drawings worked out for the design stage at which the comparison is to be carried out. The most reliable data can be obtained from the working drawings of a structure.

The economic considerations should be coordinated with other design objectives during the entire design process, that is, when choosing the type and layout for the structure, when dividing the structure into separate units to be prefabricated, when selecting the shape and dimensions of members, when assigning the concrete brand number and the class or reinforcing steel, when selecting the types of reinforcement, etc.

CHAPTER II

FLOOR DESIGN

II.1. BASIC FLOOR TYPES

Floors made of reinforced concrete are most popular in industrial and civil buildings and structures. This is because their manufacture can be highly industrialized, the material consumption and the cost of manufacture and erection are relatively low, and the structure itself is sufficiently rigid, fireproof and durable in service.

There are two basic types of floor design, namely beam and girder construction and flat-slab construction. The former refers to floors formed by slabs or floor panels supported by and acting together with beams running in one or two directions. In the flat-slab construction, flat slabs are carried directly by columns flared at the top to form what are called capitals. Either type may be built of precast members, cast in-situ or erected as a combination of precast and in-situ members. Precast and in-situ floors differ in design approach, so, when classifying them, it is necessary to mention the manner of manufacture. Accordingly, we may distinguish precast slab and girder construction, in-situ beam and girder construction, in-situ ribbed-slab construction with slabs spanning in two directions, precast/in-situ beam and girder construction, precast flat-slab construction, in-situ flat-slab construction, and precast/in-situ flat-slab construction.

According to the aspect ratio, floor slabs may be classed as follows. When $l_2/l_1 > 2$ (Fig. II.1*a*), we speak of a one-way slab subjected to bending along its shorter side; the bending moment induced along the longer side is small and may be neglected. When $l_2/l_1 \leqslant 2$ (Fig. II.1*b*), we speak of a two-way slab with two-way reinforcement, subjected to bending in two directions.

In mass construction, use is mostly made of precast floor types because their manufacture can be highly industrialized. In-situ

floors are used rather seldom, chiefly in buildings erected to other than type designs.

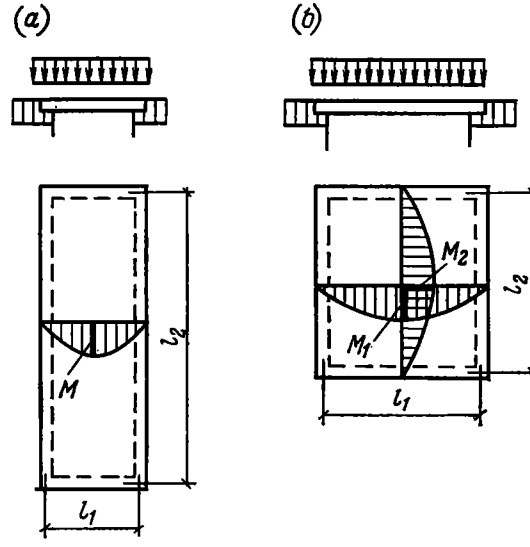

Fig. II.1. (*a*) One-way slab subjected to bending along its shorter side and (*b*) two-way slab subjected to bending in two directions

The type of floor construction is chosen from economic considerations in each particular case according to the purpose of a building, magnitude and type of loading, local conditions, etc.

II.2. SLAB AND GIRDER CONSTRUCTION

1. Arrangement of Structural Members

The slab and girder floor construction consists essentially of floor slabs (or panels) supported by girders (Fig. II.2*a*). In turn, the girders are carried by columns or bearing walls. There may be longitudinal girders (those directed along the building) and transverse girders (Fig. II.2*b*). Together with the columns, the girders make up the skeleton of the building.

In the transverse direction, a floor may have two or three spans in civil buildings, and five or six spans in industrial buildings. In the latter, the span of a girder depends on the general layout, floor type and dead load, being from 6 to 9 m with columns spaced 6 m apart. A girder span in civil buildings depends on the column grid size which may range between 2.8 and 6.8 m, being a multiple of 0.2 m.

The arrangement of structural members consists in selecting the direction of girders, assigning the span and the spacing between girders, and choosing the type and size of slabs. Here, it is necessary to take into account:

(1) the magnitude of live load, the purpose of a building, and the general layout;

(2) the general arrangement of all structural members in the building. In buildings where the transverse spatial rigidity is provided by frames with rigid joints, the girders should be run in the transverse

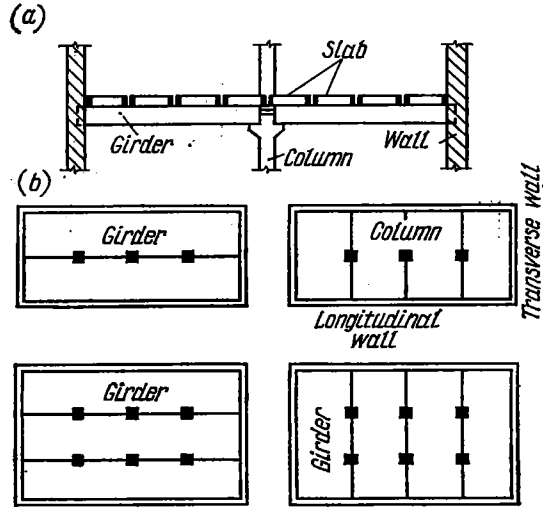

Fig. II.2. Arrangement of slabs and girders in the slab and girder floor

direction, and the slabs in the longitudinal direction. In civil buildings, the girders may be directed along, and the slabs across, the building. In either case, the columns should be appropriately spaced;

(3) the technical and economic parameters of the floor type. The amount of concrete and steel should be minimal, and the members should have a maximum weight and size compatible with the lifting power of erecting cranes and the load-carrying capacity of vehicles.

As already noted, several alternative designs are usually worked out and compared, and the least expensive is then adopted for construction.

The overall amount of concrete and steel per reinforced concrete floor is the sum of the amounts of concrete and steel in slabs, girders and columns. Slabs require about 65% of reinforced concrete, which is why primary attention should be focussed on the optimal design of slabs.

2. Floor Slabs

Selection of Optimal Cross Section. In order to save concrete and to reduce the weight of slabs, use is made of hollow-core or ribbed slabs (Fig. II.3*a*). A large part of the concrete on the tensile side of a slab is eliminated, leaving only the ribs or joists wide enough to contain reinforcing cages and resist shearing stresses. In the span

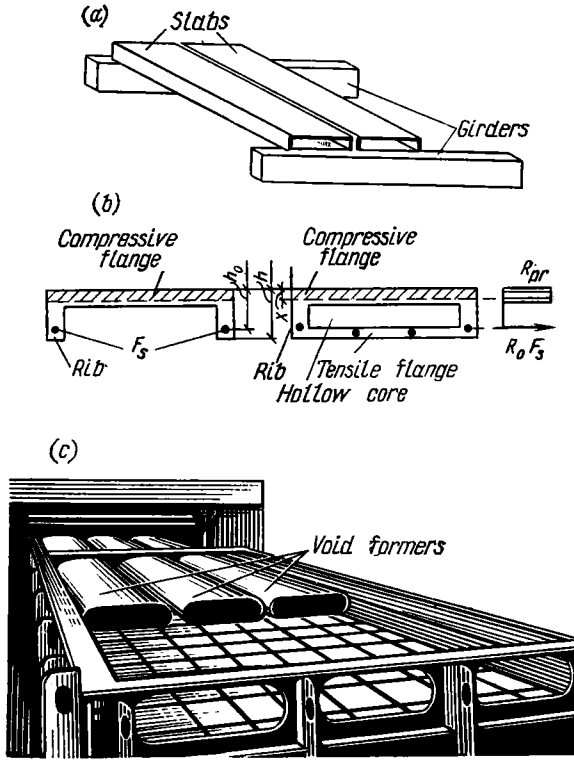

Fig. II.3. Floor slabs

between girders, such slabs behave in bending as T-beams (Fig. II.3*b*). The top flange of a slab is also subjected to local bending between ribs. If it is necessary to make a flat ceiling, the hollow cores are closed from the bottom.

Hollow cores in slabs may be oval, round or of some other shape. Wide slabs may contain several adjacent hollow cores (Fig. II.3*c*).

The general principle of design for any floor slabs may be stated as follows. Leave just enough concrete to form vertical ribs resisting

shearing forces and remove as much of the rest as the maker's plant and capabilities permit.

According to their cross section, floor slabs may be classed into those with oval, round and vertical voids, ribbed slabs with their

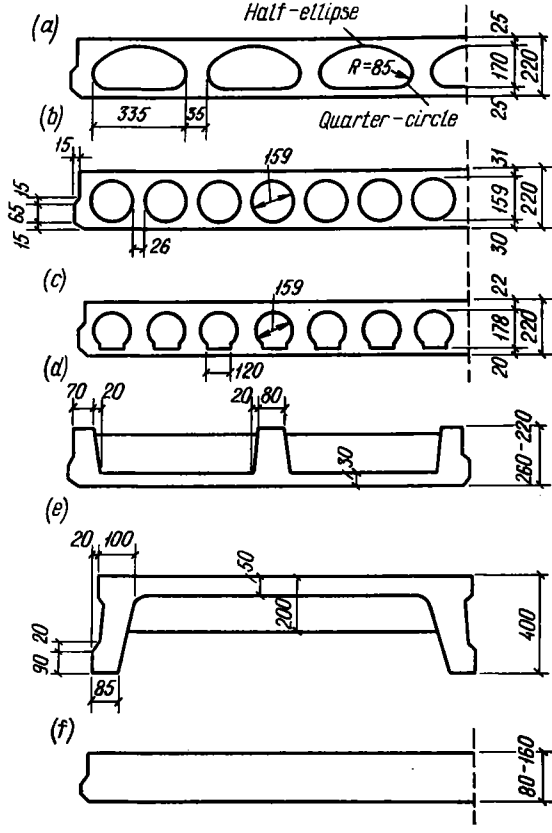

Fig. II.4. Cross sections of floor slabs

joists facing up (with floor covering placed on joists) or down, and also solid slabs (Fig. II.4*a* through *f*).

In hollow-core slabs, the minimum thickness of topping ranges between 25 and 30 mm, and the minimum thickness of ribs is from 30 to 35 mm. In ribbed slabs with their joists facing down, the minimum thickness of topping is from 50 to 60 mm.

If the length of slabs is specified in advance, their width should be chosen such that their weight would not exceed the lifting power of erecting cranes (which is usually 3 or 5 tonnes, or, sometimes,

greater). Slabs 3.2 m wide and 6 m long can cover a whole room; if they contain voids, the weight of such slabs is 5 or 6 tonnes. Hollow-core and solid slabs which form flat ceilings are used in civil buildings, whereas ribbed slabs with their joists down are used in industrial buildings for basic live loads of above 5 kN/m^2.

The main parameters of floor slabs are the reduced concrete depth determined by dividing the volume of concrete in a slab by its surface area, and the amount of reinforcing steel (Table II.1).

TABLE II.1. Floor Slab Parameters at a Nominal Span of 6 m and a Basic Load of 6 to 7 kN/m^2

Slab type	Reduced depth, cm	Amount of steel per square metre, kg		
		nonprestressed	prestressed	
			bars	wire
With oval voids	9.2	8	4.3	3.4
With vertical voids	10.2	8.5	4.7	3.7
With round voids	12	8.5	4.7	3.7
Ribbed with ribs up	8	9.1	5	4
Solid	12-16	14-16	12-14	10-11

As regards the amount of concrete used, the best choice is oval-void slabs whose reduced concrete depth is 9.2 cm, whereas that of round-void slabs is 12 cm. Oval-void slabs, however, are rather difficult to manufacture at a factory because, after the void formers have been removed, the voids in the finished product tend to collapse. That is why round-void slabs are adopted as a standard, and oval-void slabs may be made only if a factory has appropriate equipment and adequate know-how. With further progress in the factory manufacture of hollow-core slabs it may be expected that still better slab designs will be made available.

Ribbed slabs with their joists facing up having a relatively small reduced concrete depth of 8 cm are less adapted to industrialized manufacture, because they require additional floor topping. As a result, the cost of the ribbed-slab floor is higher.

In ribbed slabs with their joists facing down, the reduced concrete depth is 10.5 cm, and the amount of reinforcing steel per square metre ranges between 8.3 and 21.5 kg, depending on live load.

Prestressed floor slabs are made of M-300 and M-400 concrete, whereas nonprestressed slabs are manufactured of M-200 and M-300 concrete.

Slab Design. The effective span of a slab, l_0, is taken as the centre-to-centre distance between its supports (Fig. II.5*a* through *c*); if the slab is supported by the top faces of girders, $l_0 = l - b/2$

(where b is the width of a girder); and, if it is carried by girder flanges, $l_0 = l - a - b$ (where a is the flange overhang). If one end of a slab is supported by a girder and the other by a bearing wall, its effective span is taken as the centre-to-centre distance between the support on the wall and that on the girder.

The depth of a slab, h, should be chosen to ensure the necessary strength and stiffness (so that limiting sags would not be exceeded).

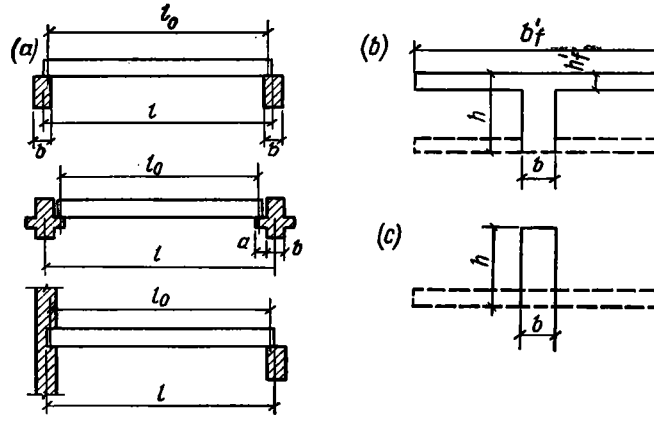

Fig. II.5. Effective spans and cross sections of floor slabs

With spans ranging between 5 and 7 m, the depth of the slab section is mostly governed by rigidity requirements. The slab section depth satisfying both strength and rigidity requirements can approximately be determined by the following formula

$$h = cl_0R_s(\theta g^b + p^b)/E_sq^b \qquad \text{(II.1)}$$

where $c = 18$ to 20 for hollow-core slabs, and 30 to 34 for ribbed slabs with the flange in the compression zone, the greater values being specified for class A-II reinforcement, and the lower for class A-III reinforcement; g^b is the long-time basic live load per square metre of the floor; p^b is the short-time basic live load per square metre of the floor;

$$q^b = g^b + p^b$$

θ is the factor taking care of the increase in sag under long-time load, equal to 2 for hollow-core slabs and 1.5 for ribbed slabs with the flange in the compression zone.

The depth of prestressed slabs may also be approximately determined by Eq. (II.1) or assumed as

$$h = (1/20 \text{ to } 1/30)\, l_0 \qquad \text{(II.2)}$$

For strength analysis in terms of bending moment, the width of the rib, b, is assumed as the total width of all slab ribs, and the design width of the compressed flange as the total width of the slab. If the topping is thin, that is $h_f'/h \leqslant 0.1$, the flange width taken in calculations should not exceed

$$b_f' = 12\,(n-1)\,h_f + b \qquad \text{(II.3)}$$

where n is the number of ribs in the cross section of the slab.

If in a ribbed slab with its joists facing down and containing lateral joists $h_f'/h < 0.1$, the flange width is taken as the total width of the slab.

Thus, the strength analysis of slabs reduces to that of a T-section with the flange in the compression zone. In most cases, the neutral axis lies within the compressed flange, so, on determining

$$A_0 = M/R_{pr}b_f'h_0^2$$

the values of ξ and η are looked up in the table and a check is made to see if $x = \xi h_0 \leqslant h_f'$. Then, the cross-sectional area of the tensile steel is found as

$$F_s = M/R_s\eta h_0$$

When $x = \xi h_0 > h_f'$ and the neutral axis crosses the rib, allowance should be made for compression in the rib.

The design width of a slab with its ribs facing up is taken equal to the total width of the ribs, and the section is treated as rectangular.

The transverse reinforcement for a slab is designed to carry shearing forces, with the design rib width, b, assumed to be equal to the total width of all ribs in the section.

In hollow-core slabs with a depth of 300 mm or less, transverse reinforcement may be dispensed with, provided

$$Q \leqslant k_3R_{ten}bh_0^2/c$$

where the right-hand side should be not less than $k_1R_{ten}bh_0$ and not more than $2R_{ten}bh_0$; k_3 is taken as 1.2 for heavy concrete, and 0.8 for porous-aggregate concretes; k_1 and c have been explained in Chapter III of Volume 1.

Incipient cracking, crack-opening and sagging analyses are carried out according to the category of crack-resistance of a given slab (see Ch. VII of Volume 1).

In analysis for sagging, hollow-core sections are replaced by equivalent I-sections. For round-void slabs, the equivalent I-section is found by equating the area of a round hole of diameter d to the area of a square hole of side h_1 (Fig. II.6*a*). Hence,

$$h_1 = d\sqrt{\pi}/2 = (\text{approx.})\,0.9d$$

Oval-core sections are reduced to equivalent I-sections (Fig. II.6*b*) by replacing oval voids by rectangular voids of the same area and

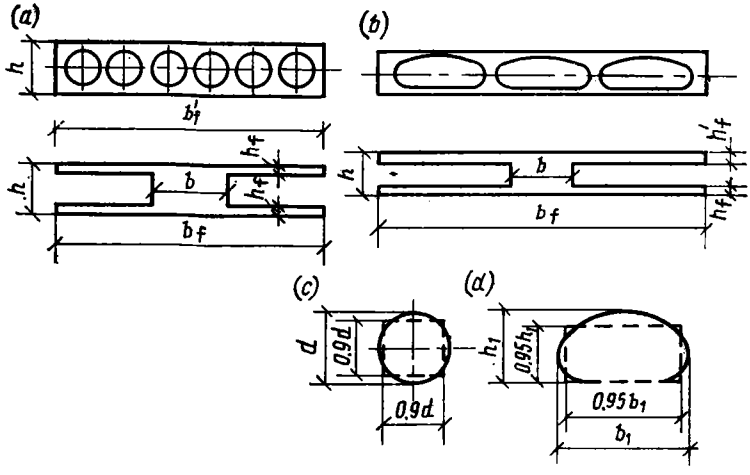

Fig. II.6. Equivalent cross sections for sagging analysis

moment of inertia so that their centroids would coincide. On designating the width and depth of the equivalent rectangle as b_1 and h_1, and the area and moment of inertia of the oval as F and I, we get

$$F = b_1 h_1;\ I = b_1 h_1^3/12 = F h_1^2/12$$

Hence,

$$h_1 = \sqrt{12I/F} \text{ and } b_1 = F/h_1$$

For hollow-core slabs with h ranging between 15 and 25 cm and cores up to 50 cm wide, the equivalent sections can be found as shown in Fig. II.6*c* and *d*.

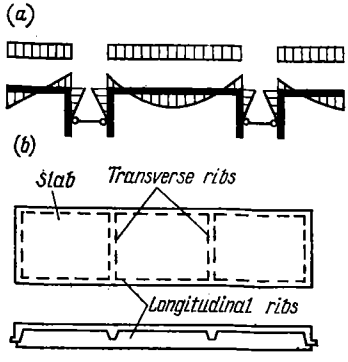

Fig. II.7. Loading system for slab flanges

The slab flange behaves in local bending as a slab of span l_0 equal to the clear distance between the ribs, partly fixed on the supports. In ribbed slabs with their ribs down, such a fixation is provided by concreting the joints, which prevents the ribs from angular deflection (Fig. II.7*a*). Here, the bending moment is defined as

$$M = ql_0^2/11$$

In ribbed slabs with intermediate transverse ribs, the bending moments in the flange may be determined as in a two-way slab supported on four sides (Fig. II.7*b*).

Reinforcement and Layout of Slabs. Slabs are reinforced with welded-wire fabric and reinforcing cages made of plain wire and hot-rolled deformed bars (Fig. II.8). Longitudinal prestressed reinforcement may consist of class A-IV, A-V, A_T-IV or A_T-V bars,

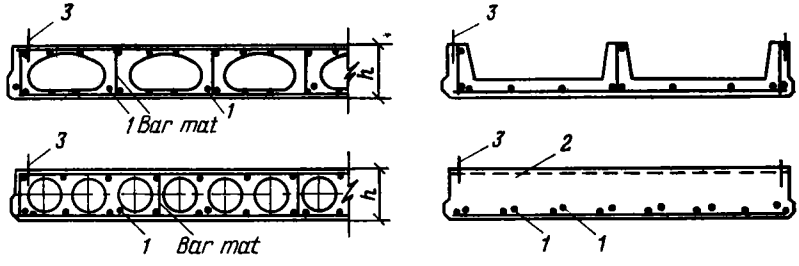

Fig. II.8. Reinforcement of floor slabs
1—prestressed reinforcement; *2*—narrow fabric placed on the topside; *3*—lifting eyes

high-strength wire or wire strands. If the slab span is less than 6 m, prestressed reinforcement may be omitted.

The longitudinal load-bearing reinforcement is distributed over the entire width of the bottom flange in hollow-core slabs and in the ribs of ribbed slabs. In hollow-core slabs, the distance between longitudinal bars should not exceed 400 mm. Transverse bars are joined to longitudinal nonprestressed distribution and load-bearing bars to form welded-bar mats which are placed in the ribs of slabs. In round-core slabs, welded-bar mats may be located only near supports at every two or three ribs.

The ends of longitudinal nonprestressed bars in ribbed slabs are welded to anchors made of angles or plates to hold the bars in place at the supports.

Solid slabs of heavy or porous-aggregate concrete are reinforced with prestressed longitudinal steel and welded fabric.

Lifting eyes are placed at the four corners of a slab and welded to the main reinforcement. Near lifting eyes, solid slabs are reinforced with additional top fabric. A likely reinforcement for a ribbed slab used in industrial buildings is shown in Fig. II.9. The nominal width of such a slab is assumed as 1.5 m.

Slabs of all types are joined by welding together embedded steel parts and concreting the gaps between the slabs (Fig. II.10*a*). The longitudinal side faces of slabs have cavities intended (after the joints have been concreted) to form intermittent keys which enable the slabs to resist shear in the vertical and horizontal directions by

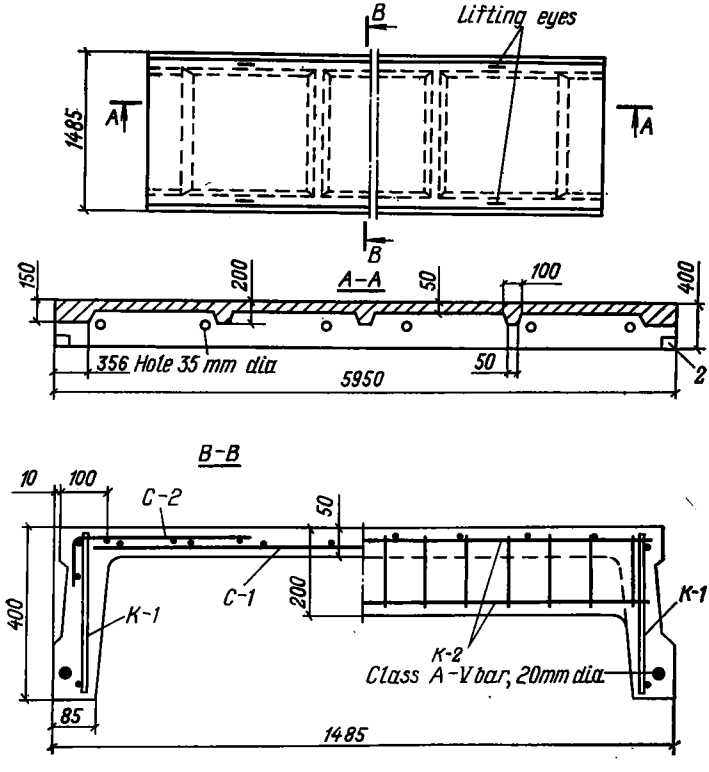

Fig. II.9. Reinforcement of a ribbed floor slab

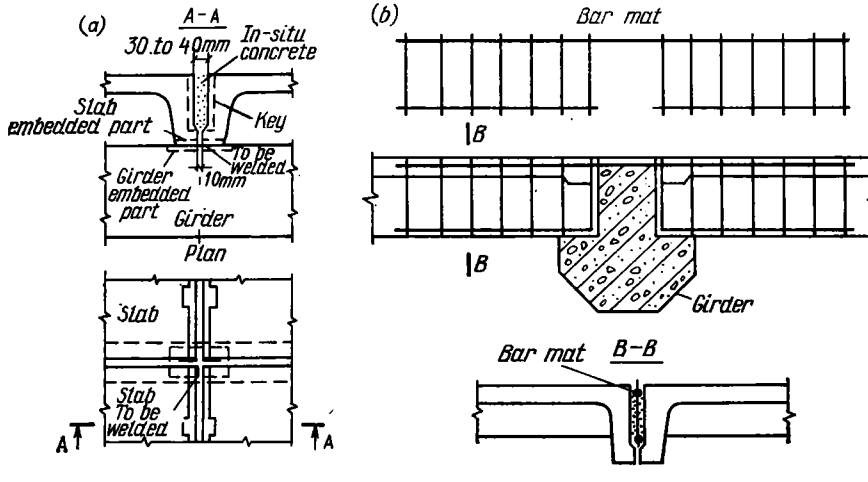

Fig. II.10. Joints between slabs

composite action. Thus connected, a floor acts as a rigid horizontal diaphragm.

If the live load applied to a floor exceeds 10 N/m², it is advisable to turn ribbed slabs into continuous slabs at the time of joint concreting. For this purpose, the joints between the ribbed slabs are reinforced with additional mats placed over the girders (Fig. II.10*b*). In terms of service loads, such slabs are analysed and designed as continuous.

3. Girder Design

Analysis of a Multispan Girder. The girder of a multispan floor is a member of a frame. When it is simply supported and has equal spans, the girder may be regarded as a continuous beam. Then, it is possible to take into account the plastic strain which causes the distribution and equalization of bending moments between the individual sections.

Analysis of Statically Indeterminate Reinforced Concrete Structures with Allowance for the Distribution of Forces. At a certain load, the stresses in the mild tensile steel reach the yield point. The plastic strain (yield) developing in a reinforced concrete structure produces a portion, known as a plastic hinge, where the local strain is considerable.

In a statically determinate structure, say, a simply supported beam (Fig. II.11*a*), the plastic hinge permits the beam parts to deflect, and this results in a significant sag. This reduces the depth of the compression zone, the zone is stressed up to $\sigma_c = R_{pr}$, and the structure fails.

A statically indeterminate structure behaves differently (Fig. II.11*b*). Here, the deflection of the beam parts due to a plastic hinge, and the accompanying sag and the build-up of stresses in the compression zone are opposed by redundant constraints produced by the fixation at the supports. This brings about Stage II-a of the stress-strain behaviour where $\sigma_s = \sigma_y$ but $\sigma_c < R_{pr}$. So, as the load keeps increasing, the structure will not collapse at the plastic hinge until new plastic hinges are formed and the redundant constraints are cancelled. In a statically indeterminate system, the development of a plastic hinge is equivalent to eliminating a redundant constraint and reducing the degree of static indeterminacy by one. The first plastic hinge appearing in the above fixed beam turns it into a system statically indeterminate to the first degree; it will take three plastic hinges (at the supports and in the span) to destroy its rigidity completely.

In the general case, a system with n redundant constraints loses its rigidity when $n + 1$ plastic hinges are formed.

After a plastic hinge has appeared in a statically indeterminate structure, the increasing load causes the distribution of bending moments among the individual sections. As this takes place, the strain in the plastic hinge increases but the bending moment remains unchanged

$$M = R_s F_s z_c$$

After the plastic hinge has formed, the arm of the internal couple, z_c, rises insignificantly with increasing load and is assumed practically constant (Fig. II.11*c*).

Consider the manner in which the bending moment is distributed, using a fixed beam as an example. As a plastic hinge appears at one

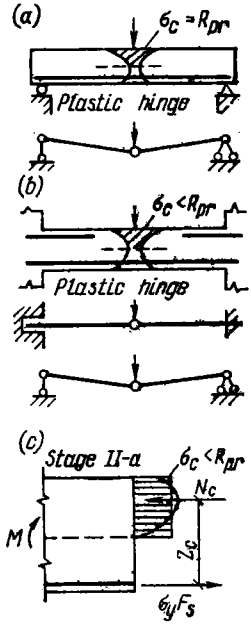

Fig. II.11. Formation of a plastic hinge in reinforced concrete beams

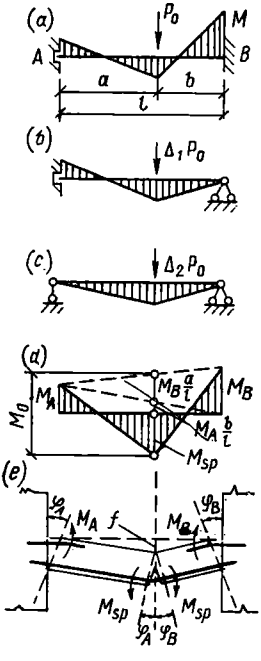

Fig. II.12. Bending moment distribution diagrams for a statically indeterminate beam

of the supports at the load P_0 (Fig. II.12*a*), the beam becomes fixed at one end and simply supported at the other, (Fig. II.12*b*), and remains such as the load keeps increasing.

When the load increases by $\Delta_1 P_0$, a plastic hinge forms at the other support and the beam becomes simply supported (Fig. II.12*c*).

Another increase in the load by $\Delta_2 P_0$ causes the beam to lose its rigidity, that is, to fail.

Let us designate the ultimate design moments in the design sections (at the plastic hinges) as M_A (at support A), M_B (at support B) and M_{sp} (in the span) (Fig. II.12*d*). The ultimate design force is

$$P = P_0 + \Delta_1 P_0 + \Delta_2 P_0 \tag{II.4}$$

At the state of ultimate equilibrium, that is, immediately before failure, the bending moments may be found statically or kinematically.

Static approach. The bending moment in the span is defined as

$$M_{sp} = M_0 - M_A b/l - M_B a/l$$

Hence, the equation for equilibrium can be written as

$$M_{sp} + M_A b/l + M_B a/l = M_0 \tag{II.5}$$

where $M_0 = Pab/l$ is the moment in a statically determinate simply supported beam.

Equation (II.5) suggests that the sum of the span moment and the fractions of the support moments corresponding to this section is equal to the moment in a simply supported beam, M_0. Also, it can be seen that the load-bearing capacity of a statically indeterminate structure does not depend on the relation between the support and span moments, and the sequence of plastic hinge formation. This sequence may be any, provided the condition for equilibrium is satisfied. A change in the moment relation, however, affects the magnitude of load at which the first and the last hinges are formed, and also the crack width at the first plastic hinge.

Kinematic approach. In the state of ultimate equilibrium, the beam is regarded as a system of free bodies connected to each other by plastic hinges (Fig. II.12*e*). If a force, P, produces a sag, f, in the beam, then the angular deflection of the members will be as follows

$$\varphi_A = \operatorname{tg} \varphi_A = f/a \text{ and } \varphi_B = \operatorname{tg} \varphi_B = f/b \tag{II.6}$$

The virtual work of the force, P, in displacing the system through f is

$$W_P = Pf \tag{II.7}$$

The virtual work of the internal forces, that is, the bending moments at the plastic hinges, can be written as

$$W_M = \Sigma \varphi M = (\varphi_A + \varphi_B) M_{sp} + \varphi_A M_A + \varphi_B M_B \tag{II.8}$$

or, recalling the above expressions for φ_A and φ_B

$$W_M = f (M_{sp} l/ab + M_A/a + M_B/b) \tag{II.9}$$

The equation for the virtual work can be written as

$$W_P = W_M$$

or

$$Pf = f\,(M_{sp}l/ab + M_A/a + M_B/b)$$

whence the design ultimate force is

$$P = M_{sp}l/ab + M_A/a + M_B/b \qquad \text{(II.10)}$$

Multiplying both sides of Eq. (II.10) by ab/l gives Eq. (II.5) found statically.

The analysis and design of statically indeterminate reinforced concrete structures on the basis of equalized moments facilitates the assignment of reinforcement. This is especially important for joints between precast members at supports. This also makes it possible to standardize reinforcement and, if necessary, to use the same type of welded-wire fabric or bar mats in cases where the bending moments as determined by the elastic method differ. When a floor is exposed to live loads, the amount of steel found by the equalized-moment method may be reduced by 20 to 30% from the figure found by the elastic method.

The crack width in the first plastic hinges is limited by keeping the equalized moment at not less than 70% of that found by the elastic method.

In order to provide the conditions under which the ultimate-equilibrium method is applicable, that is, the formation of plastic hinges and the development of sufficient local strain in the state of ultimate equilibrium, the following measures should be taken:

(1) the structure should be designed so as to preven its failure due to shear in the compression zone or the crushing of the concrete caused by the principal compressive stresses;

(2) the reinforcement of the sections where plastic hinges are to form should be limited so that the relative depth of the compression zone would be $\xi \leqslant 0.35$;

(3) use should be made of reinforcing steel with a definite yield or welded-wire fabric made of plain wire.

As far as dynamic loads (earthquake shocks, blast shocks, etc.) are concerned, it is good practice to design statically indeterminate reinforced concrete structures with allowance for plastic hinges as well.

In a structure reinforced with a steel having no definite yield point, after one of the bending moments has reached the proof ultimate value, $M_{0.2}$, at the proof yield stress, $\sigma_{0.2}$, the moment will not cease growing, but will increase at a lower rate. In this case, the load-bearing capacity of a structure is decided by the limit of elongation of the reinforcing steel or the ultimate strength of the concrete in the compression zone.

The redistribution of forces in a statically indeterminate reinforced concrete structure occurs already at an earlier loading stage due to changes in the rigidity of the sections at the supports and in the span produced by incipient cracking and crack opening in the tension zone. Although this has no pronounced effect on the distribution of forces in the state of ultimate equilibrium, that is, prior to the formation of plastic hinges, it markedly affects the behaviour of the structure under service loads and, therefore, should be taken into account in design calculations.

For continuous beams, there is a simplified method for taking into consideration the distribution of forces. The bending moments at the supports are calculated as in the elastic method and multiplied by correction factors taking care of the difference in rigidity between the support and span sections. Then, using the above moments at the supports, the bending moments in the span are found. With distributed loads or several concentrated loads, the correction factors for the bending moments at the supports are

—for the middle supports of multispan beams

$$\lambda = 3/(2 + \beta)$$

—for the middle support of a two-span beam

$$\lambda = 1.5/(0.5 + \beta)$$

—for the first intermediate support of a multispan beam, λ is taken as the mean of the above two values. Here, $\beta = B_{sp}/B_{sup}$ is the ratio of the rigidity of cracked sections in the span and at the supports.

In more detail, the analysis of statically indeterminate reinforced concrete structures with allowance for the distribution of forces is covered in appropriate manuals.

The analysis of a continuous girder as an elastic system involves the following distribution of bending moments. The effective span of the girder is taken equal to the centre-to-centre distance between the columns; the first effective span where one end of the girder is supported by a bearing wall is reckoned from the support axis at the wall to the axis of the column. The load imposed on the girder by the slabs may be either uniformly distributed (with hollow-core or solid slabs) or concentrated (with ribbed slabs). If there are more than four concentrated forces acting in the span of the girder, they may be replaced by an equivalent distributed load. For a trial determination of the self-weight of the girder, the dimensions of its cross section are taken as

$$h = 1/10l \text{ to } 1/15l \quad \text{and} \quad b = 0.3h \text{ to } 0.4h$$

If the spans of a multispan beam are equal or differ from one another by not more than 20%, the bending moments and shearing forces are looked up in tables (see Appendix X.11):

for uniformly distributed loads,

$$M = (\alpha g + \beta p)\, l^2; \qquad Q = (\gamma g + \delta p)\, l \tag{II.11}$$

for concentrated loads,

$$M = (\alpha G + \beta P)\, l; \qquad Q = \gamma G + \delta P \tag{II.12}$$

where α and β are the tabular coefficients for M induced by dead and live loads, respectively; γ and δ are the tabular coefficients for Q induced by dead and live loads, respectively.

When the live loads are applied at every other span, the bending moments are maximal in the spans being loaded; if the live loads

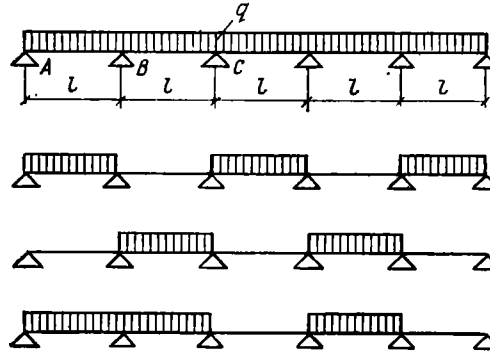

Fig. II.13. Load configuration for a multispan beam

are applied at two adjacent spans and then at a one-span interval, the bending moments are maximal (in absolute values) at the support (Fig. II.13). In a multispan girder, it is advisable to reduce the amount of steel at the supports and simplify the joints. So, for the distribution of bending moments in the girder, the diagram for the moments induced by dead and some unfavourable combinations of live loads must be supplemented by triangular diagrams with ordinates at the supports arbitrary in sign and value (Fig. II.14). Here, the ordinates of the equilized moment diagram should be at least 70% of those calculated by the elastic method. The various loading combinations are then used to construct the resultant diagrams for M and Q.

There exists another simplified method for the design of a multispan girder based on equalized moments. Here, the design equalized moment diagram is the moment diagram for an elastic multispan beam, obtained for maximum bending moments in the span (that is, for the load applied at every other span).

The design section of the girder at the support is that at the face of the column. Here, the bending moment is

$$M_f = M - Qh_{col}/2 \qquad \text{(II.13)}$$

In absolute value, M_f is greater on the span-side, where the girder is subjected only to the dead load; so, the value of Q in Eq. (II.13)

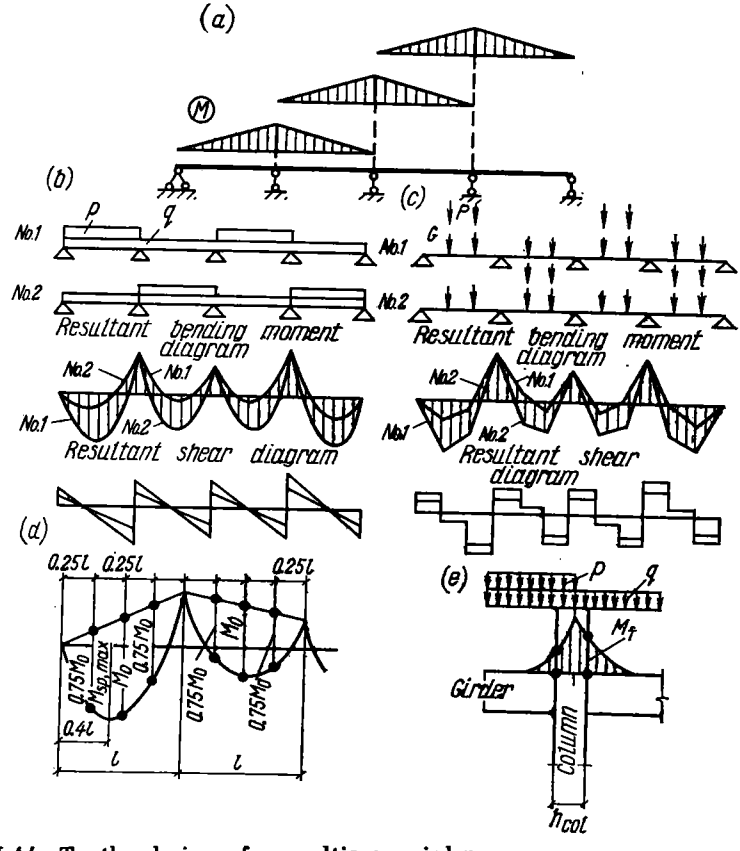

Fig. II.14. To the design of a multispan girder

(a) supplementary bending moment diagrams; (b) to the construction of the bending moment diagram due to uniformly distributed load; (c) the same, due to concentrated load; (d) to the construction of the bending moment diagram due to uniformly distributed load; (e) to determining the design moment in the girder at the column side

should correspond to the loading of this span. The cross-sectional dimensions of the girder are then refined to fit the value of M_f, and, for ξ = (approx.) 0.35, we ge

$$h_0 = 1.8\sqrt{M_f/R_{pr}b}$$

The cross-sectional area of the longitudinal steel in the girder is chosen to fit the values of M in four normal sections, namely in the first and middle spans, at the first intermediate support and at the middle support. In determining the steel area necessary to give the required value of Q, three diagonal sections are of interest: those to the left and right of the first intermediate column and at the last support.

Reinforcement and Layout of a Continuous Girder. Girders can be rectangular, T-section with the flange facing up, and T-section

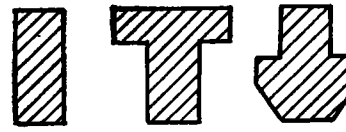

Fig. II.15. Cross sections of prefabricated girders

with the flange facing down (Fig. II.15). When floor slabs are supported by the bottom flange of a T-section girder, the overall depth of the floor is reduced.

The joints in girders are usually designed to lie directly at the side faces of columns. The bending moment in a joint near a support causes tension in the top and compression in the bottom part of the girder (Fig. II.16*a*). At a joint, the girder may bear against a reinforced concrete cantilever protruding from the column or a removable steel table installed for the ease of erection.

At any joints, the tensile force in the top part of the girder is taken up by connection bars welded during erection to steel embedded parts or stick-outs.

During erection, the tensile connection bars may be placed in ducts left in the column (Fig. II.16*b* and *d*), or they may be concreted in the column during its manufacture. In the latter case, they should have stick-outs 120 mm long which are to be welded at the site to the stick-outs in the girder (Fig. II.16*c* and *e*). If a joint carries a considerable bending moment at a support, and there are more than two bars to be welded, the alignment of the joint can be improved by inserting an additional bar welded to the column and girder stick-outs in a reusable copper mould (Fig. II.16*c*).

The compressive force in the bottom part of the girder may be transferred through the welded joints between the steel embedded parts of the girder and those of the column (Fig. II.16*b* and *d*) or through the concrete of the joint (Fig. II.16*c* and *e*).

Joints with cantilevers are capable of resisting considerable bending moments and shearing forces, so they are used where the floors are to carry heavy live loads, mainly in industrial buildings.

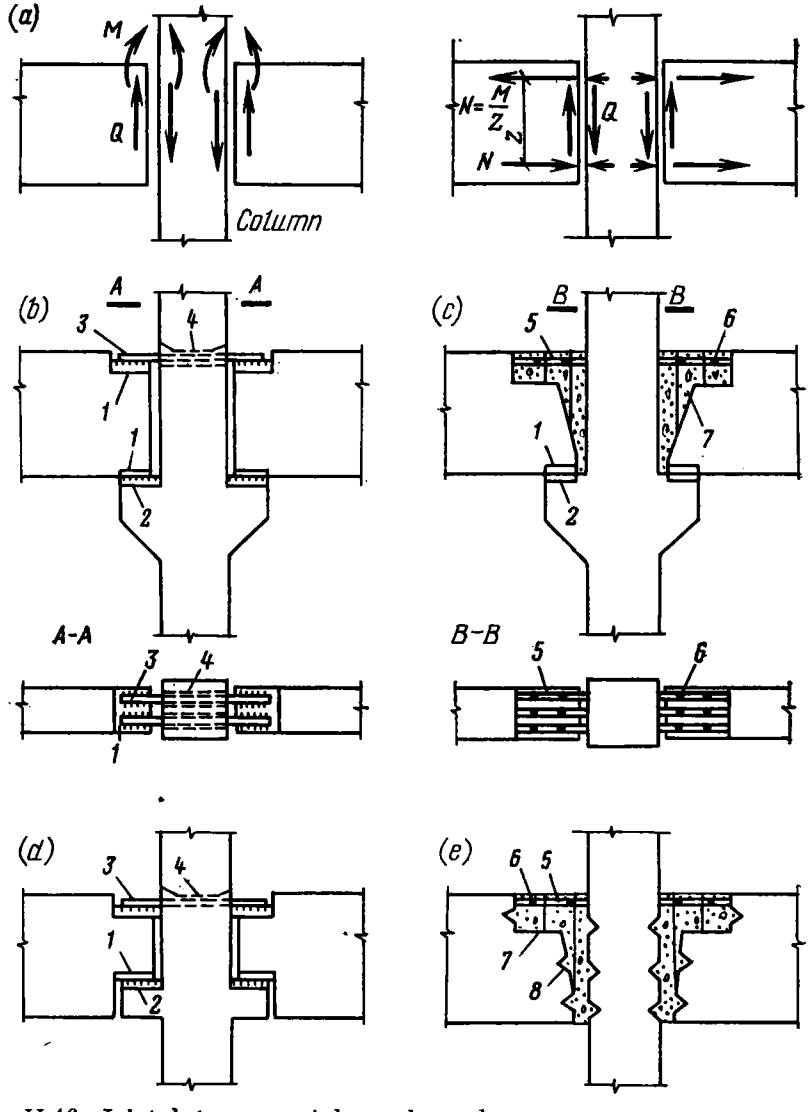

Fig. II.16. Joint between a girder and a column

1—embedded steel part of the girder; *2*—embedded steel part of the column; *3*—connection bars; *4*—ducts in the column; *5*—additional bar; *6*—to be welded in a copper mould; *7*—in-situ concrete; *8*—prismatic cavity for concrete keys

Concealed cantilever joints (with a part of the girder cut back) are difficult to manufacture because they require additional bar mats and embedded parts in the cantilevers, which increases the

amount of steel and makes the manufacture more laborious. Besides, the load-bearing capacity and rigidity of the girder at the support are reduced (Fig. II.16*d*).

In non-cantilever joints (Fig. II.16*e*), as experiments have shown, shearing forces are taken up by the concrete of the joint and the concrete keys formed in the prismatic cavities in the side of the column and the end of the girder. For this type of joint, it is advisable to weld the stick-outs of the compressive steel in the girder to the angles of the supporting bed of the column (see Fig. I.12).

The cross-sectional area of tensile connection bars is given by

$$F_s = M_f/R_s z \tag{II.14}$$

where z is the arm of the internal couple in the joint, equal to the distance between the centroids of the top and bottom embedded parts or welded joints.

In concreted joints, the cross-sectional area of connection bars may be assumed equal to that of the girder reinforcing steel at the support, on the assumption that their strength is the same:

$$F_s = F_{s,sup} R_{s,sup}/R_s \tag{II.15}$$

where $R_{s,sup}$ is the design strength of the steel at the support.

When a connection bar is welded to an embedded part in the girder, the force taken up by one fillet weld is

$$N_w = 0.85 h_w l_w R_w \tag{II.16}$$

Hence, the total length of connection bar welds is

$$\Sigma l_w = 1.3N/0.85 h_w R_w \tag{II.17}$$

Here, the factor 1.3 is included to ensure the reliability of all welded joints for a girder designed on the basis of equalized bending moments. The minimum length of a connection bar weld ranges between $4d$ and $5d$.

The dimensions of the cantilever (Fig. II.17) depend on the force Q exerted by the girder on the support. Here, the girder is supposed to be supported at the free end of the cantilever by a platform whose length is

$$d = Q/b_g R_{pr} \tag{II.18}$$

where b_g is the width of the girder.

The minimum cantilever arm with allowance for the gap, s, between the girder end and the column side ($l_{cant} = d + s$) is taken as $l_{cant} = 200$ to 300 mm. In this case, the distance from the column side to the force, Q, is

$$a = l_{cant} - d/2 \tag{II.19}$$

In short cantilevers ($l_{cant} \leqslant 0.9\ h_0$), the angle, γ, made by the compressive side of the cantilever and the horizontal should not exceed 45°. The depth of the cantilever section should be $h_{cant} = 0.7h_g$ to $0.8\ h_g$ at the column side, and $h_{cant} \geqslant h/2$ at the free end.

The section depth at the support for a short cantilever must be such that

$$Q \leqslant 1.25\ k_3 R_{ten} b h_0^2/a \tag{II.20}$$

where Q should not exceed $2.5\ R_{ten} b h_0$; and k_3 is taken as 1.2 for heavy concretes, and 0.8 for porous-aggregate concretes.

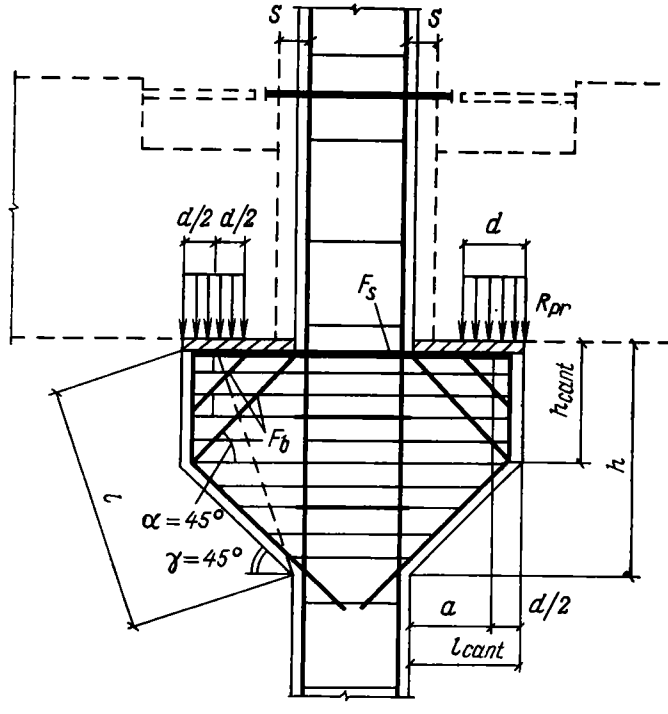

Fig. II.17 Reinforcement of a column cantilever

The cross-sectional area of the longitudinal steel in a cantilever is chosen to match the bending moment at the column side, increased by 25%

$$F = 1\ 25\ M/R_s \eta h_0 \tag{II.21}$$

where

$$M = Qa \tag{II.22}$$

Short cantilevers of depth $h > 2.5\ l_{cant}$ are reinforced with horizontal stirrups and bent bars. The spacing between stirrups should be not more than 150 mm nor more than $h/4$, the diameter of bent bars should not exceed 25 mm or 1/15 of the bent length, whichever is the smaller.

The total area of the bent bars crossing the top half of portion l (see Fig. II.17) should be not less than $0.002bh_0$ nor less than

$$F_b = (Q - Q_c)/R_{s,stir} \sin \alpha \qquad \text{(II.23)}$$

where α is the angle between the bent bars and the horizontal usually taken as 45°, and Q_c is determined according to $c = a + h_0/3$ (see Ch. III, Volume 1).

For concreted joints with the bottom reinforcement of the girder welded to the embedded parts of the cantilever, the design force, Q,

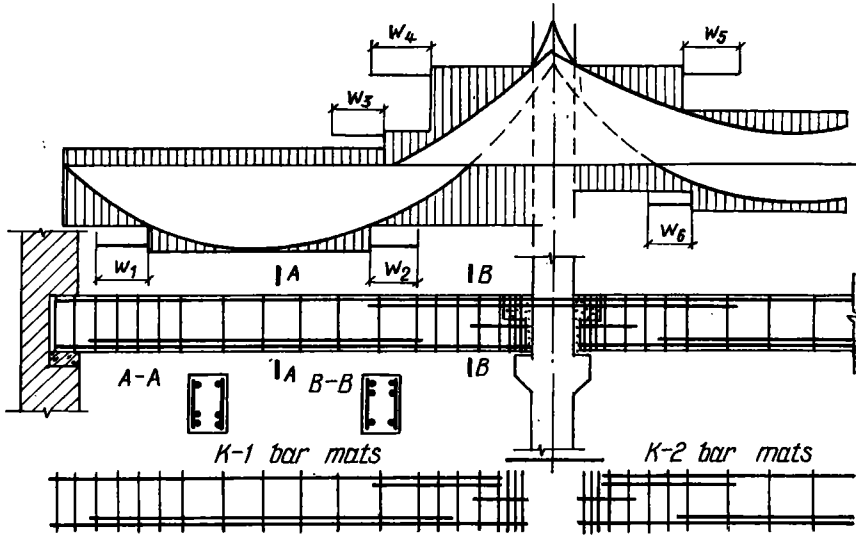

Fig. II.18. Reinforcement of and the steel diagram for a girder

exerted on the cantilever by the girder loaded after the joint has been concreted may be reduced by 25%.

In unconcreted joints, the compressive force, N, is transmitted through welded joints; the length of the welded joints between the bottom embedded parts of the girder and those in the cantilever is given by

$$\Sigma l_w = 1.3\ (N - T)/0.7h_w R_w \qquad \text{(II.24)}$$

where $T = Qf$ is the force of friction; $f =$ (approx.) 0.15 is the coefficient of friction for steel against steel.

As a rule, girders are reinforced with two welded bar mats (Fig. II.18). If a girder is intended to carry heavy loads, a third bar mat may be provided in the mid-span. The cross-sectional area and the number of longitudinal bars are determined by analysing the design sections at the supports and in the span for bending moments. The ordinates of the resultant diagram of bending moments decrease on moving away from these sections, so the steel area may be reduced, too.

In order to save reinforcing steel, some longitudinal bars are cut off in accordance with the resultant diagram of materials. The girder section where a particular tensile bar is no longer required by design is called the theoretical cut-off point of this bar. The cut-off bars should extend at least the distance, w (determined by the equations given in Ch. III, Volume 1) beyond the theoretical cut-off point.

To make sure that a girder contains no excessive steel and also to check the girder cross sections for strength, it is customary to construct what is known as the steel diagram (material diagram). The ordinates of the steel diagram are calculated as the moments due to the internal forces at a given cross section of the girder

$$M_{sec} = R_s F_s z_c \tag{II.25}$$

where F_s is the cross-sectional area of the tensile steel in the section in question and z_c is the arm of the internal couple.

At the theoretical cut-off points, the steel diagram appears as a staircase with vertical risers. Where the steel diagram considerably departs from the bending moment diagram, the strength of the girder is excessive (that is, too much tensile steel is placed there); in contrast, where the stepped steel diagram crosses the bending moment diagram, the strength is insufficient.

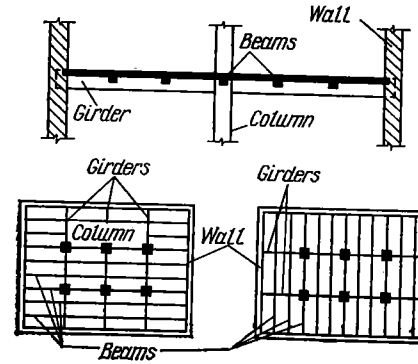

Fig. II.19. Arrangement of girders, beams and slabs in the beam and girder floor

II.3. IN-SITU BEAM AND GIRDER CONSTRUCTION

1. Arrangement of Structural Members

A floor of the beam and girder construction consists essentially of girders, beams and a slab used in the shorter direction (Fig. II.19). All structural members are cast in-situ of M-200 concrete.

In this type of construction, the concrete is eliminated from the tension zone, leaving only ribs or joists carrying the tensile steel.

The topping of the ribs—the slab—is subjected to local bending along the span equal to the distance between the beams.

The beams are carried by, and monolithically connected with, the girders which, in turn, rest on columns and exterior bearing walls.

The girders may be placed along or across a building at every 6 or 8 metres. The beams are distributed so that the axis of one of

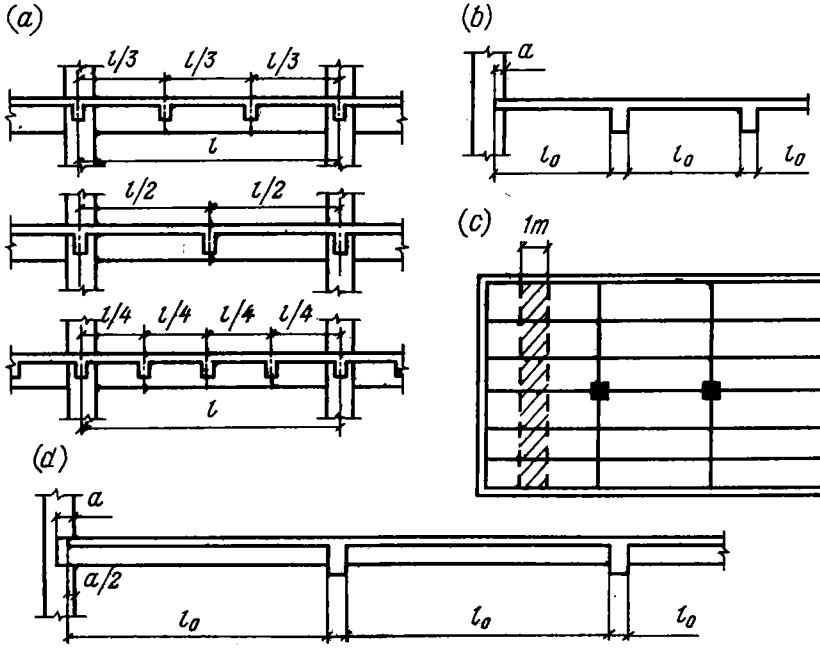

Fig. II.20. Beams, girders and slabs of the beam and girder floor

them passes through the axis of a column (Fig. II.20*a*). The beam span may range between 5 and 7 m, and the slab span between 1.7 and 2.7 m.

From economic considerations, the slab should be as thin as possible. The minimum slab thickness is 6 cm for industrial buildings and 5 cm for civil buildings. If a floor is intended to carry heavy live loads, the slab may be made thicker. For example, at a live load of 10 to 15 kN/m^2 and a slab span of 2.2 to 2.7 m, the slab thickness is taken as 8 to 10 cm (depending on the amount of reinforcement).

As a rule, the beam depth is taken as 1/12 l_b to 1/20 l_b, and that of girders as 1/8 l_g to 1/15 l_g. The width of girders and beams, designated b, may range between 0.4 h and 0.5 h.

2. Design of Slabs, Beams and Girders

The effective span of a slab, l_0, is taken equal to the clear distance between the beams. If one end of a slab is carried by a bearing wall, the effective slab span is assumed equal to the distance from the axis of the support on the wall to the rib side (Fig. II.20*b*). In plan, the design of a slab is carried out for a strip 1 m wide (Fig. II.20*c*).

The effective span of a beam, likewise designated l_0, is taken equal to the clear distance between the girders, or, if one end of a beam

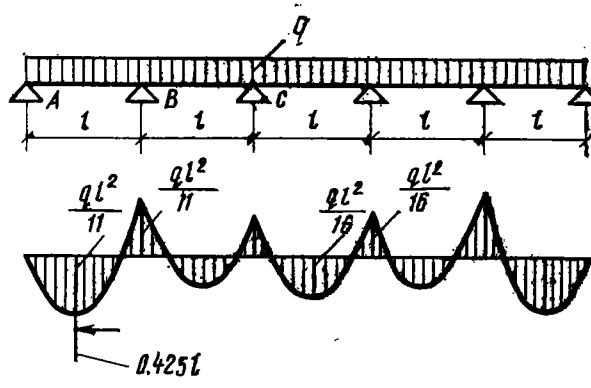

Fig. II.21. To the design of a multispan slab and beams

rests on a bearing wall, to the distance between the axis of the support on the wall to the girder side (Fig. II.20*d*).

Bending moments in multispan slabs and beams having equal spans or spans differing by not more than 20% from one another are determined with allowance for the distribution of moments, assuming that the moment in the span is equal to that at the nearest intermediate support. With a uniformly distributed load, the bending moments, M_{sup}, at the middle supports of a multispan beam (Fig. II.21) are equal to each other. Using Eq. (II.5) which describes the equilibrium of forces in the section in the mid-span, we get

$$M_{sp} + M_{sup}/2 + M_{sup}/2 = ql^2/8$$

Hence,

$$M = M_{sp} = M_{sup} = ql^2/16 \qquad \text{(II.26)}$$

In the first span, the maximum bending moment is induced in the section lying within a = (approx.) 0.425 l of the free support. In this case,

$$M_0 = qa\,(l - a)/2 = 0.123ql^2$$

Using Eq. (II.5) and recalling that $M_A = 0$, we obtain

$$M_{sp} = 0.123ql^2 - 0.425M_B$$

If we assume the bending moment at the first intermediate support as

$$M_B = ql^2/14 \tag{II.27}$$

we shall get the bending moment in the first span:

$$M_{sp} = ql^2/11 \tag{II.28}$$

Recalling that $M = M_{sp} = M_B$, we may write

$$M = ql^2/11.6 \tag{II.29}$$

Rounding off the denominator so that the error in the bending moment is less than +5%, the bending moment at the first intermediate support and in the first span will be

$$M = ql^2/11 \tag{II.30}$$

For slabs supported around the periphery by monolithically connected beams and girders, the bending moments in the middle spans

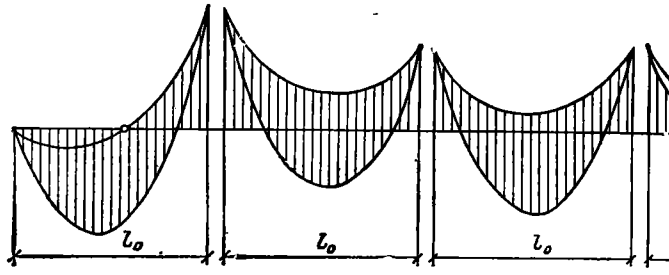

Fig. II.22. Resultant bending moment diagram for a beam

and at the middle supports (determined in the state of ultimate equilibrium without allowance for thrust) are reduced by 20%.

For beams, the resultant bending moment diagram is plotted for two loading systems (Fig. II.22), namely:

(1) the total load $q = g + p$ in the odd spans and the fictitious dead load $g' = g + 0.25\ p$ in the even spans;

(2) the total load $q = g + p$ in the even spans and the fictitious dead load $g' = g + 0.25p$ in the odd spans.

The fictitious load $g' > g$ is included into design calculations in order to determine the actual negative bending moments in the beam spans. The girders provide additional fixation which prevents the beam supports from angular deflections, thereby reducing the effect

that the live load applied to the loaded spans has on the unloaded spans.

When plotting the resultant bending moment diagram, the moments at the supports and in the span are determined by the above formulas, and the minimum bending moments in the spans are found from the parabolas plotted for g' and passing through the peaks of the bending moment ordinates.

The shearing forces in the beam are assumed as follows:

—at the first (last) free support

$$Q_A = 0.4ql_0 \tag{II.31}$$

—on the left of the first intermediate support

$$Q_B^l = 0.6ql_0 \tag{II.32}$$

—on the right of the first intermediate support and at the remaining supports

$$Q_B^r = 0.5ql_0 \tag{II.33}$$

When calculating the necessary cross-sectional area of the members, the first thing to do is to find the cross-sectional area of the beam required to resist the bending moment at the first intermediate support. Since the design is carried out in terms of equalized moments, $\xi = 0.35$. At the support, the beam is subjected to a negative bending moment, the slab is in the tension zone, and the design should be carried out as for a rectangular section with the effective depth taken as

$$h_0 = 1.8\sqrt{M/R_{pr}b}$$

Once the final standard cross-sectional dimensions, b and h, are determined, the next step is to decide on the amount of load-bearing reinforcement for four design normal sections: in the first and the middle spans as for a T-section, and at the first intermediate and the middle supports as for a rectangular section. The amount of steel required to resist the negative bending moment in the middle span is determined as for a rectangular section.

The amount of transverse reinforcement is calculated for three diagonal sections: those to the left and right of the first intermediate support (Q_B^l and Q_B^r) and at the first (or last) free support (Q_A).

The design procedures given for girders in the precast slab and girder construction hold for girders in the in-situ beam and girder construction.

The girder is subjected to the concentrated load from the beam supports (which is determined on the assumption of a continuous beam only in the case of two-span beams). In addition, account should be taken of the self-weight of the girder.

At the points where the beams and girders intersect over a column, there are three layers of reinforcement, namely the top reinforcement

of the slab, beam and girder. So at the support, the concrete cover of the girder is taken equal to $a = 6$ to 9 cm, as a consequence, the effective depth of the girder is $h_0 = h$ — (6 to 9 cm), depending on the number of reinforcement rows.

When determining the cross-sectional dimensions of the girder on the basis of bending moments, it is necessary to keep in mind that it resists the positive moment in the span as a T-beam with a flange width of $b'_f = l/3$, and the negative moment at the support as a rectangular beam of width b.

3. Reinforcement and Arrangement of Slabs, Beams and Girders

According to the bending moment diagram, multispan slabs are reinforced with rolled welded-wire fabric having load-bearing wires

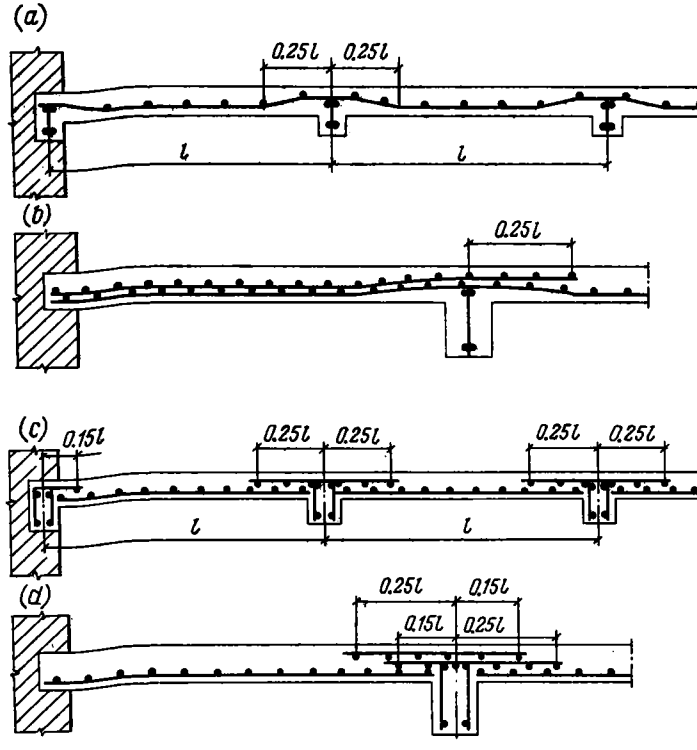

Fig. II 23. Reinforcement of a multispan slab

running in the transverse direction; in the formwork, the fabric is unroll d across the beams (Fig. II.23*a*). The fabric is bent up

within $0.25l$ of the axis of a support (where the bending moments are zero) and placed over the top bar mats of the beams. In the first span, the main fabric is covered by an additional piece which extends $0.25l$ beyond the supports (Fig. II.23*b*). If a stronger load-bearing reinforcement (6 mm or more in diameter) is required, the slabs are separately reinforced in the span and at the supports with rolled welded-wire fabric with transverse load-bearing wires (Fig. II.23*c* and *d*).

In the span, the beams are reinforced with bar mats (as a rule, two bar mats are used) which, prior to installation in the formwork, are

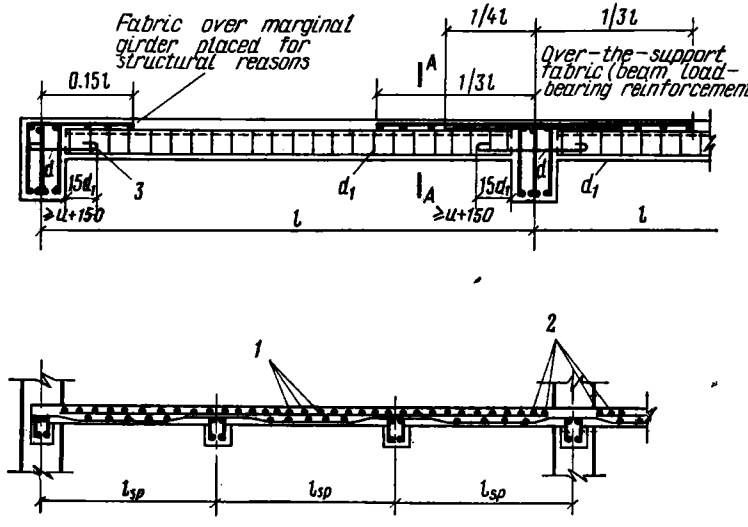

Fig. II.24. Reinforcement of a beam
1—load-bearing bars of the over-the-support fabric; *2*—additional bars at the column; *3*—two bars of $d \geqslant 1/2d_1$ and not less than 10 mm, whichever is the smaller

assembled into reinforcing cages by welding horizontal transverse bars to them. These cages extend as far as the girder faces where they are tied together at the bottom by joint bars (Fig. II.24). At the supports, the beams are reinforced with welded-wire fabric with transverse load-bearing wires. For this purpose, rolled welded-wire fabric is placed over the girder, or flat fabric pieces are distributed over the entire length above the girders. Near the columns, the fabric is cut off to be replaced by additional bars or pieces of fabric having the same cross-sectional area as the load-bearing wires within the column width. At the supports, the effective cross-sectional area of the beam tensile steel is taken equal to the total area

of the load-bearing wires in the over-the-support fabric located between the axes of adjacent beams.

With large-span beams, the tensile steel at the supports may be provided by two partly overlapping fabric pieces. At the first (or last) support, the beams are reinforced with additional welded-wire fabric placed for structural reasons.

The cut-off points for the over-the-support fabric are located according to the negative moment diagram. Where $p/g \leqslant 3$, one of the

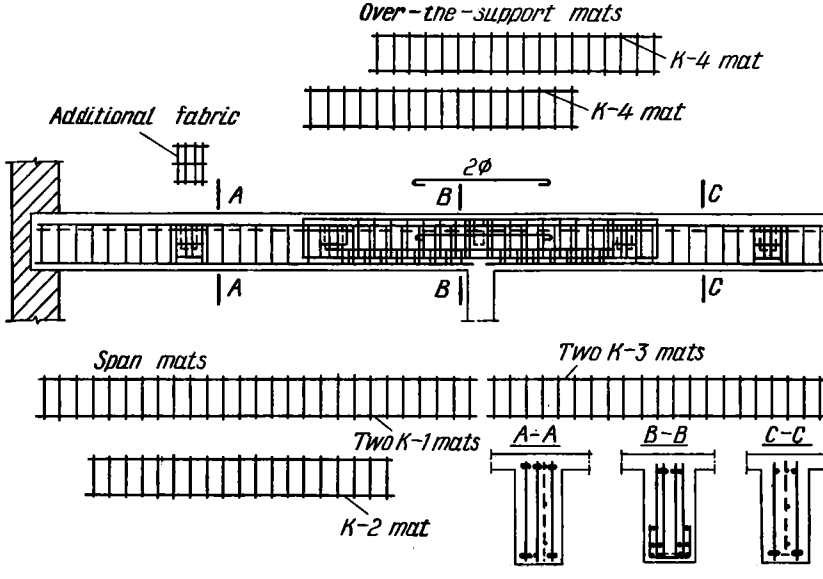

Fig. II.25. Reinforcement of a girder

fabric pieces is cut off within $0.25l$ and the other within $0.33l$ of the support axis. Past the cut-off points in the span, negative moments are taken up by the top reinforcement of the beam.

In the span, the girders are reinforced with two or three bar mats assembled into a reinforcing cage prior to installation in the formwork. Two bar mats extend as far as the column face, and the third (if any) is cut off according to the bending moment diagram. It is also possible to cut off some bars in the span. At the supports, the girder is reinforced with independent cages inserted through the reinforcing cages of the columns (Fig. II.25). The cut-off points for the bar mats and separate bars are located by the reinforcement diagram.

Load is applied to the girder through the compression zone at the beam support in the middle of the girder depth (Fig. II.26). This

local concentrated load is taken up by suspenders formed by the transverse reinforcement of the girder and the additional fabric

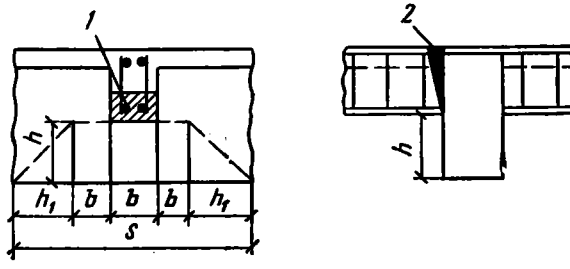

Fig. II.26. Transmission of load to a girder
1—actual area of concentrated load transmission; *2*—crack in the tension zone

pieces at the supports of the beams. The cross-sectional area of the suspender reinforcement is determined by the following equation

$$F_{susp} = P/R_s \tag{II.34}$$

The extent of the region where the transverse steel carrying the concentrated load should be taken into account (see Fig. II.26) is determined as

$$s = 2h_1 + 3b \tag{II.35}$$

II.4. IN-SITU RIBBED-SLAB FLOOR WITH SLABS SPANNING IN TWO DIRECTIONS

1. Arrangement of Structural Members

This type of floor construction consists essentially of slabs subjected to bending in two directions and girders supporting the slabs. All structural members are cast in-situ and monolithically connected.

The slabs may be from 4 to 6 m long in either direction; the ratio of slab sides, l_2/l_1, allowable in practice ranges between 1 : 1 and 1.5 : 1. The girders are designed of the same depth, and placed between columns in two directions (Fig. II.27*a*). Constructions using no intermediate columns and small slabs (less than 2 m) are known as caisson floors (Fig. II.27*b*). According to its dimensions in plan and load, the slab thickness may range between 5 and 14 cm but it should not be less than $0.02l_1$.

Floors with two-way slabs are mainly used for aesthetic reasons, for example, in lobbies, halls, etc. For the same column grid size, they require more concrete and steel than the beam and girder construction.

Experiments have shown that the ultimate load at failure is the same for rectangular and diagonal reinforcement (Fig. II.28*a* and *b*). Rectangular-mesh fabric, however, is easier to manufacture, so it is adopted as a standard slab reinforcement.

The mechanism of two-way slab failure under uniformly distributed load is illustrated in Fig. II.28*c* and *d*. On the bottom slab sur-

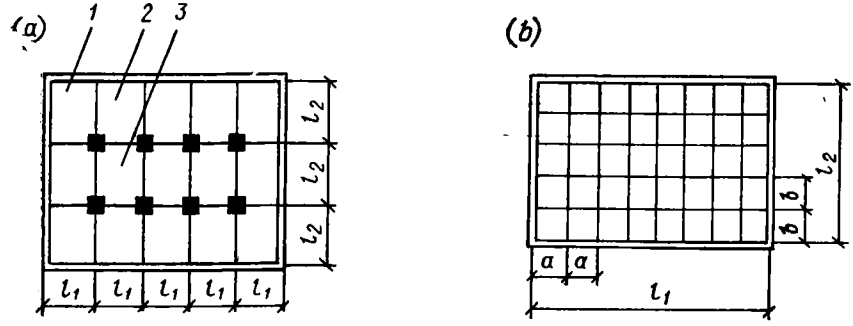

Fig. II.27. Arrangement of structural members in the ribbed-slab floor with the slabs spanning in two directions
1—corner slab; *2*—first slab; *3*—middle slab

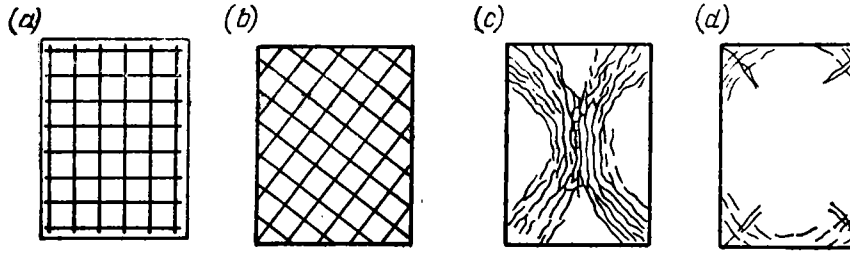

Fig. II.28. Reinforcement and failure of slabs spanning in two directions

face, cracks run along the bisectors. On the top surface of the fixed slabs, cracks are parallel to the sides and rounded at right angles to the slab diagonals in the corners.

For the proper design of their load-bearing capacity and arrangement of reinforcement, it is important to know the character of failure of two-way reinforced concrete slabs.

2. Design and Arrangement of Two-Way Slabs

Like slabs used in the beam and girder construction, two-way slabs (Fig. II.29) are reinforced with welded-wire fabric. If the span of a slab exceeds 2.5 m, the slab is independently reinforced in the

span and at the supports. In the span, use is made of flat or rolled fabric pieces placed in the bottom of the slab and joined across the width by full-strength welds. At the supports, fabric with transverse load-bearing wires is unrolled along the slab length. To save steel, two fabric pieces of different sizes, but with the same steel area are

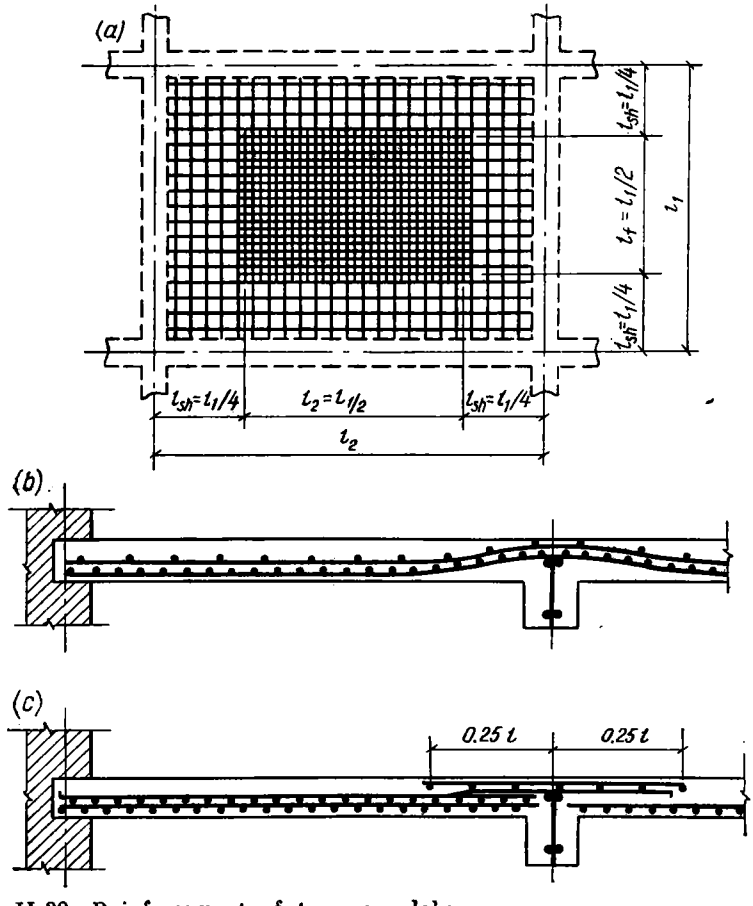

Fig. II.29. Reinforcement of two-way slabs

installed in the bottom of the slab span (Fig. II.29*a*). The shorter fabric piece is cut off within the following distance of the support

$$l_{sh} = 1/4\ l_1$$

where l_1 is the shorter span of the slab. For a simply supported slab

$$l_{sh} = 1/8\ l_1$$

With slabs shorter than 2.5 m in span, use is made of rolled welded-wire fabric. Fabric with longitudinal load-bearing wires up to 5 mm in diameter is unrolled along the shorter side of the two-way slab similarly to the slab used in the beam and girder construction (see Fig. II.23*a*). Here, the distribution steel acting in the longer direction is taken into account in design calculations.

In multispan slabs, the bending moment in the first span is greater than in the middle spans. So in the first span, the main fabric is covered with additional rolled (Fig. II.29*b*) or flat (Fig. II.29*c*) welded-wire fabric.

Two-way slabs are calculated kinematically in the state of ultimate equilibrium. In this state the slab is regarded as a system of

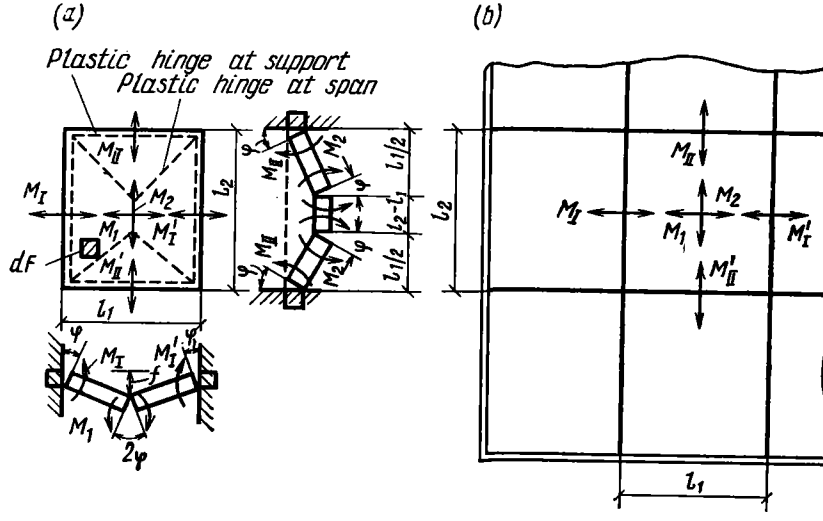

Fig. II.30. To the design of two-way slabs in the state of ultimate equilibrium

flat members connected by plastic hinges which form along the bisectors of the slab corners in the span and along the girders at the supports (Fig. II.30*a*). The bending moments in the slab, M, depend on the area of the steel, F_s, crossed by a plastic hinge. They are determined per metre of slab width by the following formula

$$M = R_s F_s z_c$$

The work done by external and internal forces in moving the system in the state of ultimate equilibrium is described by an equation different for each type of reinforcement, so the respective bending moments induced by a uniformly distributed load are determined likewise differently.

In the general case, the slab resists bending moments in the span, M_1 and M_2, and at the supports, M_{I}, M'_{I}, M_{II} and M'_{II} (Fig. II.30*b*). In the state of ultimate equilibrium, the slab sags under load, and its flat surface becomes a pyramid with triangular and trapezoidal faces. The height of the pyramid is equal to the maximum slab sag, f, and the angular deflection of the members is

$$\varphi = \operatorname{tg} \varphi = 2f/l_1 \tag{II.36}$$

Since the slab sags, the external load moves and does the work equal to the product of the load intensity, q, and the volume of the body produced as a result of the motion

$$W_q = qv = qfl_1 (3l_2 - l_1)/6 \tag{II.37}$$

Here, the work done by the internal forces is determined as the work done by the bending moments in producing angular deflections (Fig. II.30*c*)

$$W_M = \Sigma M\varphi = (2\varphi M_1 + \varphi M_{\mathrm{I}} + \varphi M'_{\mathrm{I}})\, l_2 + (2\varphi M_2 + \varphi M_{\mathrm{II}} + \varphi M'_{\mathrm{II}})\, l_1 \tag{II.38}$$

Because $W_q = W_M$, Eqs. (II.37) and (II.38) may be equated. Then, on replacing the angular deflection φ by its value from Eq. (II.36), we get

$$ql_1^2(3l_2 - l_1)/12 = l_2 (2M_1 + M_{\mathrm{I}} + M'_{\mathrm{I}}) + l_1 (2M_2 + M_{\mathrm{II}} + M'_{\mathrm{II}}) \tag{II.39}$$

If one of the bottom fabric pieces in the slab is cut off within 0.25 l of the support, the cross-sectional area of the steel crossed by a linear plastic hinge in the marginal zone is halved and Eq. (II.39) takes the form

$$ql_1^2 (3l_2 + l_1)/12 = l_2 (2M_1 + M_{\mathrm{I}} + M'_{\mathrm{I}}) + l_1 (3M_2/2 - M_1/2 + M_{\mathrm{II}} + M'_{\mathrm{II}}) \tag{II.40}$$

If the slab is reinforced with rolled fabric having longitudinal load-bearing wires bent up within 0.25 l_1 of the supports, the marginal portions have no bottom steel. Then

$$ql_1^2 (3l_2 - l_1)/12 = l_2 (2M_1 + M_{\mathrm{I}} + M'_{\mathrm{I}}) + l_1 (M_2 - M_1 + M_{\mathrm{II}} + M'_{\mathrm{II}}) \tag{II.41}$$

The right-hand sides of Eqs. (II.39) through (II.41) include the bending moments per unit of slab width, namely two moments in the span, M_1 and M_2, and four moments at the supports, M_{I}, M'_{I}, M_{II} and M'_{II}. Using the relations between the design bending moments given in Table II.2, the problem can be reduced to one unknown.

TABLE II.2. Ratios of Design Bending Moments in Two-Way Slabs

l_2/l_1	M_2/M_1	M_I/M_1 and M'_I/M_1	M_{II}/M_1 and M'_{II}/M_1
1-1.5	0.2-1	1.3-2.5	1.3-2.5
1.5-2	0.15-0.5	1-2	0.2-0.75

If the slab is simply supported at one or more ends, the respective bending moments in Eqs. (II.40) and (II.41) are taken as zero.

The effective spans, l_1 and l_2, are taken equal to the clear distance between the girders or to the distance from the axis of the support on the wall to the face of the girder (in the case of simple supports).

In the state of ultimate equilibrium, a thrust arises in two-way slabs connected monolithically with the girders supporting them, and this increases the load-bearing capacity of the slabs. So when calculating the slab steel reinforcement, the bending moments found by calculation should be reduced by 20% in the middle spans and at the middle supports, by 20% in the first spans and at the first intermediate supports if $l_{ed}/l < 1.5$, and by 10% in the first spans and at the first intermediate supports if $1.5 \leqslant l_{ed}/l \leqslant 2$. Here, l is the effective span of the slab at right angles to the edge of the floor, and l_{ed} is the effective span of the slab parallel to the floor edge.

The amount of reinforcement for slabs is determined as for a rectangular section. The load-bearing reinforcement in the direction of the shorter span is placed below the steel running in the direction of the longer span. Because of this, the effective depth of the slab section is different for the two directions, the difference being equal to the bar diameter.

3. Design and Arrangement of Girders

The load from the slabs is applied to the girders as a load distributed over load areas taking the shape of triangles and trapezoids (Fig. II.31*a*).

To find this load, the slab angles are bisected, and the bisectors are drawn until they intersect (Fig. II.31*b*). The product of the load $g + p$ (per square metre) and the respective load area gives the total load applied to the girder span carrying slabs on either side:

—for the girder of span l_1

$$R_1 = (g + p)\, l_1^2/2 \qquad \text{(II.42)}$$

—for the girder of span l_2

$$R_2 = (g + p)\, l_1\, (2l_2 - l_1)/2 \qquad \text{(II.43)}$$

In a simply supported girder, the respective bending moments induced by such a load are

$$M_1 = (g + p)\, l_1^3/12 \qquad \text{(II.44)}$$

$$M_2 = (g + p)\, l_1\, (3l_2^2 - l_1^2)/24 \qquad \text{(II.45)}$$

Also, it is necessary to take into account the uniformly distributed load, q, due to the self-weight of the girder and part of the live

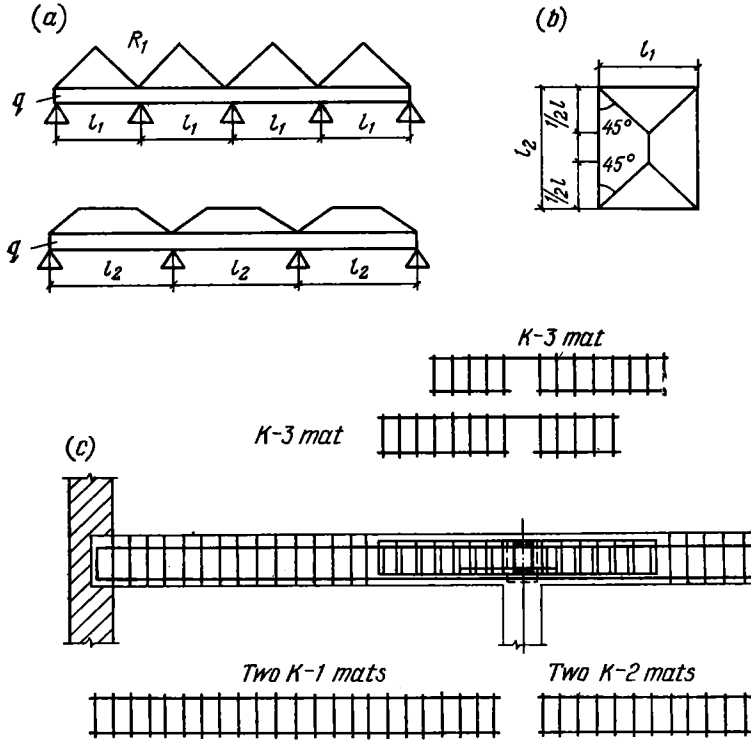

Fig. II.31. Loading systems for and reinforcement of girders in the ribbed-slab floor with two-way slabs

load carried by the floor, the latter being determined over a load strip equal to the girder width b.

The effective spans of the girders are taken equal to the clear space between the columns or to the distance from the axis of the support on the bearing wall (in the case of simple supports) to the face of the first column. For simplicity, the effective span of the girder is assumed as the clear space between the slab ribs (which slightly increases the effective span).

The bending moments with allowance for their distribution are as follows:

—in the first span and at the first intermediate support

$$M = 0.7M_0 + ql^2/11 \tag{II.46}$$

—in the middle spans and at the middle supports

$$M = 0.5M_0 + ql^2/16 \tag{II.47}$$

where M_0 is determined by Eqs. (II.44) and (II.45).

In a three-span girder, the bending moment in the middle span should be taken to be not less than the bending moment in a fixed girder

$$M = 0.4M_0 + ql^2/24 \tag{II.48}$$

Here, the cross-sectional area is determined and the reinforcing steel is arranged as in the girder of the beam and girder construction. At the supports, the girders are reinforced with cages where some bottom bars are removed to provide independent reinforcement (Fig. II.31*c*).

II.5. PRECAST/IN-SITU BEAM AND GIRDER CONSTRUCTION

1. Basic Principles of Precast/In-Situ Structures

Precast/in-situ structures consist of both precast members and in-situ parts. The hardened concrete of the latter combines the structure into an interacting system.

The precast members serve as a frame for in-situ concrete, and they carry the main reinforcing steel, usually prestressed. Additional reinforcement may be installed on this frame during erection. Precast members are made of high-strength concrete, whereas in-situ members are cast of ordinary (M-150 or M-200) concrete.

A distinction of precast/in-situ structures is that strains in the in-situ concrete develop after the precast members have been strained, and no cracking takes place in the in-situ concrete until the prestressed precast members have cracked. Experiments have shown that prestressed precast and in-situ members can interact also when made of porous-aggregate concretes.

It should be remembered that the use of precast/in-situ structures involves two different processes, namely the erection of precast members and the concreting of in-situ parts. So they may be used only where warranted.

2. Precast/In-Situ Floors

For spans up to 9 m long, use may be made of floors containing prestressed reinforced concrete boards serving as a frame and as the tension zone for a reinforced concrete girder (Fig. II.32). These members give support to tray-like reinforced concrete members used

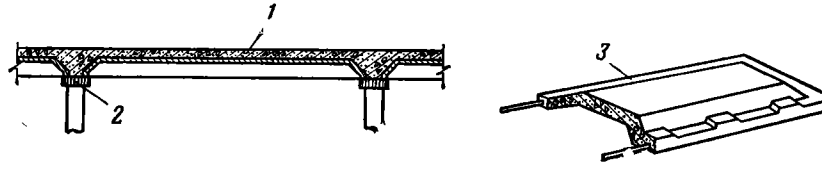

Fig. II.32. Precast in-situ floor
1—in-situ concrete; *2*—prestressed reinforced concrete board; *3*—prefabricated membe

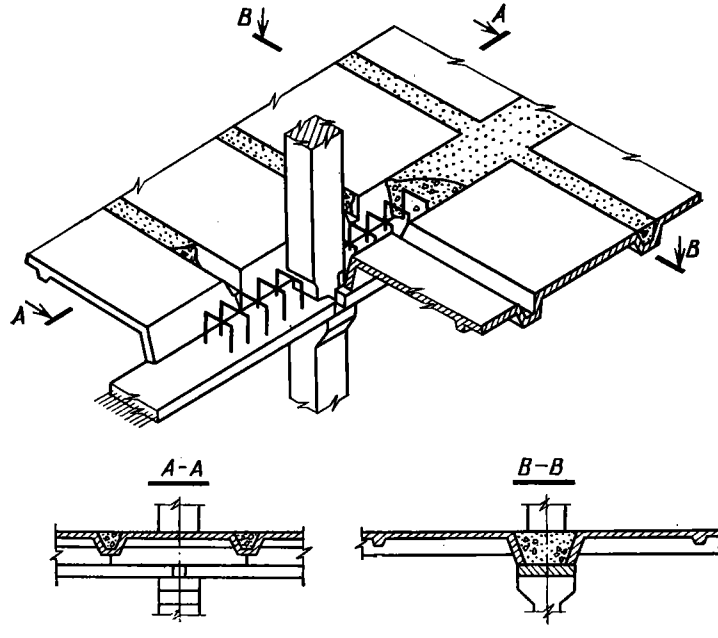

Fig. II.33. Precast/in-situ beam and girder floor with a frame made of reinforced concrete slabs

as formwork for the in-situ concrete. Multispan floors of this type are reinforced with additional steel installed above the supports.

In a floor made up of prestressed boards and tray-like slabs (Fig. II.33), the in-situ concrete accounts for 30% of the total concrete volume.

The in-situ concrete is placed into cavities formed between the side faces of adjacent slabs. The girders and beams are rendered continuous by the reinforcing steel placed over the supports during erection. For a better bond between the precast members and in-situ concrete, stirrups are provided in the reinforced concrete boards serving as the bottom of the girder.

Precast/in-situ beam and girder floors are designed with allowance for the distribution of bending moments, which makes it possible to reduce the amount of steel placed during erection. The validity of moment equalization for multispan precast/in-situ members has been checked experimentally.

II.6. FLAT-SLAB CONSTRUCTION

1. Precast Flat-Slab Floors

A precast flat-slab floor consists essentially of a system of precast flat slabs supported directly by column capitals (Fig. II.34). The main purpose of the capitals is to provide rigid connection between the floor and the columns, reduce the effective span of the slabs, and provide a suitable support for the slabs. The column grid size is usually 6 m square.

An advantage of the flat-slab construction over the beam and girder floor is that it has no protruding ribs and, as a consequence, the electric wiring and other services can be laid directly on the flat ceiling. Owing to the smaller overall depth of the flat-slab floors, the overall height of multi-storey buildings is decreased and the amount of wall materials can be reduced.

Flat-slab floors are mainly used in multi-storey ware-houses, cold-storage houses, meat-packing plants and other industrial buildings intended for heavy live loads. For a live load of 10 kN/m^2 and more, the flat-slab construction is less expensive than the respective beam and girder structure.

The flat-slab floor consists essentially of three groups of members, namely capitals, drop panels, and slabs. The capitals are carried by the flared portions of columns and take up the load from the drop panels which run in two mutually perpendicular directions and act as girders. To provide a multispan system, the drop panels are connected to one another at the top by welding together embedded steel parts. The slabs are supported on four sides by the flanges of the drop panels, and are subjected to bending in two directions as two-way slabs. After the embedded parts have been welded, the slab joints are concreted.

The precast flat-slab floor behaves as a two-way ribbed-slab construction. Here, the drop panels act as wide girders. The floor slabs

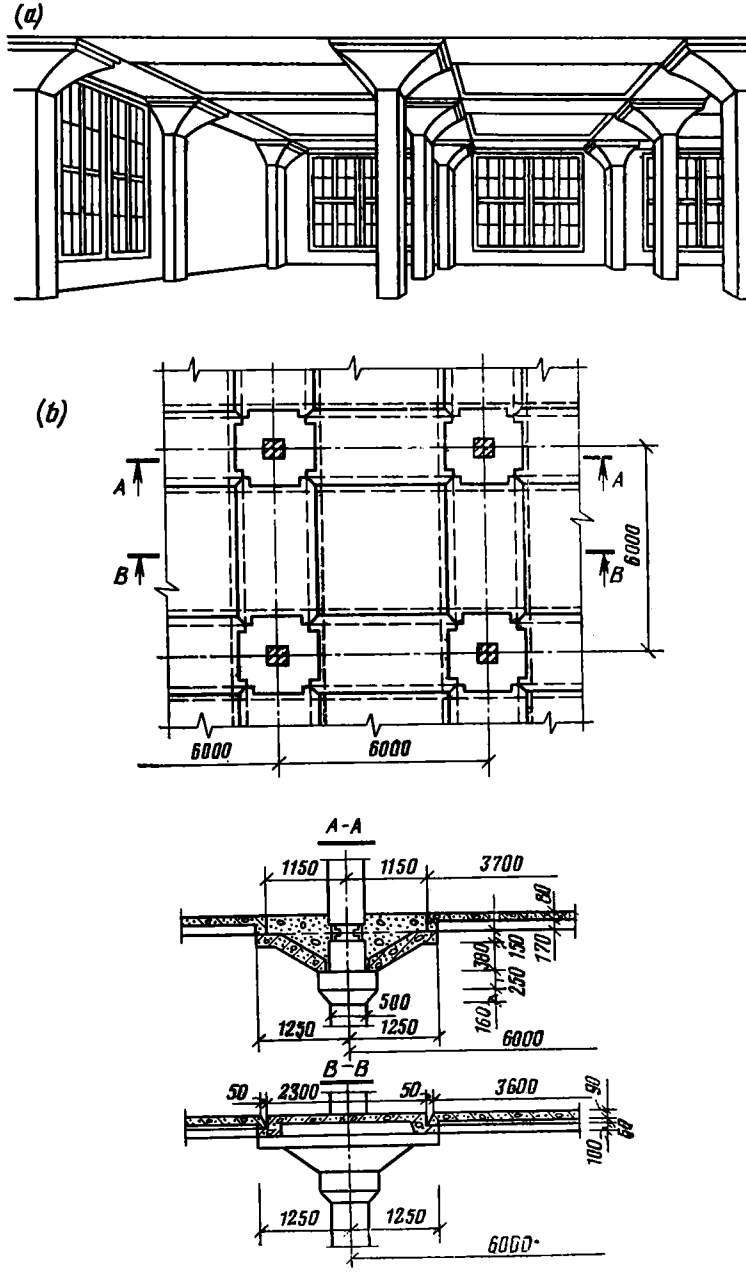

Fig. II.34. Precast flat-slab floor with ribbed slabs
(*a*) general view; (*b*) plan and sections

may be of ribbed (Fig. II.34) or hollow-core (Fig. II.35) construction, and the capitals may be hollow or solid. The columns have separate members for each floor.

Experiments have shown that the drop panels have a low strain in the transverse direction, so the longitudinal load-bearing reinforcement may uniformly be distributed over the cross section.

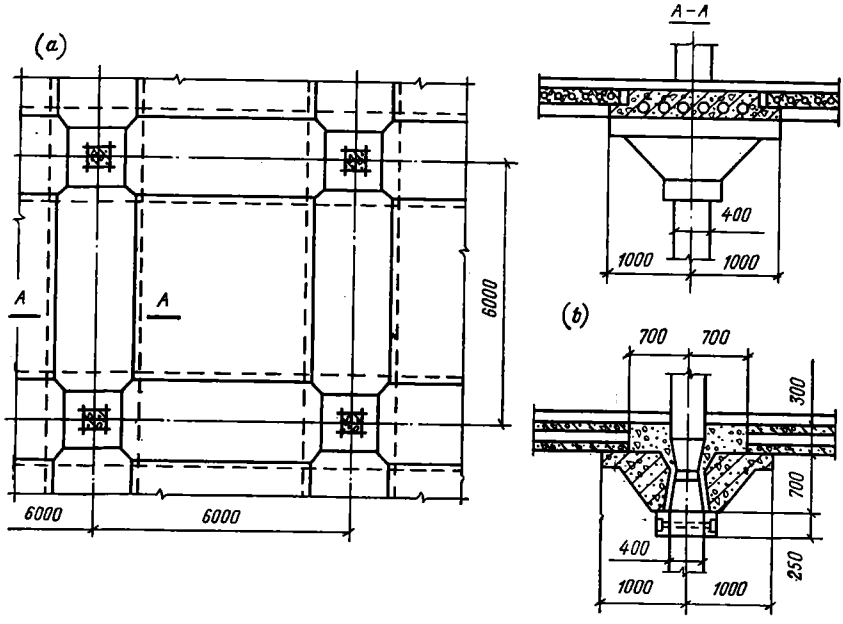

Fig. II.35. Precast flat-slab floor with hollow-core slabs
(a) plan and section; (b) capital components

The bending moment in the span of a square drop panel is determined with allowance for the partial fixation at the edges and elastic pliability of the supporting contour. The bending moments in the span and at the supports of the drop panels are calculated as for a multispan beam taking into account the distribution of moments

$$M_{sup} = M_{sp} = ql^2/16$$

where q is the reduced uniformly distributed load per metre run of the drop panel.

The effective span of drop panels is taken equal to the clear distance between the capital edges multiplied by 1.05.

The capitals are calculated in both directions to carry the load induced by the forces transmitted by the supports and the bending moments of the slabs. The design reinforcement is placed on the

top of the capital, whereas the capital walls are reinforced from structural consideration. In addition, the capitals are designed to carry erection loads as cantilevers.

The columns of the skeleton are designed to resist the longitudinal compressive force, N, due to the load applied to the above-lying floors, and the bending moment, M, induced by the live load applied to one side of the floor.

2. In-Situ Flat-Slab Construction

The in-situ flat-slab floor consists of a solid slab supported directly by columns with capitals (Fig. II.36*a*). The capital is designed

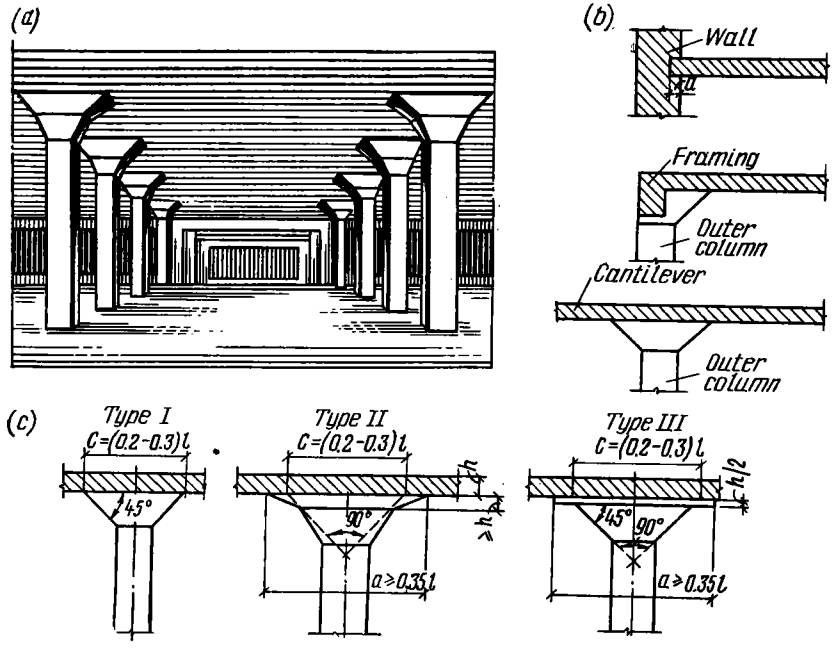

Fig. II.36. In-situ flat-slab floor
(a) general view; (b) slab supported on the outer walls or columns; (c) slab supported on the column capitals

so as to (a) provide adequate rigidity at the joint between the in-situ slab and the column; (b) protect the slab against crushing along the perimeter of the capital; and (c) reduce the effective span of the flat slab and distribute the bending moments across its width more uniformly.

In flat-slab floors, the columns can be arranged in a square grid or in a rectangular, equi-span grid. In a rectangular grid, the ratio of the longer to the shorter span is chosen to be $l_2/l_1 \leqslant 1.5$. The most rational design is one with a column grid size 6 m square. Around the periphery of a building, a flat slab may be supported by bearing walls and framing, or it may extend beyond the capitals of the outer columns (Fig. II.36*b*).

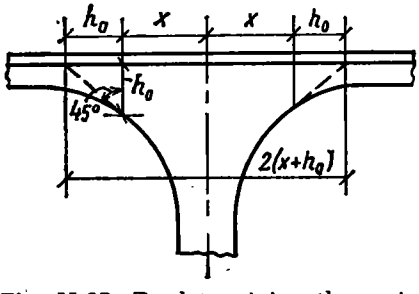

Fig. II.37. To determining the capital dimensions

In industrial buildings, three types of capitals are used (Fig. II.36*c*). Type I is intended for light loads, whereas types II and III are for heavy loads. In the three types, the intercept made by the capital bevels on the bottom face of the slab is taken on the assumption that the support reaction propagates in concrete at 45°. This intercept is taken as $c = 0.2l$ to $0.3l$. The dimensions and shape of the capitals should be chosen so as to prevent the flat slab against crushing along the perimeter of the capital. For this purpose, the capital strength at any distances x and y from the axis of the column (Fig. II.37) should satisfy the following condition

$$Q \leqslant R_{ten} b h_0$$

where

$$Q = q\,[l_1 l_2 - 4\,(x + h_0)\,(y + h_0)] \tag{II.49}$$

$$b = 4\,(x + y + h_0) \tag{II.50}$$

For square capitals, $x = y$.

The thickness of an in-situ flat slab should be sufficient to provide the requisite rigidity, that is, $h = 1/32l_2$ to $1/35l_2$ (where l_2 is the longer span in the case of unequal slab sides); for flat slabs made of porous-aggregate concrete, $h = 1/27l_2$ to $1/30l_2$.

The flat-slab floor is designed by the ultimate equilibrium method. Experience shows that the most unfavourable (design) loading for the flat slab is the load distributed along a strip across its span, and the load distributed over its entire surface area. With such loading systems, linear plastic hinges may be located in two ways.

With the strip load, three linear plastic hinges form in the state of ultimate equilibrium, which connect the members where the slab is likely to break (Fig. II.38*a*). In the span, the plastic hinge forms along the axis of the loaded slab, and cracks open from the bottom. At the supports, the plastic hinges lie within the distance c_1 of the column axes. This distance depends on the shape and size

of the capitals; here, cracks open from the top. In the outer slabs simply supported by the bearing wall along their outer edge, only two linear hinges form, namely in the span and at the support near the first row of intermediate columns.

With the load distributed over the entire floor area, linear hinges in the middle slabs are mutually perpendicular and run parallel to

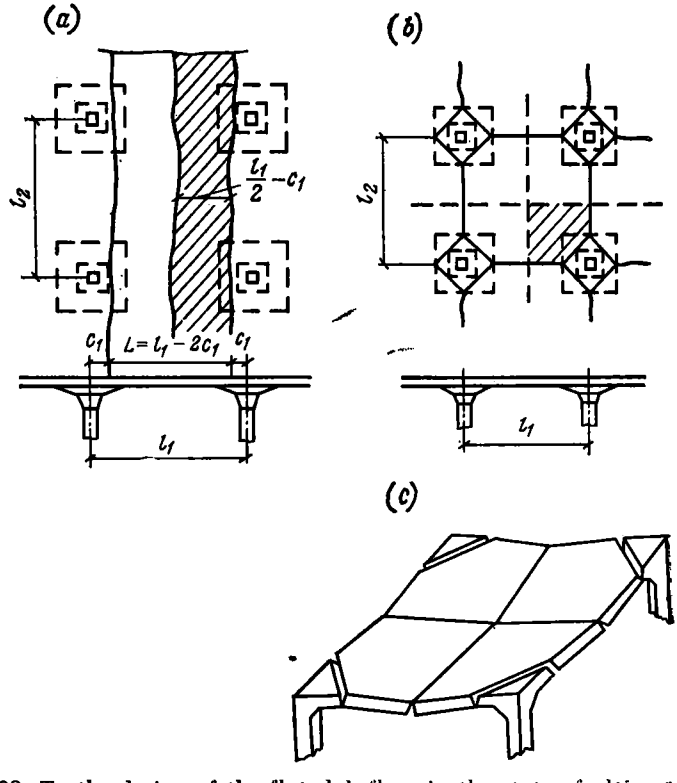

Fig. II.38. To the design of the flat-slab floor in the state of ultimate equilibrium

the column rows. In this case, cracks open from the bottom. Each slab is divided by the plastic hinges into four free bodies which rotate around the supporting linear plastic hinges whose axes are at 45° to the column rows (Fig. II.38*b* and *c*). In the middle slabs, cracks open only above the supporting plastic hinges, whereas along the column lines they propagate through the entire thickness of the slab. In the outer slabs, the position of linear plastic hinges may vary according to the type of support (simple supports on bearing walls, supports on semicapitals, framing beams, etc.).

When a middle slab is loaded so that the strip load breaks it into two free bodies connected by three linear plastic hinges, it is calculated by equating the sum of the bending moments in the span and

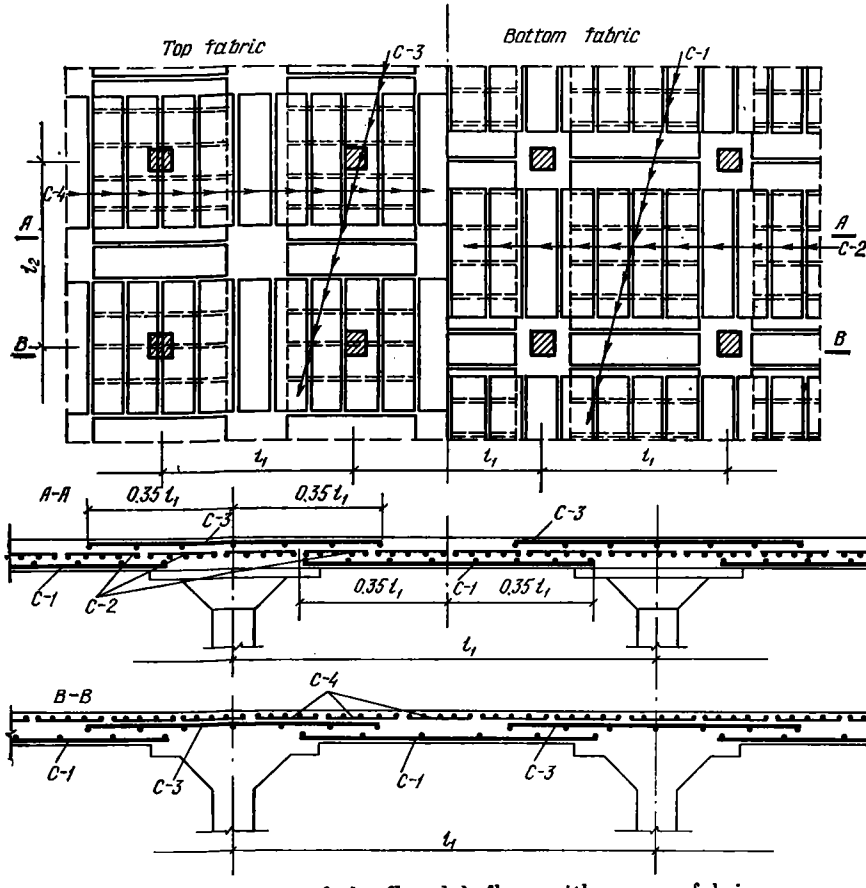

Fig. II.39. Reinforcement of the flat-slab floor with narrow fabric

at the support taken up by the slab in the plastic hinges, $M_{sup} = R_s F_{sup} z_{sup}$ and $M_{sp} = R_s F_{sp} z_{sp}$, to the bending moment in a beam of width l_2 and span $l_1 - 2c_1$, that is

$$q l_2 (l_1 - 2c_1)^2/8 \leqslant R_s (F_{sup} z_{sup} + F_{sp} z_{sp}) \qquad \text{(II.51)}$$

The same applies to the other direction:

$$q l_1 (l_2 - 2c_2)^2/8 \leqslant R_s (F_{sup} z_{sup} + F_{sp} z_{sp}) \qquad \text{(II.52)}$$

Here, $q = g + p$ is the load per square metre of the slab; c_1 and c_2 are the distances from the plastic hinges at the supports to the

axes of the nearest columns in the directions of l_1 and l_2, respectively; F_{sup} is the cross-sectional area of the steel within one slab in the plastic hinge at the support; F_{sp} is the cross-sectional area of the steel within one slab in the plastic hinge in the span; and z_{sup} and z_{sp} are the arms of the internal couples in the plastic hinges at the support and in the span, respectively.

On putting $\theta_{sup} = F_{sup}/F_{s1}$ and $\theta_{sp} = F_{sp}/F_{s1}$ (where $F_{s1} = F_{sup} + F_{sp}$) and substituting them into Eq. (II.51), we get

$$ql_2(l_1 - 2c_1)^2/8 \leqslant R_s F_{s1} z_{sp}(\theta_{sup} z_{sup}/z_{sp} + \theta_{sp}) \qquad \text{(II.53)}$$

For a square slab subjected to the load distributed over its entire surface area and having the same amount of steel in either direction,

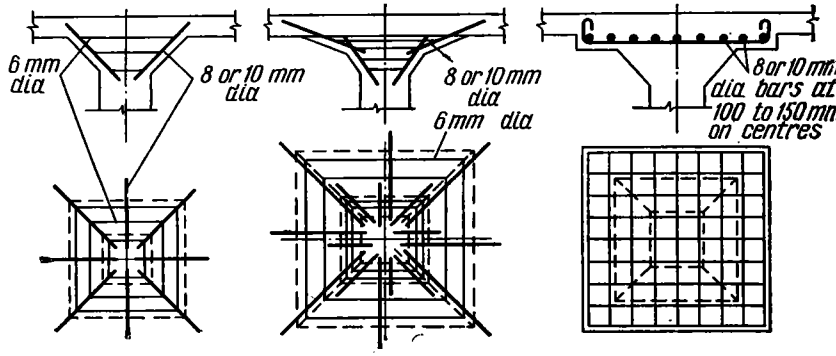

Fig. II.40. Reinforcement of column capitals

$F_s = F_{s1} = F_{s2}$, the condition for equilibrium is as follows

$$ql^3[1 - 2c/l + 4(c/l)^3/3]/8 \leqslant R_s F_s z_{sp}(\theta_{sup} z_{sup}/z_{sp} + \theta_{sp}) \qquad \text{(II.54)}$$

where c is a leg of the right-angled triangle breaking off a quarter of the slab.

When designing the middle slabs, it is recommended to assume $\theta_{sup} = 0.5$ to 0.67, $\theta_{sp} = 0.5$ to 0.33, and c_1/l_1 and c_2/l_2 anywhere between 0.08 and 0.12.

When designing the outer slabs, it should be envisaged that the slab can break up into parts in any one of several ways, according to the manner in which they are supported.

The reinforcement for in-situ flat slabs may be rolled welded-wire fabric or mats. The moments in the span are carried by the fabric (or mats) placed on the underside, and the moments at the supports by the fabric (or mats) placed on the topside.

If narrow welded-wire fabric with longitudinal load-bearing wires is used, in areas where tensile stresses exist in two directions, the reinforcement must be placed in two layers at right angles to each other (Fig. II.39).

Near columns, the top fabric pieces are separated, or holes are provided in the fabric and additional bars are installed so as to make up for the interrupted reinforcement.

The capitals are reinforced for structural reasons, mainly to resist shrinkage and temperature forces (Fig. II.40).

3. Precast/In-Situ Flat-Slab Floors

In this type of construction, the frame for the in-situ concrete is provided by precast slabs spaced over columns and in the spans (Fig. II.41). During erecti n, the capitals may be fastened to the

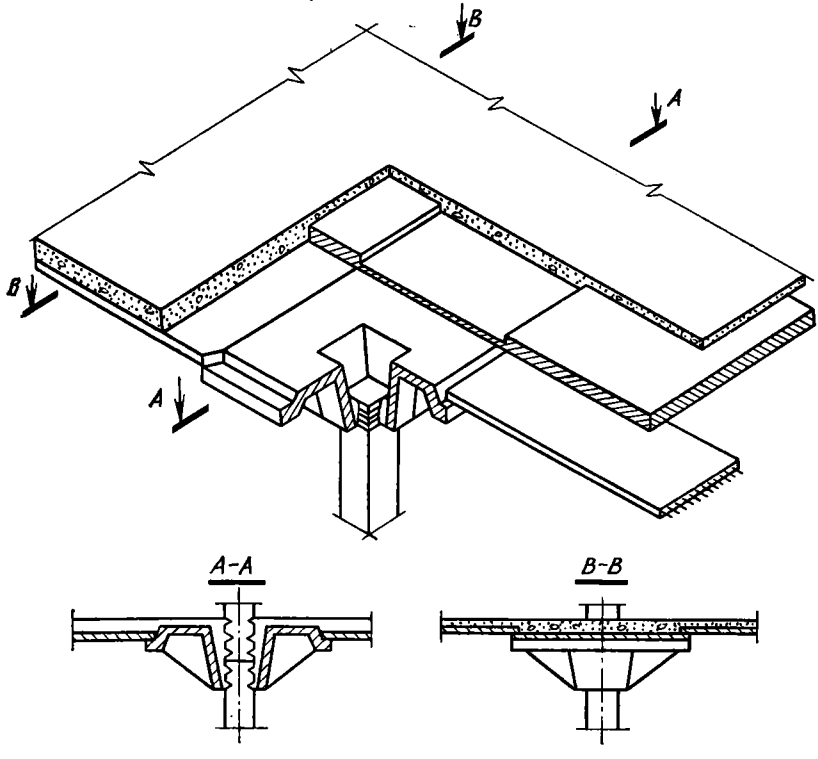

Fig. II.41. Precast/in-situ flat-slab floor

columns with removable stirrups. The bond between the capitals and the columns is then formed after the floor has been concreted and concrete keys have been made on the column surface.

The column capitals give support to the supporting slabs 5 to 6 cm thick placed in two mutually perpendicular directions. Between

these slabs is installed the span slab of the same thickness supported on four sides. The precast slabs are prestressed and reinforced with high-strength steel.

The precast frame of the floor is concreted with two layers of in-situ concrete 4 to 5 cm thick in the span and 9 to 10 cm thick at the supporting slabs. To make the structure continuous, welded-wire fabric is installed where support moments are at work. In this type of floor, the in-situ concrete accounts for 50% of the entire concrete volume.

The total amount of concrete and steel in precast/in-situ or in-situ floors exceeds that in precast flat-slab floors using ribbed or hollow-core slabs.

CHAPTER III

REINFORCED CONCRETE FOUNDATIONS

III.1. GENERAL

Engineering structures, including industrial and civil buildings, widely use reinforced concrete foundations (for vertical loads only, the usual name given is *footing*). There are three basic types of foundations (Fig. III.1), namely an *isolated* or *independent footing* (which supports a single column, pier or other concentrated load*), a *strip* or *continuous footing* (which supports a bearing wall** or a row of three or more columns***), and a *mat* or *raft foundation* (which extends under the entire building area and supports all the wall and column loads from the building). Foundations bearing on soil over a sufficient area so that undue settlement does not occur are called *spread foundations* (these are predominantly covered in this text), those in which the load is distributed into the soil by piles combined at the top by a reinforced concrete raft (or pile cap) are known as *pile foundations.*

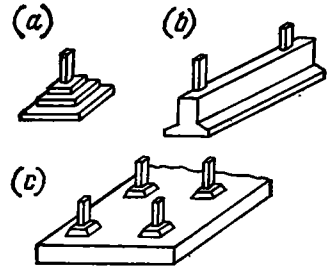

Fig. III.1. Types of reinforced concrete footings
(a) isolated footing; (b) strip footing; (c) mat foundation

Isolated footings are used to support relatively light loads with columns spaced widely apart.

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* One which supports a column is called a *column footing.—Translator's note.*

** A footing that supports a bearing wall is termed a *wall footing.—Translator's note.*

*** A foundation consisting of continuous footings placed at right angles to each other is known as a *two-way continuous foundation.—Translator's note.*

Strip footings are used when isolated column footings are placed close to one another, which is usually the case if the bearing capacity of the soil is low and the structure is designed to carry heavy loads. Strip footings are also advisable for inhomogeneous soils and nonuniformly distributed external loads because they make the otherwise unequal settlement of the foundation material more uniform.

Raft or mat foundations are used if the bearing capacity of a strip footing is inadequate or the deformation of the foundation material exceeds the allowable value. They make the settlement of the foundation material still more even. Such foundations are used on weak and inhomogeneous soils, and also with heavy and nonuniformly distributed loads.

The cost of a foundation accounts for 4 to 6% of the total cost of a building. It may be reduced by a careful foundation design. For large structures, the type of foundation is adopted from several alternative designs by comparing them for cost, amount of materials required, and labour.

According to the manner of manufacture, we may distinguish precast and in-situ foundations.

III.2. COLUMN FOOTINGS

1. Precast Footings

Depending on their size, precast column footing can be made as integral unit or composed of several blocks. The size of integral precast footings (Fig. III.2) is relatively small. They are made of M-150, M-200 or M-300 heavy concretes, and are installed on a layer of compacted sand and gravel 10 cm thick. The footings are reinforced with welded-bar mats placed at the bottom. The minimum concrete cover is 35 mm. For a footing installed directly on soil, the concrete cover should be at least 70 mm.

Precast columns are built in special sockets provided in footings for a depth equal to h_{col} to 1.5 h_{col} (Fig. III.2*c*). The thickness of the baseplate in a socket should be not less than 200 mm. The gap width between the column and the socket walls should be at least 50 mm at the bottom, and 75 mm at the top. During erection, the column is secured in the socket with liners and wedges or a jig, and straightened. Then, the gaps are filled with M-200 fine-aggregate concrete.

Large-size precast footings consist of several units (Fig. III.3). They take more materials than integral footings. When these footings are designed to carry considerable moments and thrusts, their component blocks should be connected to one another by welded dowels, anchors, embedded parts, etc.

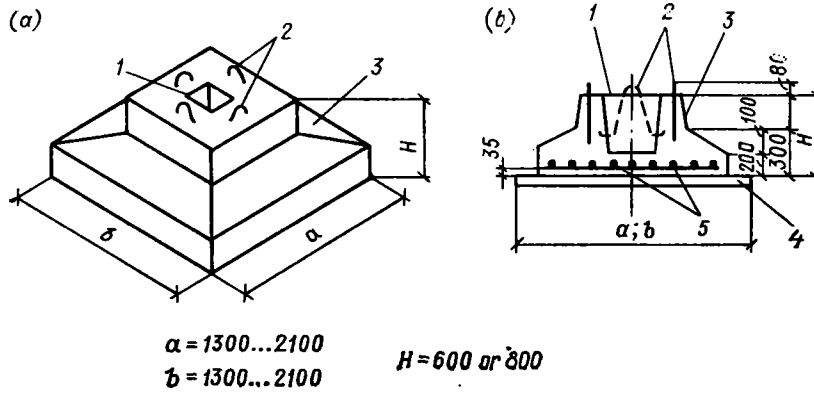
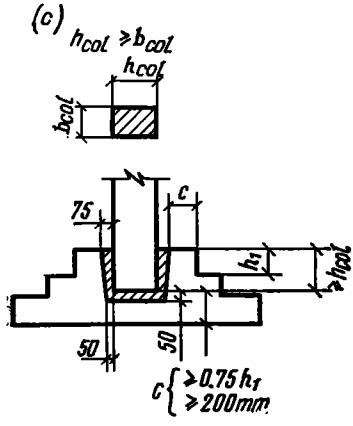


Fig. III.2. Integral precast column footings

(*a*) general view; (*b*) cross section; (*c*) connection between precast column and footing; *1*—column socket; *2*—lifting eyes; *3*—footing; *4*—compacted sand and gravel; *5*—welded-bar mat



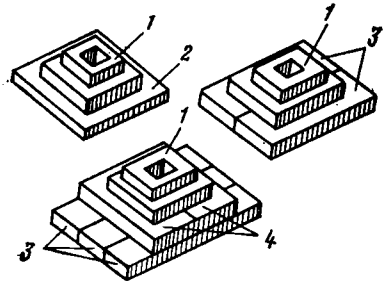


Fig. III.3. Composite footings

1—socket; *2*—integral footing base; *3*—composite footing base; *4*—column base members

2. In-Situ Footings

In-situ isolated footings are used in precast and in-situ buildings and structures of skeleton construction. Standard in-situ footings intended to support precast columns have unified dimensions (which are multiples of 300 mm): from 1.5 × 1.5 m to 6.6 × 7.2 m in plan and 1.5, 1.8, 2.4, 3, 3.6 and 4.2 m in depth (Fig. III.4). These foot-

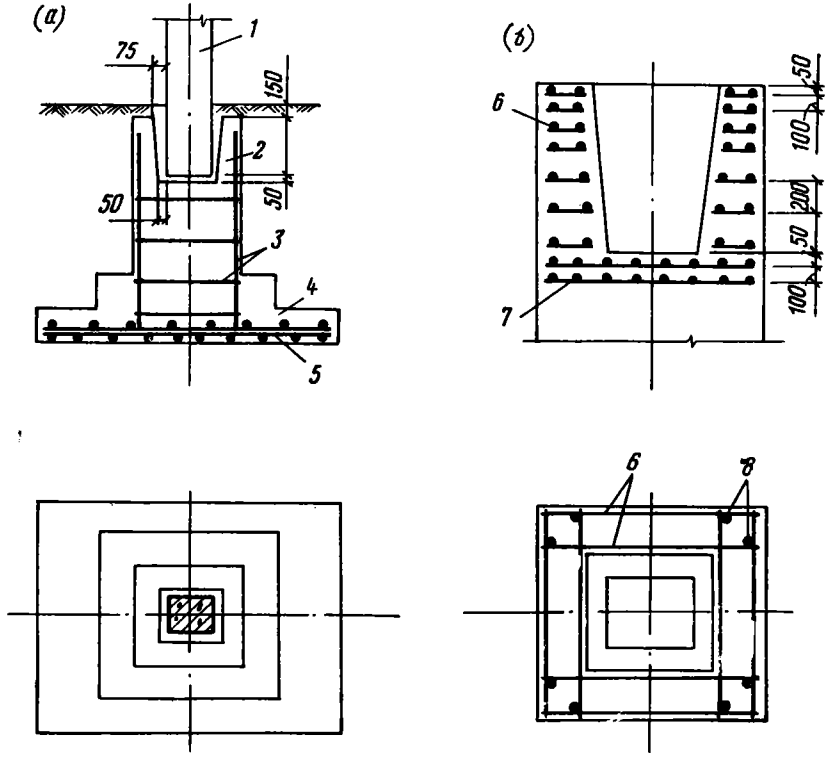


Fig. III.4. In-situ isolated footing intended to carry a precast column
(*a*) general view and distribution of reinforcing steel; (*b*) reinforcement distribution in th socket; *1*—precast column; *2*—socket; *3*—socket reinforcing cage; *4*—footing base; *5*—footing-base bar mats; *6*—socket welded-bar mats; *7*—indirect reinforcement of socket; *8*—vertical bars of socket reinforcing cage

ings are composed of an extended footing base with a length-to-depth ratio of 1/2 reinforced with a double welded-bar mat, and a deep socket reinforced with a cage.

The most common in-situ footings used for in-situ columns are shown in Fig. III.5. They may be stepped or sloped in construction, the former using a formwork which is easier to install. The overall

depth of a footing, H, is taken so as to avoid stirrups and bent up bars. Within a footing, the pressure exerted by the columns is transmitted at 45° from the vertical. This should be kept in mind when assigning the dimensions to the top steps (Fig. III.5*c*).

Like precast footings, in-situ footings are reinforced with welded-bar mats placed only at the bottom. If a footing is more than 3 m

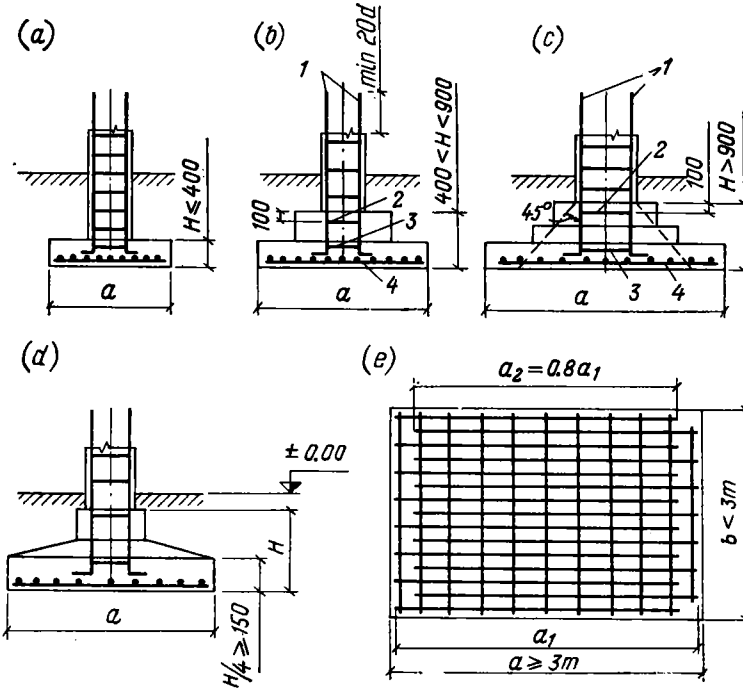


Fig. III.5. In-situ isolated footings intended to carry in-situ columns
(*a*) one-step footing; (*b*) two-step footing; (*c*) three-step footing; (*d*) sloped footing; (*e*) non standard bar mats placed at the bottom; *1*—dowels; *2*—second stirrup of cage; *3*—first stirrup of cage; *4*—welded-bar mats

wide, use may be made of non-standard welded-bar mats in which one half of the bars are made shorter than usual by 1/10 of the footing width to save metal (Fig. III.5*e*).

For connection with an in-situ column, the footing has dowels whose cross-sectional area is equal to the design area of the column reinforcement at the top face of the footing. Within the footing, the dowels are tied up by stirrups into a reinforcing cage which is installed on concrete or brick pads.

The footing dowels should be long enough for splicing to be done as prescribed in Sec. I.3 (see Volume 1) and above the floor level.

The column reinforcement may be lap-spliced with the dowels without welding in compliance with the general requirements for such joints. In columns subjected to axial compression or eccentrical compression with small eccentricities, the reinforcing steel is joined to the dowels at one place; the reinforcement of columns in eccentrical compression with large eccentricities should be spliced to the

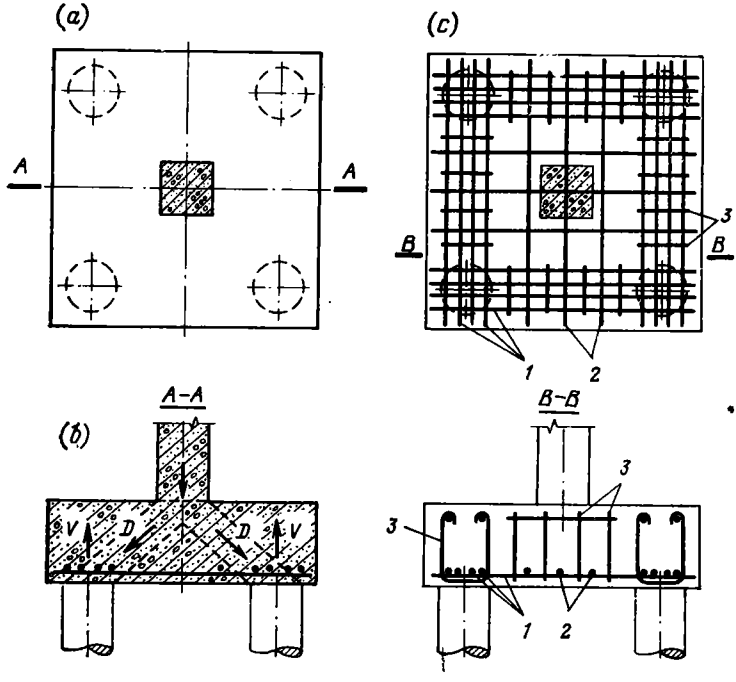


Fig. III.6. Pile cap of an isolated pile foundation
(a) plan; (b) internal forces; (c) distribution of reinforcement; *1*—marginal reinforcing bars taking up thrust; *2*—central zone reinforcement; *3*—marginal bar stirrups

dowels at least at two levels on each side of the column. In the latter case, if each side of the column cross section contains three bars, the middle bar is the first to be spliced.

The best results are achieved when the column reinforcement is joined to the footing dowels by arc welding. The joints must be designed for ease of erection and welding. If the entire cross section contains four bars, only welded joints are allowed.

In pile foundations, the load is transmitted to the piles through pile caps. A pile cap (Fig. III.6) is a rigid slab in which the forces, D, spread from the column base in all directions in plan. In the directions from the centre of the column base to the piles, these

forces are transferred directly to the piles; between the piles (Fig. III.6*b*), they must be balanced by the upward forces, V, which should be applied to the compression zone of the pile cap and thus transmitted to the piles, too. The pile cap reinforcement is designed in accordance with this loading system: the marginal reinforcing bars take up the thrust induced by the forces, D, the stirrups embracing them carry the forces, V, and are anchored in the compression zone of the pile cap, and the tensile steel placed in the neutral zone in the bottom of the pile cap resists the tensile forces arising between the piles (Fig. III.6*c*).

3. Foundation Design

In the general case, the dimensions of the footing base are assigned in accordance with relevant standards and specifications to suit the bearing and strain capacity of the soil. The general design procedures can be found in a course on foundation analysis and design. In Soviet practice, the trial footing-base dimensions for Class I and II buildings and final footing-base dimensions for Class III and IV buildings and structures resting on soils whose compressibility does not increase with depth are determined so that the average pressure at the bottom of the footing does not exceed the nominal design bearing capacity of the soil, R_0. The latter is found on the basis of soil tests and appropriate standards which give the nominal soil bearing capacity, R_0, according to the character and condition of the soil. Under the circumstances, the final dimensions of the footing are taken according to the design soil bearing capacity, R, calculated by standard formulas, including an allowance for R_0, footing-base dimensions, and embedment depth.

Experiments have shown that, in the general case, the pressure beneath the bottom of the footing is nonuniformly distributed and depends on the foundation rigidity, soil properties and average pressure intensity. In design calculations, however, the pressure is supposed to be distributed uniformly, which is of minor importance for isolated footings.

The pressure exerted on the soil by the edge of a footing loaded eccentrically in one direction should not exceed $1.2R$, and that exerted by a corner of a footing subjected to biaxial eccentrical compression should be not more than $1.5R$.

As for other reinforced concrete elements, the footing dimensions and the amount of reinforcement are chosen so that the structure will carry its design service loads.

Axially Loaded Footings. As a first trial, the required base area of an axially loaded footing (Fig. III.7) is determined as

$$F_f = ab = N/(R_0 - \gamma_{av}H_1) \qquad \text{(III.1)}$$

where N is the design force transmitted to the footing, taken with the overload factor $n = 1$ applied; H_1 is the embedment depth; and γ_{av} = (approx.) 20 kN/m³ is the average load due to the unit volume of the footing and the backfill over its steps. Unless otherwise specified, axially loaded footings are made square or nearly square in plan.

The minimum depth of a square footing is chosen so as to prevent it from punching, assuming that the punching can take place over

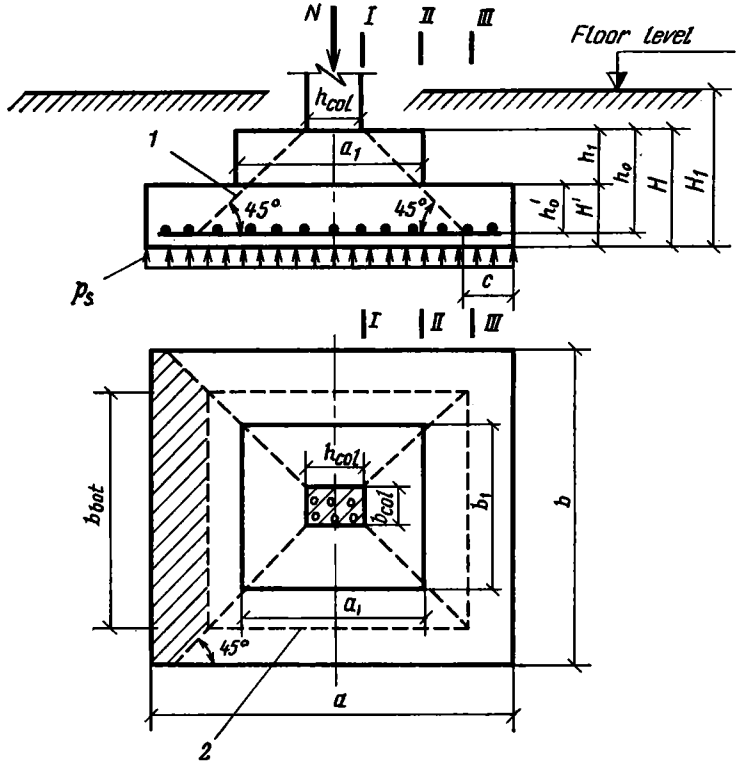


Fig. III.7. To the design of an axially loaded footing
1—punching pyramid; *2*—punching pyramid base

the surface of a pyramid whose side faces begin at the column base and make an angle of 45° with the base. The condition for strength (for heavy concretes) is as follows

$$P \leqslant R_{ten} h_0 b_m \qquad \text{(III.2)}$$

where R_{ten} is the design tensile strength of the concrete; and $b_m = 2\,(h_{col} + b_{col} + 2h_0)$ is the arithmetic mean of the perimeters

of the top and bottom bases of the punching pyramid within the effective depth of the footing, h_0.

The design punching shear is assumed to be equal to the design force, N, (with $n > 1$) acting in the column at the top of the footing, minus the soil pressure within the base of the punching pyramid

$$P = N - F_b p_s \tag{III.3}$$

where

$$p_s = N/F_f \quad \text{and} \quad F_b = (h_{col} + 2h_0)(b_{col} + 2h_0)$$

In Eq. (III.3), the weight of the footing and the backfill over its top is not taken into consideration because it does not contribute to punching. The effective depth of the footing can be calculated using the approximate formula derived from Eqs. (III.2) and (III.3)

$$h_0 = -(h_{col} + b_{col})/4 + 0.5\sqrt{N/(R_{ten} + p_s)} \tag{III.4}$$

The punching strength of rectangular footings should also satisfy the condition (III.2). Here,

$$\left.\begin{aligned} P &= Fp_s \\ b_m &= 0.5\,(b_1 + b_{bot}) \end{aligned}\right\} \tag{III.5}$$

where F is the area of the shaded section of the footing base in Fig. III.7; and b_1 and b_{bot} are the top and the bottom sides of the punching pyramid face.

The overall depth of a footing and the dimensions of the top steps are assigned in compliance with the structural requirements given above.

Under the contact soil pressure, the external parts of the footing behave as cantilevers built in the bulk of the foundation. Accordingly, they are designed as cantilevers, using the following design sections: I-I at the face of the column, II-II at the face of the top step, and III-III at the base side of the punching pyr mid.

The effective depth of the bottom step is taken such that the step will resist the shearing force without transverse reinforcement in the inclined section beginning at III-III. According to Eq. (III.66) in Volume 1, the shearing force carried by the unit width of this section (with $q_{tr} = 0$) is

$$p_s c = 2h_0\sqrt{k_2 R_{ten} p_s} \tag{III.6}$$

where, referring to Fig. III.7,

$$c = 0.5\,(a - h_{col} - 2h_0)$$

In addition, the effective depth of the bottom step should be enough to resist punching as prescribed by the condition (III.2).

The amount of reinforcement to be placed at the bottom of the footing base is determined by the analysis of normal sections I-I

and II-II. The design bending moments in these sections are

$$\left.\begin{aligned} M_{\mathrm{I}} &= 0.125 p_s (a - h_{col})^2 b \\ M_{\mathrm{II}} &= 0.125 p_s (a - a_1)^2 b \end{aligned}\right\} \qquad \text{(III.7)}$$

The cross-sectional area of the load-bearing reinforcement required by the entire width of the footing can be found as

$$F_{s\mathrm{I}} = M_{\mathrm{I}}/0.9 h_0 R_s \text{ and } F_{s\mathrm{II}} = M_{\mathrm{II}}/0.9 h_0' R_s \qquad \text{(III.8)}$$

The percentage of reinforcement in the design section should be not less than the minimum permissible percentage of reinforcement for bending members.

With a rectangular base plate, the amount of reinforcement should be found for either direction.

Example III.1. *Given*: a precast column 35 × 35 cm in cross section; design force: $N_{n=1} = 800$ kN and $N_{n>1} = 900$ kN; concrete: M-150 heavy, in air entire time ($R_{ten} = 0.63$ MPa, $m_c = 1$); reinforcement: class A-III bars, $R_s = 340$ MPa; design bearing capacity of soil (fine sand, medium density, low moisture content): $R_0 = 0.25$ MPa; depth of embedment: $H_1 = 1.75$ m.

To design: An axially loaded isolated footing.

Solution. Find a trial footing area, using Eq. (III.1) converted to newtons and metres, without correcting R_0 for the width and depth of the footing:

$$F_f = N/(R_0 - \gamma_{av} H_1) = (800 \times 10^3)/(0.25 \times 10^3 - 20 \times 1.75 \times 10^3) = 3.64 \text{ m}^2$$

We take the footing base as 1.9 × 1.9 m in plan and its area as $F_f = 3.61$ m².

Find the pressure exerted on the soil and the contact soil pressure, using Eq. (III.3)

$$p_s = N/F_f = 900/(1.9 \times 1.9) = 249 \text{ kN/m}^2 = 24.9 \text{ N/cm}^2$$

Calculate the minimum effective depth of the footing, required to resist the punching shear, by Eq. (III.4) converted to kilonewtons and metres:

$$h_0 = -(0.35+0.35)/4 + 0.5 \sqrt{900/(0.63 \times 10^3 + 249)} = 0.39 \text{ m}$$

whence

$$H_{\min} = h_0 + a_c = 39 + 4 = 43 \text{ cm}$$

The minimum depth of the footing for structural reasons (see Fig. III.2*c*) is

$$H_{\min} = h_{col} + 25 \text{ cm} = 35 + 25 = 60 \text{ cm}$$

We adopt $H = 60$ cm ($h_0 = 56$ cm), and proportion the footing as shown in Fig. III.8.

Carry out the strength analysis of the bottom step whose effective depth in the design section is $h_0' = 30 - 4 = 26$ cm.

(a) The condition (III.6) is satisfied (the relevant calculations are omitted), so no transverse reinforcement is required.

(b) Equation (III.3) gives

$$F_b = (h_{col} + 2h_0)^2 = (0.35 + 2 \times 0.56)^2 = 2.16 \text{ m}^2$$

$$P = N - F_b p_s = 900 - 2.16 \times 249 = 362 \text{ kN}$$

Referring to the condition (III.2),

$$P = 362\,000 \text{ N} < R_{ten} h_0' b_m = 0.63 \times 26 \times 504\,(100) = 825\,000 \text{ N}$$

where $b_m = 4\,(a_1 + h_0') = 4\,(100 + 26) = 504$ cm. As is seen, the condition (III.2) is satisfied.

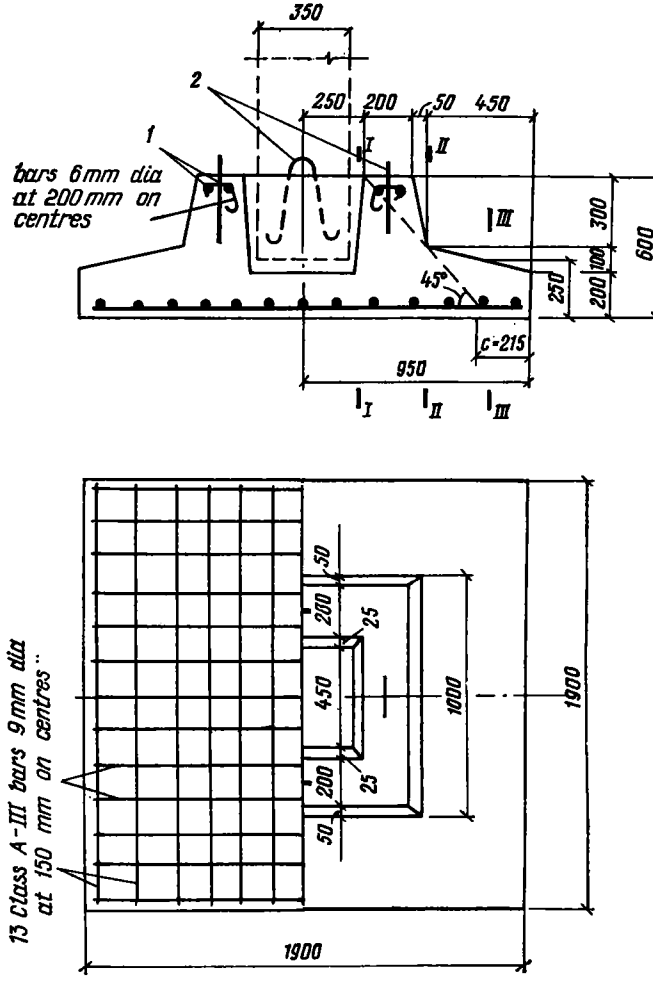


Fig. III.8. To Example III.1

Determine the bending moments in sections I-I and II-II, using Eq. (III.7) converted to newtons and centimetres:

$$M_{\text{I}} = 0.125 p_s (a - h_{col})^2 b = 0.125 \times 24.9\,(190 - 35)^2 190 = 142 \times 10^5 \text{ N cm}$$

$$M_{\text{II}} = 0.125 p_s (a - a_1)^2 b = 0.125 \times 24.9\,(190 - 100)^2 190 = 48 \times 10^5 \text{ N cm}$$

Using Eqs. (III.8), find the steel area

$$F_{sI} = M_I/0.9h_0R_s = [142 \times 10^5\,(0.01)]/(0.9 \times 56 \times 340) = 8.29\ \text{cm}^2$$

$$F_{sII} = M_{II}/0.9h_0'R_s = [48 \times 10^5\,(0.01)]/(0.9 \times 26 \times 340) = 6.05\ \text{cm}^2$$

We adopt a nonstandard welded-bar mat consisting of thirteen class A-III bars 9 mm in diameter spaced 15 cm apart in either direction ($F_s = 8.27\ \text{cm}^2$). The percentage of reinforcement in the design sections is

$$\mu_I = (F_s \times 100)/b_1h_0 = (8.27 \times 100)/(100 \times 56) = 0.15\%$$

$$\mu_{II} = (F_s \times 100)/bh_0' = (8.27 \times 100)/(190 \times 26) = 0.17\%$$

which is greater than $\mu_{min} = 0.1\%$ required by appropriate standards.

If the above calculations show that the trial footing area is insufficient, the footing construction should be corrected as appropriate.

Eccentrically Loaded Footings. It is good practice to make these footings with the base extended in the plane of the applied overturning moment. In the case of uniaxial eccentrical loading, the trial pressure exerted by the edge of the footing (Fig. III.9) should be determined on the assumption that the pressure is linearly transmitted to the soil in the direction of the moment, using the following equations

$$p_{1,2} = (N_f/ab)\,(1 \pm 6e/a)$$

$$\text{with } e = M_f/N_f \leqslant a/6 \qquad \text{(III.9)}$$

$$p_1 = 2N_f/by = 2N_f/3b\,(0.5a - e)$$

$$\text{with } e = M_f/N_f > a/6 \qquad \text{(III.10)}$$

In these equations,

$$N_f = N + \gamma_{av}H_1F_f \text{ and } M_f = M + QH \qquad \text{(III.11)}$$

where N, M and Q are the design normal force, overturning moment and shearing force (with $n = 1$) acting at the top of the footing, respectively; N_f and M_f are the force and the moment at the bottom of the footing, respectively.

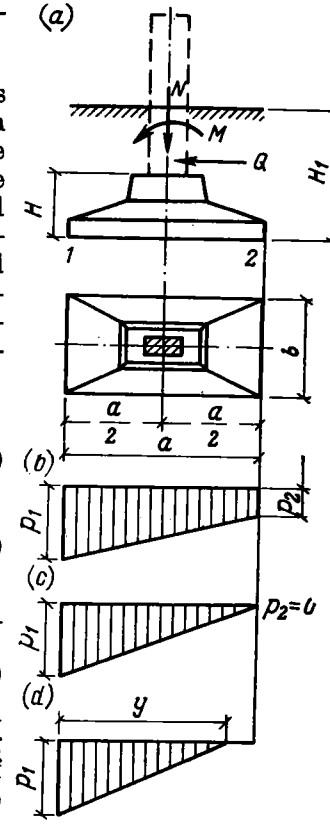


Fig. III.9. To the design of an eccentrically loaded footing

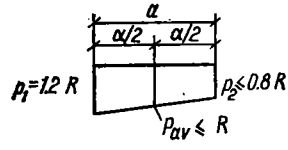
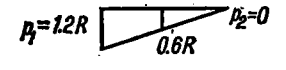
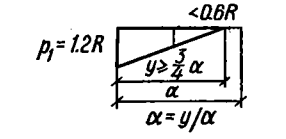
According to relevant standards, the pressure exerted by the edge of the footing should not exceed $1.2R$, and the average pressure should be

$$p_{av} = N/F_f \leqslant R$$

Since the footing is loaded eccentrically, the pressures under the edges in the plane of the overturning moment are unequal. The maximum safe difference between these pressures depends on the superstructure. Experience has shown that for one-storey industrial buildings which support cranes having a lifting power of more than 75 tonnes and for open trestles, p_2 should be not less than $0.25p_1$ (Fig. III.9*b*); for industrial buildings which are intended to support cranes with a lifting power of less than 75 tonnes, the pressure may be distributed as shown in Fig. III.9*c*; and for no-crane industrial buildings, designed to resist additional load combinations, the pressure may be distributed as shown in Fig. III.9*d*, with not more than 1/4 of the footing base left unloaded ($y \geqslant 3/4\ a$).

With allowance for the above requirements, the dimensions of the footing may be calculated by the equations given in Table III.1.

TABLE III.1. Equations for Footing-Base Dimensions of Rectangular Isolated Footings Eccentrically Loaded in One Direction

Scheme No.	Pressure diagram under footing base	Equations
1		$a = e_0\,(2+\sqrt{1.055K-2.5})$ $K = N/(1.2R-\gamma_{av}H_1)\,me_0^2$ $b = ma,\ m<1$
2		$a = 6e_0\,(1-\gamma_{av}H_1/0.6R)$ $b = N/a\,(0.6R-\gamma_{av}H_1)$
3		$a = 5e_0\,(0.6R\alpha-\gamma_{av}H_1)/R\alpha\,(1.5-\alpha)$ $b = N/a\,(0.6R\alpha-\gamma_{av}H_1)$

In the table, m is the specified ratio of the footing-base sides; $\gamma_{av} = 20$ kN/m³ is the average load due to the unit volume of the footing and the backfill over its top; H_1 is the depth of embedment; e_0 is the eccentricity of the force at the bottom of the footing (minus the self-weight of the footing and the backfill over it); all quantities in the equations are expressed in tonnes of force and metres.

An eccentrically loaded footing is designed in the same manner as an axially loaded footing. Here, the pressure on the soil is assumed to be exerted by design forces minus the self-weight of the footing

and the backfill, that is, with the second term in the first of Eqs. (III.11) dropped.

The bending moments acting in the cantilever parts of the footing may be determined with the trapezoidal pressure diagrams replaced by triangular diagrams of the same area.

Example III.2. *Given*: an eccentrically loaded footing; $N_{n=1} = 2\,000$ kN (200 tf); $e_0 = 0.15$ m; $Q = 0$; nominal bearing capacity of soil (fine sand, medium density, low moisture content) $R_0 = 0.25$ MPa (25 tf/m²); $H_1 = 2$ m; $p_2 \geqslant 0.25p_1$.

To find: Trial footing-base dimensions.

Solution. Assume $\gamma_{av} = 20$ kN/m³ (2 tf/m³).

The conditions of the problem correspond to scheme 1 of Table III.1.

Choose $m = 0.8$, then (recalling that the equations of Table III.1 are written for quantities in tf and m)

$$k = N/(1.2R - \gamma_{av}H_1)me_0^2 = 200/[(1.2 \times 25 - 2 \times 2)\,0.8 \times 0.15^2] = 426$$

$$a = e_0(2 + \sqrt{1.055k - 2.5}) = 0.15\,(2 + \sqrt{1.055 \times 426 - 2.5}) = 3.48 \text{ m}$$

$$b = ma = 0.8 \times 3.48 = 2.79 \text{ m}$$

We take the trial dimensions as $a = 3.5$ m and $b = 2.8$ m without correcting R_0 for the width and depth of the footing.

Check to see if the above dimensions are adequate. The design force at the bottom of the footing according to Eq. (III.11) is

$$N_f = N + \gamma_{av}H_1F_f = 200 + 2 \times 2 \times 3.5 \times 2.8 = 239 \text{ tf } (2\,390 \text{ kN})$$

The eccentricity at the bottom is

$$e = Ne_0/N_f = (200 \times 0.15)/239 = 0.126 < a/6 = 350/6 = 0.583 \text{ m}$$

Equation (III.9) as converted to newtons and centimetres gives the edge pressure at the bottom of the footing

$$p_1 = N_f(1 + 6e/a)/ab = 2\,390\,000\,(1 + 6 \times 126/350)/(350 \times 280)$$
$$= 29.7 \text{ N/cm}^2$$

which is $< 1.2R_0 = 1.2 \times 0.25 = 0.3$ MPa;

$$p_2 = N_f(1 - 6e/a)/ab = 2\,390\,000\,(1 - 6 \times 125/350)/(350 \times 280)$$
$$= 19.1 \text{ N/cm}^2$$

which is $> 0.25R_0 = 0.25 \times 0.255 = 0.062$ MPa.

Calculate the average pressure

$$p_{av} = N_f/ab = 2\,390\,000/350 \times 280 = 24.4 \text{ N/cm}^2$$

which is $< R_0 = 0.25$ MPa.

As is seen, the chosen trial dimensions of the footing base satisfy the given conditions. They are further used in the design of the footing and the foundation material. If necessary, the trial dimensions may be corrected.

III.3. STRIP FOOTINGS

1. Wall Footings

Strip footings intended to carry bearing walls are most commonly made of precast members, namely pad-slabs and footing blocks (Fig. III.10*a*). Pad-slabs may be of constant or variable thickness,

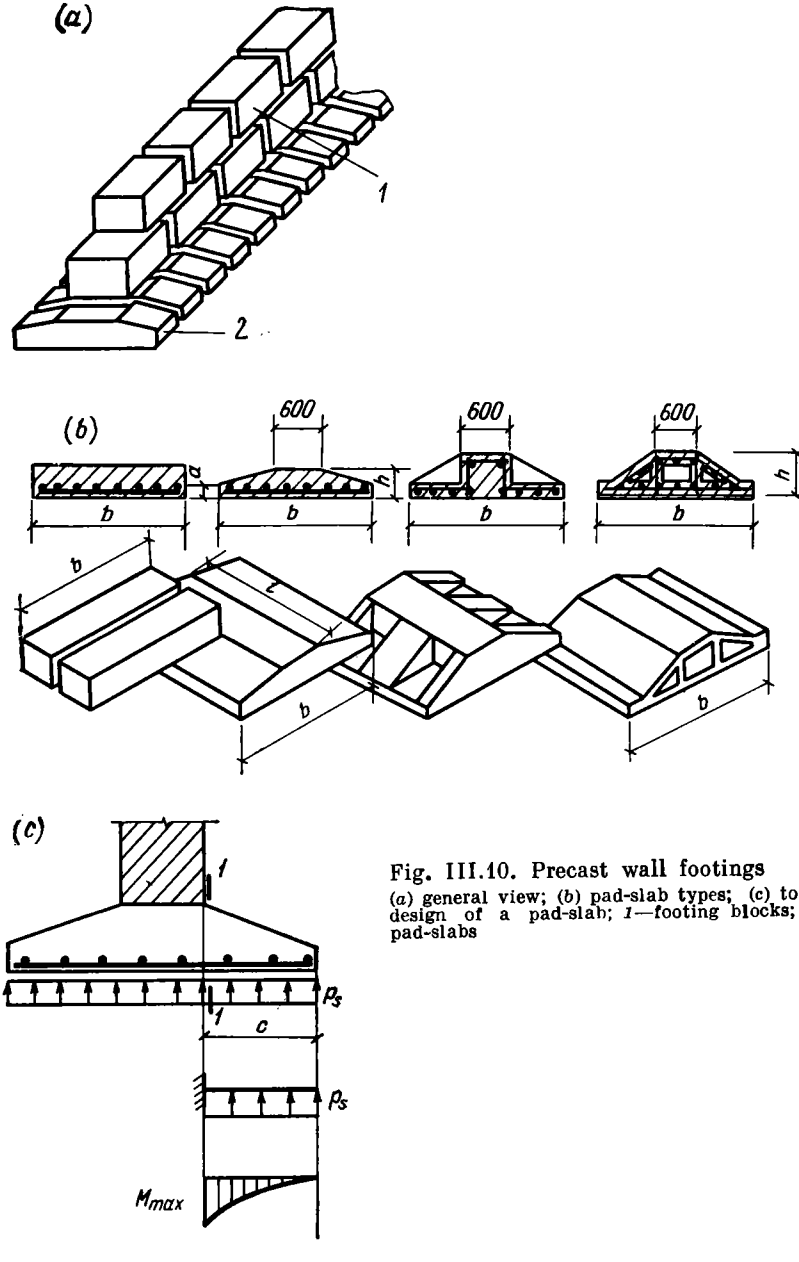


Fig. III.10. Precast wall footings
(*a*) general view; (*b*) pad-slab types; (c) to the design of a pad-slab; *1*—footing blocks; *2*—pad-slabs

they may also be solid, ribbed or hollow-core (Fig. III.10*b*). They may be installed touching one another or with gaps between them. Design needs to be carried out only for those pad-slabs whose projections behave as cantilevers resisting the contact soil pressure p_s (without taking the weight of the pad-slab and the backfill over its top into account). The amount of reinforcing steel is taken to carry the following bending moment

$$M = 0.5 p_s c^2$$

where c is the cantilever arm (Fig. III.10*c*, section I-I).

The thickness h of a continuous pad-slab is taken to resist the shearing force, $Q = p_s c$. It should be enough to carry the design service loads without transverse reinforcement.

2. Strip Footings for Columns

As already noted, strip footings intended to carry rows of columns consist of independent strips which may run either along or at right angles to the column rows (Fig. III.11*a*), or both (Fig. III.11*b*), in which case we speak of two-way strip foundations. Strip footings may be cast in-situ or composed of precast members. They are of T-section with its flange facing down. With cohesive soils, strip footings may sometimes be of T-section with its flange facing up (see alternative cross section I-I in Fig. III.11*a* and *b*). In this case, the scope of excavation and formwork is reduced, but mechanized excavation is more difficult to carry out.

The projections of the T-section flange behave as cantilevers with their ends fixed in the rib. The flange should be thick enough to carry the shearing forces without transverse or bent up bars. With small arms, the flange is taken to be of constant depth; otherwise, its depth is made to increase towards the rib.

In the longitudinal direction, an isolated strip footing behaves in bending as a beam subjected to the concentrated column loads from above and to the distributed contact soil pressure from beneath.

Rib reinforcement is similar to that in multispan beams. The amount of longitudinal load-bearing reinforcement is taken on the basis of the normal-section analysis for bending moments, and that of transverse bars (stirrups) and bent up bars is determined by the inclined-section shear analysis. In order to increase the foundation rigidity, the cross-sectional area is chosen to be adequate with a low percentage of reinforcement, but it should not be less than the minimum safe area for bending members. When designing a footing, it is necessary to keep in mind that it may be nonuniformly loaded in the course of erection, and that the soil settlement may be differential. For this purpose, the ribs are reinforced on either side with

top and bottom longitudinal bars with μ ranging between 0.2 and 0.4%.

Strips are reinforced with welded- or tied-bar mats (Fig. III.11*c* and *d*). The cross section of a rib should contain at least two welded-bar mats for a rib width of $b \leqslant 400$ mm, three mats for b ranging

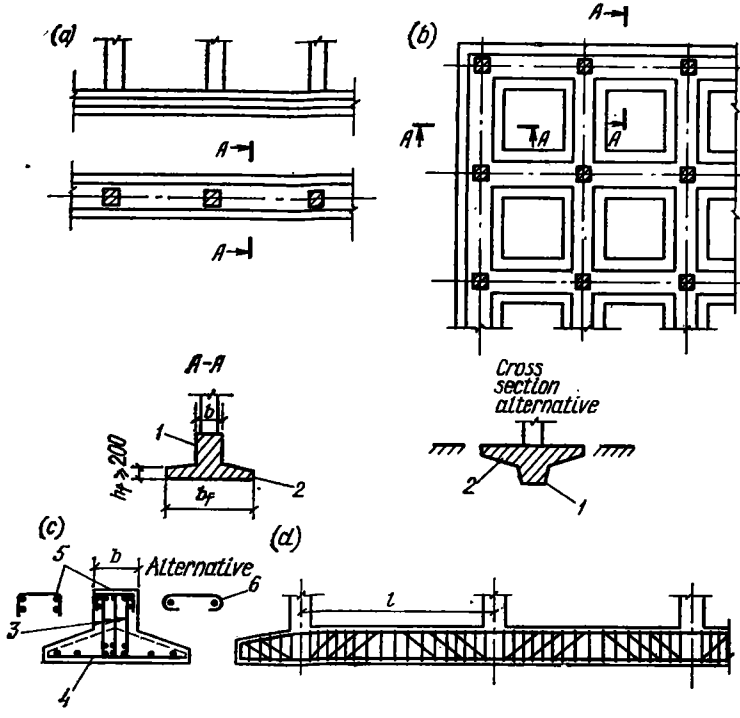


Fig. III.11. In-situ strip footings intended to carry rows of columns
(*a*) independent strips; (*b*) two-way strips; (*c*) distribution of reinforcing steel over the cross section; (*d*) distribution of reinforcing steel in the longitudinal direction; *1*—rib; *2*—flange *3*—welded-bar mats; *4*—bottom bar mats; *5*—top tray-like welded-wire fabric; *6*—top flat welded-wire fabric

between 400 and 800 mm, and four mats for b exceeding 800 mm. It is recommended to reinforce the top longitudinal bars of the welded mats along the entire length with welded-wire fabric (tray-like or flat with transverse wire ends hooked) in the transverse direction and also with transverse bars spaced not wider than 20*d* apart (where d is the diameter of the longitudinal bars) in the longitudinal direction.

If the ribs are reinforced with tied-bar mats, the cross section should contain at least four vertical stirrups for b ranging between 400 and 800 mm and six for b exceeding 800 mm. The closed stirrups

should be at least 8 mm in diameter and spaced not wider than 15d apart.

The longitudinal load-bearing bars may be spaced according to the general rules; in heavy footings, the spacing should be not less

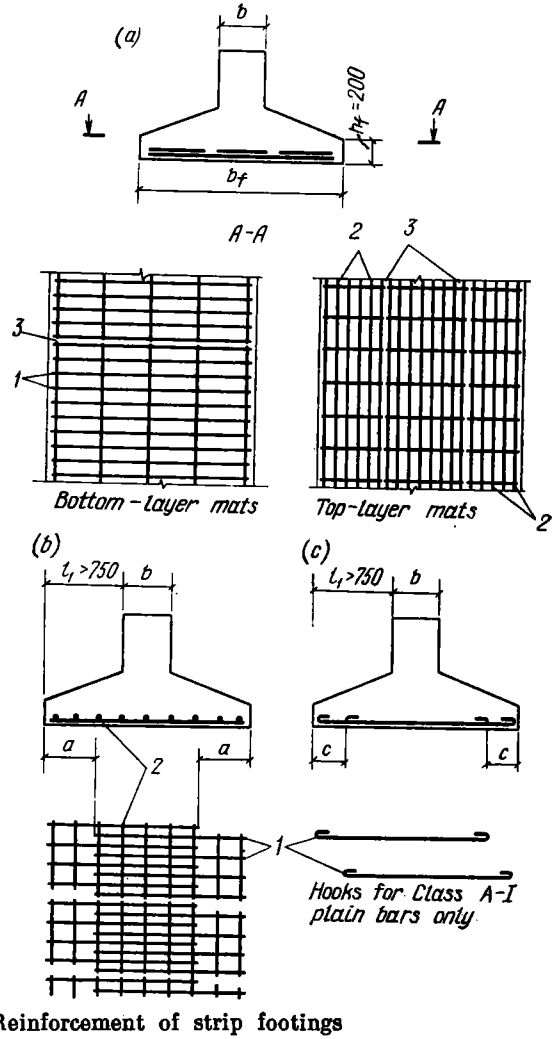


Fig. III.12. Reinforcement of strip footings

(a) standard narrow welded-wire fabric; (b) non-standard welded-wire fabric; (c) tied-wire fabric; *1*—flange load-bearing bars; *2*—strip load-bearing bars; *3*—fabric splices

than 100 mm, so that coarser aggregate could be used in the concrete. The design cross-sectional area of the reinforcing steel includes

that of the longitudinal bars and wires in the mats and fabric. Some bottom longitudinal load-bearing bars (up to 30%) may be distributed over the entire flange width.

Figure III.12 shows flanges reinforced with welded- and tied-bar mats (and separate bars). It is good practice to use wide welded-wire fabric with the load-bearing reinforcement running in two directions. In this case, the longitudinal wires are used as the strip reinforcement, and the transverse wires as the flange reinforcement. Narrow fabric is placed in two layers (Fig. III.12*a*), with the load-bearing fabric located in the bottom layer. The fabric is placed without overlapping, except the top pieces which are lap-spliced without welding in the longitudinal direction as prescribed for splicing welded-wire fabric in the direction of the load-bearing wires. With large flange overhangs (more than 750 mm), one half of the load-bearing reinforcement may be cut off within the distance $c = 0.5l_1 - 20d$ of the face (Fig. III.12 *b* and *c*). If bending moments of the opposite sign are likely to arise in the flange, top reinforcement (shown by the dashed line in Fig. III.11*c*) should be provided.

3. Design of Strip Footings

General. To design a reinforced concrete strip footing is to determine the soil pressure at the bottom of the footing on the assumption that the footing base deforms together with the soil, to calculate the internal forces acting in the footing, to find the cross-sectional area of the strip, and to choose the required amount of steel.

The strain analysis of a foundation material for the maximum safe total settlement, average settlement, differential settlement, tilt and other factors is carried out, and the design pressure, R, exerted on the soil is determined in accordance with relevant standards.

Under load, a strip footing and the associated foundation material act together, thereby forming a single system. Their interaction gives rise to what is known as the contact soil pressure. For design purposes, we distinguish *rigid footings* whose deflections under load are negligible in comparison with the deflections of the foundation material, and *flexible footings* whose deflections are comparable with the deflections of the soil.

Relatively short strip footings of large cross-sectional area carrying closely spaced columns may be classed as absolutely rigid footings.

Long strips supporting columns spaced wider apart are regarded as flexible footings.

It is impossible to express the physical properties of soils and their bedding in terms of simple mathematical relations.

According to relevant standards, the loading system for the foundation material may be configured as an elastic linearly or nonlinearly deformed half-space, a compressed layer of finite depth, a medium which can be characterized in terms of a modulus of subgrade reaction, and so on, depending on the mechanical properties of the soil, the character of soil bedding and the characteristics of the superstructure (say, dimensions and configuration in plan, overall rigidity, and the like). Preference should be given to a linearly deformed half-space with an arbitrarily limited depth of compression, or a linearly deformed layer of finite depth, if above the limited depth of compression in the case of the half-space the material is a soil of low compressibility for which the modulus of elasticity is $E \geqslant 100$ MPa or if the footing base is large (more than 10 m in width or diameter) and the modulus of elasticity of the soil is $E \geqslant \geqslant 10$ MPa, irrespective of the depth of the low-compressibility soil.

As is stated in a course on foundation analysis and design*, the design method involving the concept of the modulus of subgrade reaction is applicable to weak soils or shallow compressed layers over a bedrock.

For strip footings whose base is relatively small as compared with the strip length, the current practice is to treat them as an elastic half-space or, if the soil conditions are as given above, as a foundation involving the modulus of subgrade reaction.

Design of Strip Footing as Beam on Elastic Half-Space. The design procedure discussed below has been proposed by B. N. Zhemochkin and A. P. Sinitsyn**. For the design purpose, the continuous connection between the beam and the foundation is replaced by concentrated rigid rods (Fig. III.13*a*). The forces in the rods are taken equal in magnitude to the resultant pressure uniformly distributed among the footing elements corresponding to each rod. As a rule, the spacings, c, between the rods are assigned to be the same throughout, and the number of elements is taken to range between 9 and 11.

Using the combined method of solving statically indeterminate system, the basic loading configuration may be obtained by separating the beam from the foundation, replacing the rods by appropriate forces, X_0 through X_4, and fixing the beam at the middle (Fig. III.13*b*). Then, the unknowns are the forces, X_0 through X_4, and the settlement of the fixation, y_0. They are found by solving

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* N. A. Tsytovich et al., Foundation Analysis and Design, Higher School Publishers, 1970 (*in Russian*).

** Zhemochkin B. N., Sinitsyn A. P., Practical Methods for the Design of Foundation Beams and Slabs on an Elastic Foundation Material, Gosstroiizdat. Moscow, 1962 (*in Russian*).

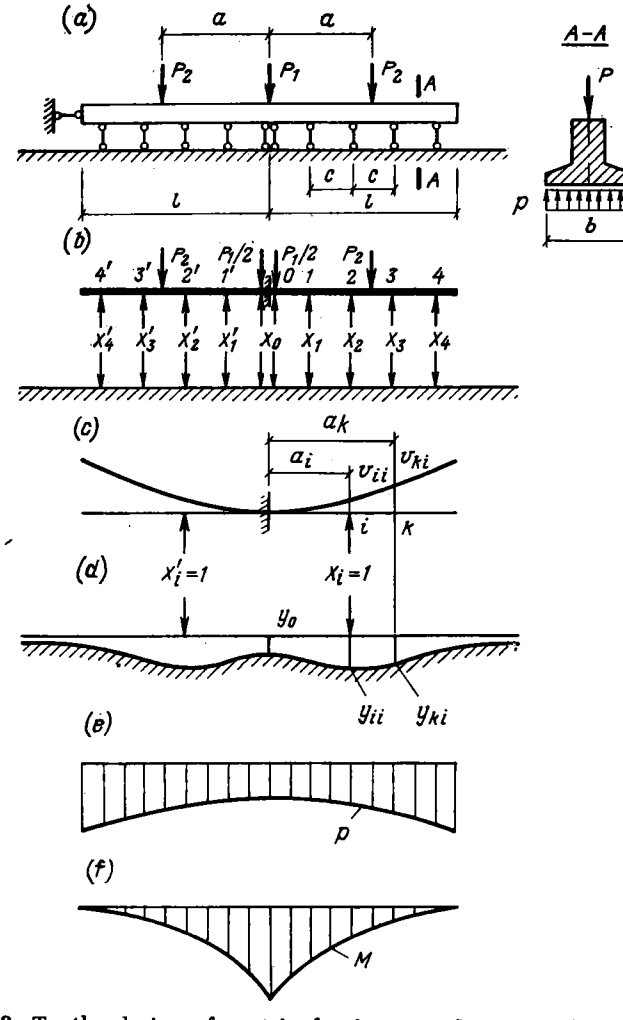


Fig. III.13. To the design of a strip footing as a beam on elastic half space (*a*) loading system; (*b*) basic design system; (*c*) strip axis deflections under $X_i = 1$; (*d*) foundation surface deflections under $X_i = 1$; (*e*) pressure diagram at the strip base; (*f*) bending moment diagram for the strip

simultaneously the following system of equations

$$\left.\begin{aligned} &\delta_{00}X_0 + \delta_{01}X_1 + \delta_{02}X_2 + \ldots + \Delta_{0P} + y_0 = 0\\ &\delta_{10}X_0 + \delta_{11}X_1 + \delta_{12}X_2 + \ldots + \Delta_{1P} + y_0 = 0\\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ &X_0 + X_1 + X_2 + \ldots = \Sigma P \end{aligned}\right\} \qquad \text{(III.12)}$$

The coefficients of the unknown forces represent the deflections in the system along the applied forces and consist of two terms

$$\delta_{kl} = v_{kl} + y_{kl} \tag{III.13}$$

The sag of the beam v_{kl} (Fig. III.13*c*) is calculated using the methods of structural mechanics

$$v_{kl} = \int (M_k M_l / EI)\, dx = a_k^2 (a_l - a_k/3)/2EI = c^3 w_{kl}/6EI \tag{III.14}$$

where

$$w_{kl} = (a_k/c)^2 (3a_i/c - a_k/c)$$

The settlement of the foundation, y_{kl} (Fig. III.13*d*) is determined as

$$y_{kl} = (1 - \mu_0^2) F_{kl}/\pi E_0 c \tag{III.15}$$

where E and μ_0 are the modulus of elasticity and Poisson's ratio of the soil, respectively; F_{kl} is the soil-settlement function at a point k under unit forces, $X_i = 1$. The values of this function calculated by the theory of elasticity are given in Table III.2.

TABLE III.2. Soil-Settlement Function, F_{kl}, for Foundation Material Treated as Elastic Half-Space

x/c	$b/c = 2/3$	$b/c = 1$	$b/c = 2$	$b/c = 3$
0	4.265	3.525	2.406	1.867
1	1.069	1.038	0.929	0.829
2	0.508	0.505	0.49	0.469
3	0.336	0.335	0.33	0.323
4	0.251	0.25	0.249	0.246
5	0.2	0.2	0.199	0.197
6	0.167	0.167	0.166	0.165
7	0.143	0.143	0.143	0.142
8	0.125	0.125	0.125	0.124
9	0.111	0.111	0.111	0.111
10	0.1	0.1	0.1	0.1

Note. Here, x is the distance from the point of load application to a given point, b is the beam width, and c is the distance between the rods.

All deflections in the system of equations (III.12) have the same factor

$$\pi E_0 c/(1 - \mu_0^2)$$

The F_{kl} function should be regarded as the function of two forces, X_i and X_i', acting simultaneously and applied at the respective distances from the point k in question. For example, for F_{32} we

should take into account that $X_2 = 1$ is within the distance c, and $X_2' = 1$ is within the distance $5c$ of point *3*. The ordinates of their influence lines, looked up in Table III.2, should be added together so that for $b/c = 1$, $F_{32} = 1.038 + 0.2 = 1.238$.

Substituting Eqs. (III.14) and (III.15) into Eq. (III.13) gives

$$\delta_{ki} = (F_{ki} + \alpha w_{ki})(1 + \mu_0^2)/\pi E_0 c \qquad \text{(III.16)}$$

where

$$\alpha = \pi E_0 c^4/6EI\,(1 - \mu_0^2) \qquad \text{(III.17)}$$

The sags, w, of the beam are taken into account only on one side, because one side of a beam fixed in the middle has no effect on the other.

With very rigid beams, EI is very large and α is nearly zero; with flexible beams, EI is smaller and α ranges between 0.1 and 1.

The moment of inertia, I, of the beam prior to normal cracking in the concrete is calculated for the entire beam section; after the onset of cracking, it is determined on replacing the concrete in the tension zone at a crack by the tensile steel area.

The deflections caused by the external forces (load) P are determined by the following formula

$$\Delta_{kP} = -\alpha w_{kP} P\,(1 - \mu_0^2)/\pi E_0 c \qquad \text{(III.18)}$$

where

$$w_{kP} = (a_k/c)^2\,(3a/c - a_k/c)$$

The ordinate of the reaction diagram is found as

$$p_i = X_i/cb \qquad \text{(III.19)}$$

The reaction diagram is stepped, so the abrupt changes in it should be smoothed over (Fig. III.13*e*).

The bending moments (Fig. III.13*f*) are determined as in a cantilever beam. For example, for point *2* we may write

$$M_2 = X_4 2c + X_3 c + 0.5X_2 c/4$$

Here, we take into account only that part of the distributed load induced by X_2 which is applied past point *2*.

In addition to the bending moments, we should calculate the shearing forces, Q, applied to the strip.

Once M and Q are found, we can design the strip footing in accordance with the general requirements for reinforced concrete structures.

If a beam is composed of several portions differing in rigidity, the design loading system remains the same (see Fig. III.13), but the component sags, v_{ki}, of the beam are determined, taking the stiffness of each particular element.

Design of Strip Footing as Beam on Elastic Foundation with Subgrade Reaction. This design method is based on the assumption that the settlement of the soil at a given point is independent of the settlements at the other points and is proportional to the pressure exerted at that point.

Accordingly, the foundation is assumed to settle only within the footing base. Actually, however, the foundation material is subjected to settlement beyond the footing base as well.

For a beam on an elastic foundation having the modulus of subgrade reaction, c_y (Fig. III.14), we may write

$$q = bp_s = bc_y y \qquad \text{(III.20)}$$

where c_y is the modulus of subgrade reaction ranging between approximately 0.3 and 1 kgf/cm³ for very soft soils, from 1 to 3 kgf/cm³ for soft soils, and between 3 and 8 kgf/cm³ for medium dense soils; b is the width of the footing; p_s is the soil pressure; and y is the settlement of the soil and the beam at the distance x from the beam end (the origin).

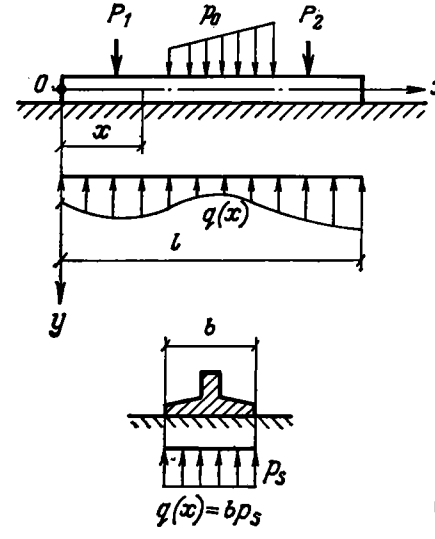


Fig. III.14. To the design of a strip footing as a beam on an elastic foundation with subgrade reaction

The second derivative of this expression is

$$q'' = bc_y y'' \qquad \text{(III.21)}$$

From a course on the strength of materials, we may write

$$y'' = -M/EI;\ q - p_0 = M'' \text{ or } q'' = M^{IV} \qquad \text{(III.22)}$$

where p_0 is the linear load function.

On substituting Eq. (III.22) into Eq. (III.21) and carrying out some simple manipulations, we get the following differential equation

$$s^4 M^{IV}/4 + M = 0 \qquad \text{(III.23)}$$

where

$$s = \sqrt[4]{4EI/bc_y} \qquad \text{(III.24)}$$

has a linear dimension and is called the linear characteristic of a beam on an elastic foundation.

If $l/s < 0.75$ (where l is the length of the beam), the beam is called *rigid* (here, the bending strain may be neglected); if $0.75 < l/s < 3$, the beam is called *short*; and if $l/s > 3$, the beam is called *long*. The above classification is not strict, so some departures are allowed in practice.

The general solution of Eq. (III.23) is

$$M = C_1 e^{-\varphi} \cos \varphi + C_2 e^{-\varphi} \sin \varphi + C_3 e^{\varphi} \cos \varphi + C_4 e^{\varphi} \sin \varphi \qquad \text{(III.25)}$$

where $\varphi = x/s$, and x is the current coordinate.

Long Beams. In a long beam loaded as shown in Fig. III.15, the moment and shearing force at the end ($x = l$) are

$$M = 0 \quad \text{and} \quad Q = dM/dQ = 0$$

These conditions are satisfied if, in Eq. (III.25), $C_3 = 0$ and $C_4 = 0$.

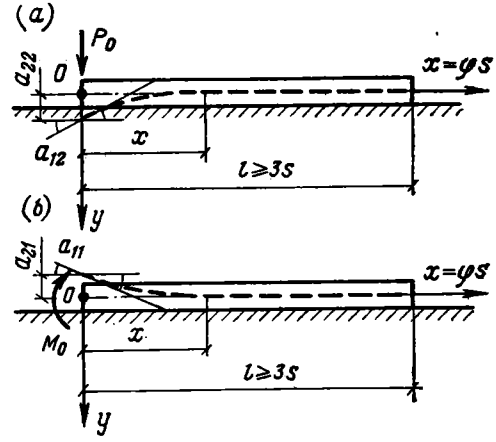


Fig. III.15. Loading systems for long beams loaded at the end
(*a*) concentrated load, loading system I; (*b*) bending moment, loading system II

As is seen, the general solution for a long beam in the adopted loading system is

$$M = C_1 e^{-\varphi} \cos \varphi + C_2 e^{-\varphi} \sin \varphi \qquad \text{(III.26)}$$

It should be noted as a final remark that the parameters at one end of a long beam have no effect on those at the other.

Differentiating Eq. (III.26) consecutively yields the expression for the shearing force, Q, and deflections, namely the settlement $w = EIy$ and the angular deflection $w' = EIy'$. Recalling Eqs. (III.20) and (III.24), we may find the footing pressure simulta-

neously with the settlement of the beam

$$q = bc_y y = bc_y w/EI = 4w/s^4 \tag{III.27}$$

Therefore,

$$Q = M' = (1/s)\,[-C_1 e^{-\varphi}(\cos\varphi + \sin\varphi) + C_2 e^{-\varphi}(\cos\varphi - \sin\varphi)] \tag{III.28}$$

$$w = qs^4/4 = M''s^4/4 = 0.5s^2\,(C_1 e^{-\varphi}\sin\varphi - C_2 e^{-\varphi}\cos\varphi) \tag{III.29}$$

$$w' = 0.5s\,[C_1 e^{-\varphi}(\cos\varphi - \sin\varphi) + C_2 e^{-\varphi}(\cos\varphi + \sin\varphi)] \tag{III.30}$$

The integration constants, C_1 and C_2, are determined from the boundary conditions.

In loading system *I* (Fig. III.15*a*) where a concentrated load, P_0, is applied at the end of the beam (that is at the origin), with $x = 0$ and $\varphi = 0$, we have $M = 0$ and $Q = -P_0$ (the shearing force is assumed to be positive if it is directed upwards). From Eqs. (III.26) and (III.28), we find that $C_1 = 0$ and $C_2 = -sP_0$, so

$$M = -sP_0 e^{-\varphi}\sin\varphi;$$

$$Q = -P_0 e^{-\varphi}(\cos\varphi - \sin\varphi) \tag{III.31}$$

$$w = 0.5s^3 P_0 e^{-\varphi}\cos\varphi;$$

$$w' = -0.5s^2 P_0 e^{-\varphi}(\cos\varphi + \sin\varphi) \tag{III.32}$$

In loading system II (Fig. III.15*b*), where a concentrated moment, M_0, is applied at the end of the beam (that is, at the origin), with $x = 0$ and $\varphi = 0$, we have $M = M_0$ and $Q = 0$. From Eqs. (III.26) and (III.28), we find that $C_1 = C_2 = M_0$ and

$$M = M_0 e^{-\varphi}(\cos\varphi + \sin\varphi);$$

$$Q = -(2/s)\,M_0 e^{-\varphi}\sin\varphi \tag{III.33}$$

$$w = 0.5s^2 M_0 e^{-\varphi}(\sin\varphi - \cos\varphi);$$

$$w' = sM_0 e^{-\varphi}\cos\varphi \tag{III.34}$$

The above equations yield the deflection influence factors for the loaded end of the beam: with $M_0 = 1$ we get the angular deflection, a_{11}, and the settlement, a_{21}, which are multiples of EI (Fig. III.15*b*), and with $Q_0 = 1$ we have the settlement, a_{22}, and the angular deflection, a_{12}, which are also multiples of EI (Fig. III.15*a*). They are as follows

$$a_{11} = s;\quad a_{12} = a_{21} = -0.5s^2;$$

$$a_{22} = 0.5s^2 \tag{III.35}$$

In loading system III (Fig. III.16*a*) which is symmetrical, one half of the load is applied to each half-beam, and the section directly beneath the load undergoes no angular deflection. As a consequence, with $x = 0$ and $\varphi = 0$, we get $Q = -P_0/2$ and $w' = 0$. Equations (III.28) and (III.30) give $C_1 = -C_2 = 0.5P_0s$, so

$$M = 0.25P_0se^{-\varphi}(\cos\varphi - \sin\varphi);$$

$$Q = -0.5P_0e^{-\varphi}\cos\varphi \qquad \text{(III.36)}$$

$$w = 0.25P_0s^3e^{-\varphi}(\sin\varphi + \cos\varphi);$$

$$w' = -0.25P_0s^2e^{-\varphi}\sin\varphi \qquad \text{(III.37)}$$

In loading system IV (Fig. III.16*b*) which is skew-symmetric, one half of the external moment is applied to each half-beam, and

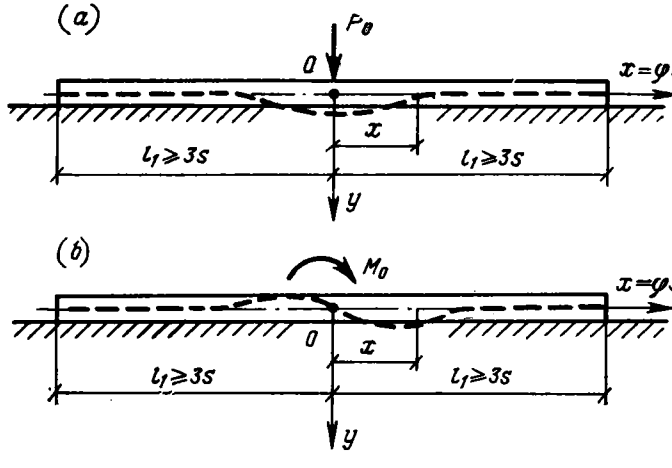


Fig. III.16. Loading systems for long beams loaded within the distance $l_1 \geqslant 3s$ of the ends
(*a*) concentrated load, loading system III; (*b*) bending moment, loading system IV

the settlement beneath the applied moment is equal to zero. So with $x = 0$ and $\varphi = 0$, we have $M = M_0/2$ and $w = 0$. Equations (III.26) and (III.29) give

$$C_1 = M_0/2 \quad \text{and} \quad C_2 = 0$$

Therefore,

$$M_1 = 0.5M_0e^{-\varphi}\cos\varphi;$$

$$Q = (-M_0/2s)\,e^{-\varphi}(\cos\varphi + \sin\varphi) \qquad \text{(III.38)}$$

$$w = 0.25s^2M_0e^{-\varphi}\sin\varphi;$$

$$w' = 0.25sM_0e^{-\varphi}(\cos\varphi - \sin\varphi) \qquad \text{(III.39}$$

When a long beam is subjected to several loads, the solution may be obtained by adding together the solutions obtained for loading systems I through IV.

Short Beams. In the case of short beams ($0.75 < l/s < 3$) the solution given by Eq. (III.25) leads to unwieldy formulas. In design practice, on the other hand, short beams are less frequent than are rigid and long beams. So we shall restrict ourselves to only one problem of practical importance. This is a beam subjected to two

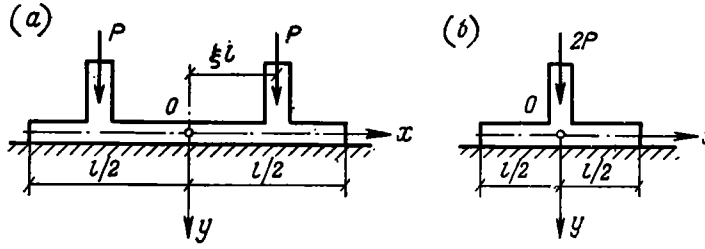


Fig. III.17. Loading systems for short beams loaded (*a*) with two symmetrical concentrated loads, and (*b*) with one concentrated load applied in the midspan

symmetrical concentrated loads (Fig. III.17*a*). Consider the approximate solution by the variational method due to Lagrange and Ritz. The equations describing the sag and angular deflections of the beam are as follows

$$y = a_1 + a_2 (x^4 - 3l^2x^2/2) \tag{III.40}$$

$$y' = a_2(4x^3 - 3l^2x) \tag{III.41}$$

Here, the constant parameters are

$$\left.\begin{aligned} a_1 &= 2P\ (1 + 9l^4A/80B)/c_y l \\ a_2 &= 2PA/c_y lB \end{aligned}\right\} \tag{III.42}$$

where

$$\left.\begin{aligned} A &= \xi^4 - 3\xi^2/2 + 0.112 \\ B &= 4.8EI/c_y + 0.0091\ l^4 \end{aligned}\right\} \tag{III.43}$$

Since the above solution is approximate, the M and Q diagrams should be plotted semi-analytically using the $q = bc_y y$ pressure diagram where the sag, y, is given by Eq. (III.40), rather than using the formulas $M = -EIw''$ and $Q = -EIw'''$ (because the third derivative of the adopted function of w shows no abrupt change).

These formulas may be used when the beam is subjected to the concentrated load, $2P$, applied in the middle (Fig. III.17*b*), if we assume $\xi = 0$.

Rigid Beams. The footing pressure of rigid beams (in which $l/s < 0.75$) is determined by the formulas of a course on the strength of materials without taking the deformation of the beams into account, because their rigidity, EI, is assumed to be infinity. At points *1* and *2* located at the ends of the beam (Fig. III.18) the soil stress is as follows:

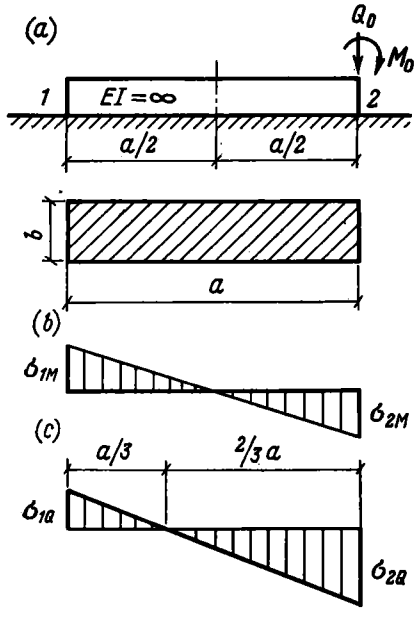


Fig. III.18. To the design of rigid beams

(*a*) loading system; (*b*) pressure diagram at the beam base, due to the bending moment applied at the edge; (*c*) pressure diagram at the beam base, due to the concentrated load applied at the edge

for M_0

$$\sigma_{1M} = -\sigma_{2M} = M_0/W = 6M_0/a^2b \tag{III.44}$$

for Q_0

$$\left.\begin{matrix}\sigma_{1Q}\\ \sigma_{2Q}\end{matrix}\right\} = -Q_0/F \pm 0.5Q_0a/W$$

$$= -Q_0/ab \pm 3Q_0/ab$$

$$= \left\{\begin{matrix}+2Q_0/ab\\ -4Q_0/ab\end{matrix}\right\} \tag{III.45}$$

The settlement of the beam at the edges is given by

$$\left.\begin{matrix}y_{1M} = -y_{2M} = \sigma_{1M}/c_y\\ y_{1Q} = -0.5y_{2Q} = \sigma_{1Q}/c_y\end{matrix}\right\} \tag{III.46}$$

and the angular deflections are

$$\left.\begin{matrix}\alpha_M = 2y_{1M}/a = 2\sigma_{1M}/ac_y\\ \alpha_Q = 3y_{1Q}/a = 3\sigma_{1Q}/ac_y\end{matrix}\right\} \tag{III.47}$$

Putting $M_0 = 1$ in Eq. (III.44), we get from the first lines of Eqs. (III.46) and (III.47) the angular deflection, a_{11}, and linear deflection, a_{21}, of point *2*, which are multiples of EI_b. Recalling Eq. (III.24), we may write

$$a_{11} = 2\,(6/a^3)\,(EI_b/bc_y) = 12s^4/4a^3 = 3s^4/a^3 \tag{III.48}$$

$$a_{21} = 6EI_b/a^2bc_y = -1.5s^4/a^2 \tag{III.49}$$

Putting $Q_0 = 1$ in Eq. (III.45), we get from the second line of Eqs. (III.46) the linear deflection of point *2*, which is a multiple of EI_b:

$$a_{22} = 4 \times 2EI_b/abc_y = 4s^4/4a = s^4/a \tag{III.50}$$

Combined Systems. These systems consisting of rigid and long beams (Fig. III.19) are rather frequent in practice. The moment,

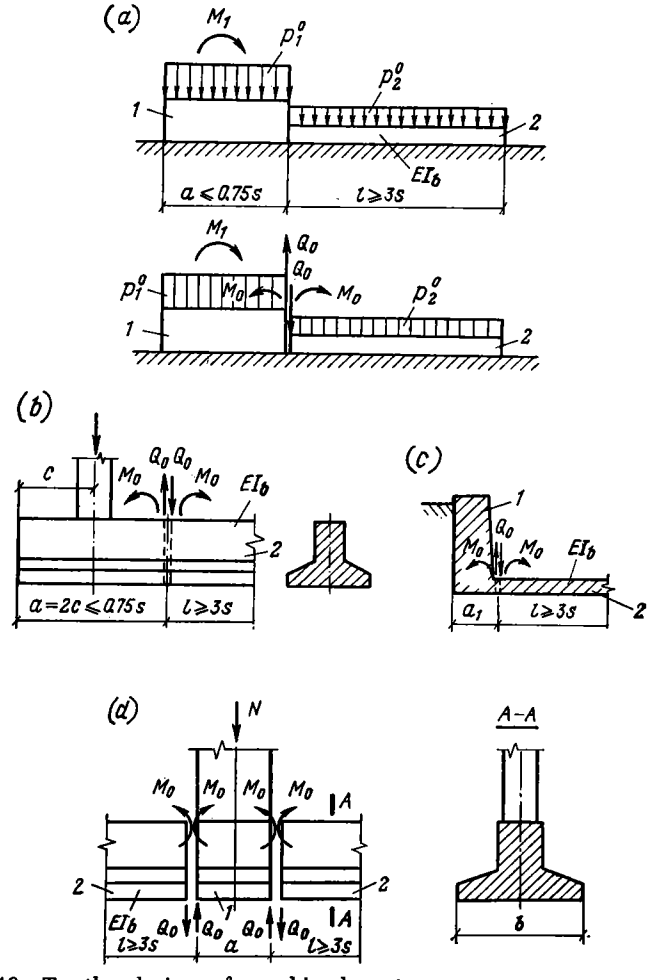


Fig. III.19. To the design of combined systems

(*a*) combined system consisting of a rigid and a long beam; (*b*) end portion of a strip footing; (*c*) junction of a retaining wall and a floor; (*d*) junction of an intermediate column with a footing beam; *1*—rigid beam; *2*—long beam

M_0, and the shearing force, Q_0, at the point of junction are determined by equating the mutual deflection of the beam ends to zero (Fig. III.19*a*):

$$\left.\begin{aligned} a_{11}M_0 + a_{12}Q_0 + a_{1p} &= 0 \\ a_{21}M_0 + a_{22}Q_0 + a_{2p} &= 0 \end{aligned}\right\} \qquad \text{(III.51)}$$

where a_{11}, $a_{12} = a_{21}$ and a_{22} are the mutual deflections taken as multiples of EI_b, found by adding together the influence factors of the end deflections of the composite beams, which are also multiples of EI_b.

Recalling Eqs. (III.35), (III.48) and (III.49), we may write for the system in question:

$$\left.\begin{aligned} a_{11} &= s + 3s^4/a^3 \\ a_{12} &= a_{21} = -s^2/2 + 1.5s^4/a^2 \\ a_{22} &= s^3/2 + s^4/a \end{aligned}\right\} \qquad \text{(III.52)}$$

The load terms taken as multiples of EI_b are the deflections due to the external loads, p_1^0 and p_2^0, and also the moment, M_1, which arise at the points where the unknown M_0 and Q_0 are applied. Recalling Eqs. (III.27), (III.48) and (III.49), we get

$$\left.\begin{aligned} a_{1p} &= -M_1 3s^3/a^2 \\ a_{2p} &= -M_1 1.5s^4/a^2 - (p_1^0 - p_2^0)\, s^4/4 \end{aligned}\right\} \qquad \text{(III.53)}$$

Solving the set of equations (III.51), we find M_0 and Q_0 which are used further to plot the moment and shear diagrams for the composite beams.

The above design procedure applies to the end of a footing beam (Fig. III.19*b*) and the junction of a retaining wall and a floor (Fig. III.19*c*). Analysis of the former design system has shown that the pressure from the footing is more uniformly transmitted to the soil if the strip extends 0.6 to 0.7s beyond the outer columns.

The junction of an intermediate column with a footing beam is a system consisting of a rigid portion beneath the column and two long beams (Fig. III.19*d*). At the points where the long beams join the rigid portion, we cut the system and apply the unknown Q_0 and M_0. For no angular deflection to take place at the end of the long beam, it is required that

$$sM_0 - Q_0 s^2/2 = 0 \qquad \text{(III.54)}$$

Similarly, for the mutual deflection of the long beams and the rigid portion at the cut-points to be zero, it is required that

$$-s^2 M_0/2 + s^3 Q_0/2 - (N - 2Q_0)\, EI_b/abc_y = 0 \qquad \text{(III.55)}$$

The last term in Eq. (III.55) is the settlement (taken as a multiple of EI_b) of the rigid portion under the action of the force $(N - 2Q_0)$.

Solving Eqs. (III.54) and (III.55) gives

$$Q_0 = N/(2 + a/s); \quad M_0 = Ns/2\,(2 + a/s) \qquad \text{(III.56)}$$

As is seen from Eqs. (III.56), Q_0 and M_0 decrease with increasing a.

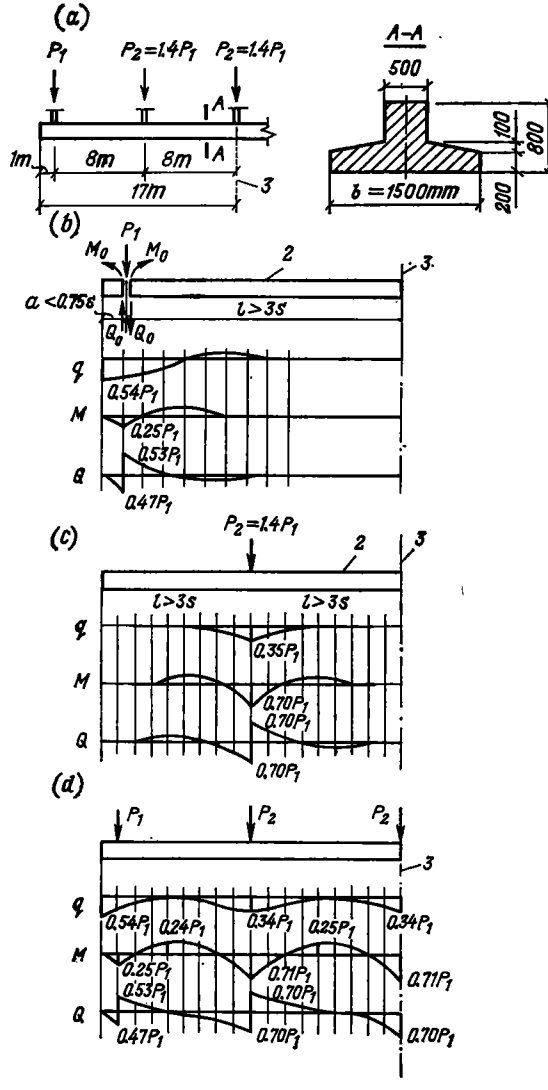


Fig. III.20. To Example III.3

(*a*) loading system; (*b*) diagrams for the marginal load; (c) diagrams for the intermediate load; (*d*) resultant diagrams; *1*—rigid beam; *2*—long beam; *3*—strip footing middle

Example III.3. *Given*: a reinforced concrete strip footing (Fig. III.20*a*); linear characteristic of the beam according to Eq. (III.24), $s = 2$ m; column cross-sectional area is small so the rigid portions beneath them may be ignored.

To find: footing pressure $q = bp_s$ and internal forces, M and Q.

Solution. With the footing exposed to the load, P_1, applied at the marginal column (Fig. III.20*b*), we have a junction composed of two portions: a rigid cantilever with $a = 1$ m (that is $a = 0.5\ s$ which is less than $0.75\ s$) and the remaining part of the beam with $l > 3\ s$ (that is, the beam is regarded as a long one). Using Eqs. (III.52), calculate the deflections of the beam ends, taken as multiples of EI_b, at the cut-point for $M_0 = 1$ and $Q_0 = 1$:

$$a_{11} = s + 3s^4/a^3 = 2 + (3 \times 2^4)/1^3 = 50$$

$$a_{22} = s^3/2 + s^4/a = 2^3/2 + 2^4/1 = 20$$

$$a_{12} = a_{21} = -s^2/2 + 1.5s^4/a^2$$

$$= -2^2/2 + (1.5 \times 2^4)/1^2 = 22$$

Determine the deflections induced by the load, using design system I (see Fig. III.15*a*) and putting $P_0 = P_1$ in Eqs. (III.32)

$$a_{1p} = -0.5\ s^2P_1 = -2P_1$$

$$a_{2p} = 0.5\ s^3P_1 = 4P_1$$

The set of equations (III.51) takes the form

$$50M_0 + 22Q_0 - 2P_1 = 0$$

$$22M_0 + 20Q_0 + 4P_1 = 0$$

Solving the above equations gives $M_0 = 0.248P_1$ and $Q_0 = -0.473\ P_1$. Substituting these values into the equations for design systems I and II (see Fig. III.15) yields for the long portion of the beam

$$q = [0.527e^{-\varphi} \cos \varphi - 0.124e^{-\varphi} (\cos \varphi - \sin \varphi)]\ P_1$$

$$M = [-1.054e^{-\varphi} \sin \varphi + 0.248e^{-\varphi} (\cos \varphi + \sin \varphi)]\ P_1$$

$$Q = [-0.527e^{-\varphi} (\cos \varphi - \sin \varphi) - 0.248e^{-\varphi} \sin \varphi]\ P_1$$

Using the above values, we plot diagrams for q, M and Q. The values of $e^{-\varphi}$ $\sin \varphi$ and $\cos \varphi$ may be looked up in an appropriate handbook.

Calculate the end pressures per unit length for the rigid portion of the beam, using Eqs. (III.44) and (III.45)

$$q_1 = 6M_0/a^2 - 2Q_0/a = 0.542P_1$$

$$q_2 = -6M_0/a^2 + 4Q_0/a = 0.404P_1$$

With the load $P_2 = 1.4P_1$ (Fig. III.20*c*) applied at the intermediate column, the beam may be treated as a system of two long beams. Using the solution for system III (see Fig. III.16*a*), we get

$$q = 0.35P_1e^{-\varphi} (\cos \varphi + \sin \varphi)$$

$$M = 0.7P_1e^{-\varphi} (\cos \varphi - \sin \varphi)$$

$$Q = -0.7P_1e^{-\varphi} \cos \varphi$$

which are used to plot the diagrams for q, M and Q.

The final solution in the form of q, M and Q diagrams is obtained by adding together the diagrams plotted for all loading systems of the strip foundation (Fig. III.20*d*).

Two-Way Strip Footings. An approximate design of two-way strip footings is carried out on the assumption that the pressure exerted by the structure is distributed over the entire soil area

beneath it. For more accurate results, strips are regarded as beams on an elastic foundation. The unknown forces of interaction between the strips running in either direction are found by equating their sags at the points of intersection. The overturning moments are negligibly small and, as a consequence, are not taken into consideration.

The design of a two-way strip footing as a system of beams running in two directions and lying on a foundation treated as an elastic half-space is too laborious. A simpler way is to design two-way beams by assuming an elastic foundation to which the concept of subgrade reaction is applicable. Here, in addition to the design systems discussed above, we should consider the junction of a rigid column base plate and four long beams (Fig. III.21). The moments and forces at the junction are determined by equations similar to Eqs. (III.56); if the long beams are equally rigid we may write

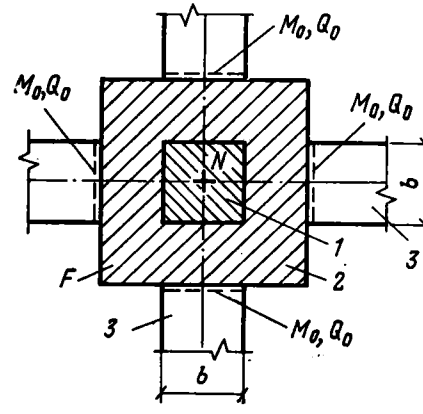


Fig. III.21. To the design of the junction of a rigid column base plate and two-way strips
1—column; *2*—column base plate; *3*—strips (long beams)

$$\left.\begin{aligned} Q_0 &= N/(4 + F/sb) \\ M_0 &= 0.5sN/(4 + F/sb) \end{aligned}\right\} \qquad \text{(III.57)}$$

where F is the area of the base plate (shaded in Fig. III.21); and b is the width of the strip base.

4. Interaction Between Structures and Footings Lying on Compliant Foundations

In the case of footings lying on compliant foundations, it must be borne in mind that footings interact with the underlying soil and also that rigid superstructures (if their rigidity is sufficiently large) may cause a redistribution of the service loads over the foundation. For example, a bunker trestle supported by strip footings (Fig. III.22*a*) should be designed with allowance for its interaction with the previously examined system consisting of the footing strips and the foundation.

As already noted, the forces, P_k, in the column (Fig. III.22*b*), which represent the load imposed on the footing, are markedly affected by the rigidity of the superstructure, designated $(EI)_s$. The

state of stress of the trestle and the footings considerably depends on the position of the live load, G, (filled cells) on the structure.

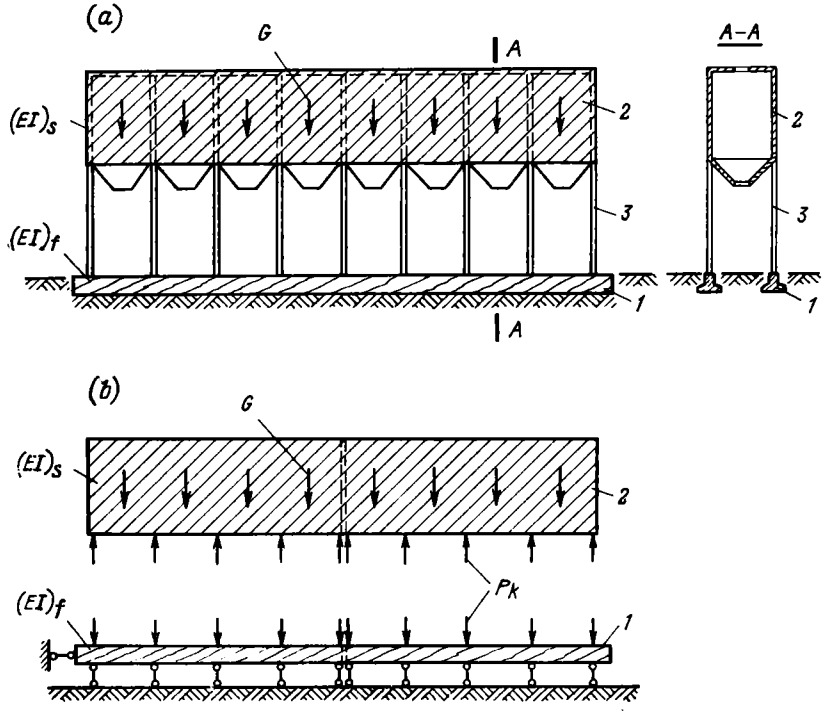


Fig. III.22. To the design of the interaction between a superstructure and a strip footing (using a bunker trestle as an example)
(*a*) general view; (*b*) loading system; *1*—strip footing; *2*—bunker; *3*—column

As a rule, the footing design which allows for interaction with the superstructure significantly cuts down the consumption of construction materials.

III.4. MAT FOOTINGS

1. General

The common types of mat footings are the flat plate construction, beam-and-slab construction, and construction using basement walls as a part of a mat (Fig. III.23). The last type of footing is most rigid. Mat footings are used for especially heavy and nonuniformly distributed loads.

The configuration and dimensions of a mat footing in plan are designed so that the resultant of the main service loads will be applied approximately to the centre of the mat.

In some practical cases, it suffices to assume that the soil pressure is approximately uniformly distributed over the entire mat bottom.

If a mat footing carries loads spaced wide and unequal distances apart, it would be more proper to design it as a slab on a compliant foundation.

Under the action of the soil pressure, the mat footing behaves as a reinforced concrete floor turned upside down in which the columns act as supports and the footing members are subjected to upward bending due to the soil pressure.

In accordance with Sec. III.3, mat footings of practical importance can best be designed as slabs on a layer of limited compression depth or, in some cases, on an elastic foundation displaying subgrade reaction. The solution of such problems is outside the scope of this text.

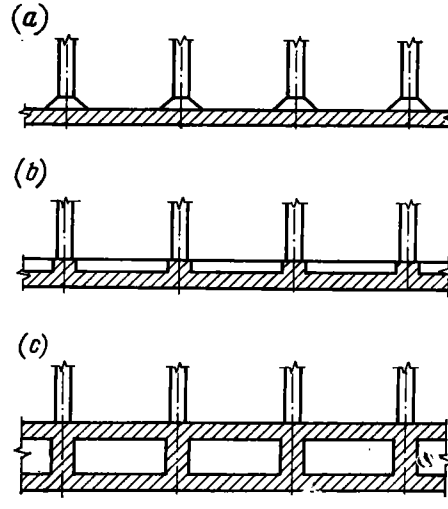


Fig. III.23. Mat footings
(*a*) flat plate construction; (*b*) beam-and-slab construction; (c) construction using basement walls as a part of a mat

In buildings and structures of considerable length (except relatively short end portions) mat footings may approximately be regarded as independent strips of unit width, lying on a compliant foundation. The design of mat footings by the subgrade-reaction method is done as discussed in Sec. III.3. The design of footings on a layer of limited depth of compression will be discussed below.

Flat plate footings are reinforced with welded-wire fabric, with the load-bearing wires running in one direction and with the fabric pieces placed one on top of another in not more than four layers. They are lap-spliced without welding in the load-bearing direction, and spliced without lapping in the other direction. The top fabric pieces are laid on supporting bars.

Beam-and-slab footings are reinforced with welded-wire fabric and welded-bar mats. Figure III.24 illustrates a likely reinforcement for a footing used in multi-storey buildings. The slab contains longitudinal and transverse fabric placed in two layers. The most stressed zone is additionally reinforced with a double layer of longitudinal fabric. Local bending is taken up by the top reinforcement

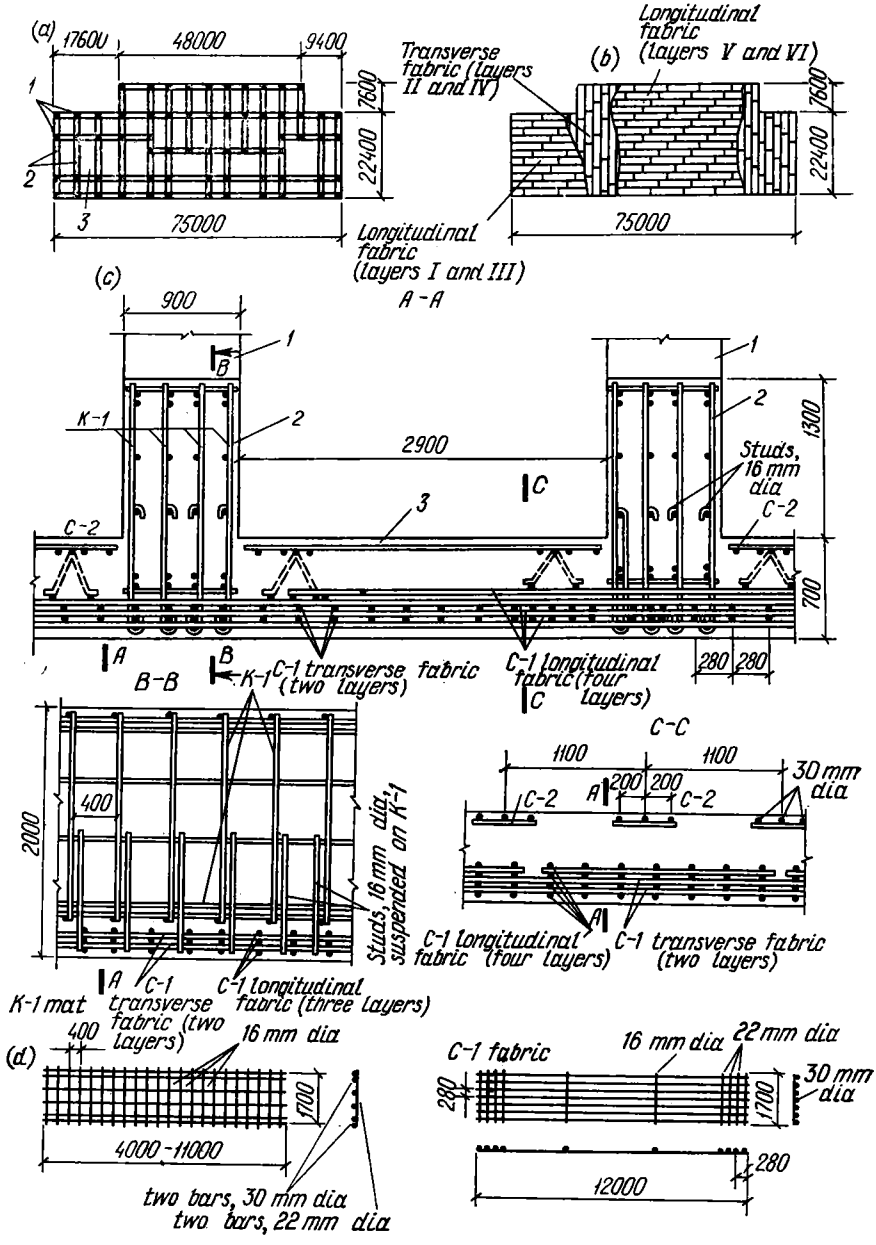


Fig. III.24. To proportioning a mat footing of the beam-and-slab construction
(*a*) plan; (*b*) arrangement of welded-wire fabric in plan; (*c*) details of footing reinforcement; (*d*) welded-bar mats and welded-wire fabric; *1*—columns; *2*—rib; *3*—slabs

grouped into three-bar mats with gaps left to provide access to the bottom reinforcement. In the beams, the bar mats are welded into reinforcing cages by means of transverse bars, and are fastened to the slab reinforcement by studs.

2. Design of Mat Footings as Slabs on Layer of Limited Compression Depth

In elastic theory, the case of a slab of unit width separated from the mat footing together with the foundation is treated as a plane problem involving plane deformation. In contrast to beams taken

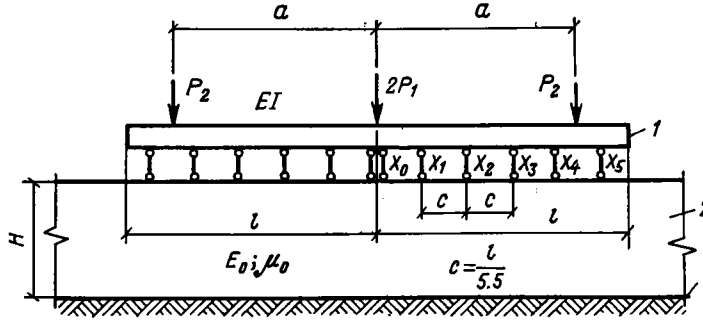


Fig. III.25. Loading system for a footing designed as a slab on a compressed layer of limited depth
1—footing slab of width $b = 1$; *2*—elastic compressed layer; *3*—perfectly rigid foundation

to be lying on an elastic half-space (see Fig. III.13), the design system adopted here (Fig. III.25) is concerned with the deformation experienced by a foundation of limited depth H and of a length usually assumed to be not more than half the length of the strip in question.

The basic design system, design procedure and equations are essentially the same as used for beams on an elastic half-space.

Here, the force displacement is found as

$$\delta_{ki} = (\alpha_1 w_{ki} + I_{ki})(1 - \mu_0^2)/\pi E_0 \qquad \text{(III.58)}$$

where w_{ki} is given by Eq. (III.14);

$$\alpha_1 = \pi E_0 c^3/6EI\,(1 - \mu_0^2) \qquad \text{(III.59)}$$

and the settlement of the foundation, Y_{ki}, is looked up in Table III.3 where its values for an elastic layer of limited depth are calculated by the technique proposed by S. S. Davydov who has developed the entire design method*.

* S. S. Davydov, Design of Structures on Elastic Foundation, Moscow Railway Engineering Institute, 1967 (*in Russian*). S. S. Davydov, Design of Footing Slabs on Combined Foundation. "Foundation Analysis and Design, and Soil Mechanics", No. 4, 1970 (*in Russian*).

TABLE III.3. Settlement Y_{hi} of Foundation Treated as Compressed Layer of Limited Depth

μ_0	H	x/c									
		0	1	2	3	4	5	6	7	8	9
0.2	c	3.628	0.179	—0.016	—0.002	—	—	—	—	—	—
	$2c$	3.938	0.903	0.112	—0.015	—0.015	—0.006	—0.001	—	—	—
	$3c$	4.735	1.547	0.455	0.1	—0.003	—0.02	—0.015	—0.008	0.003	—0.001
	$4c$	5.305	2.059	0.827	0.318	0.097	—0.009	—0.017	—0.02	0.015	—0.009
	$5c$	5.749	2.471	1.17	0.567	0.254	0.095	0.019	—0.011	0.02	—0.019
0.3	c	2.468	0.094	—0.034	—0.007	—0.002	—0.001	—	—	—	—
	$2c$	3.770	0.763	0.03	—0.053	—0.031	—0.014	—0.006	—0.003	—0.002	—0.001
	$3c$	4.565	1.391	0.334	0.019	—0.052	—0.048	—0.031	—0.018	—0.01	—0.006
	$4c$	5.135	1.897	0.687	0.207	0.015	—0.047	—0.054	—0.044	—0.031	—0.02
	$5c$	5.578	2.31	1.02	0.438	0.149	0.014	—0.041	—0.055	—0.051	—0.041
0.4	c	2.173	—0.075	—0.087	—0.029	—0.012	—0.006	—0.002	—0.001	—	—
	$2c$	3.461	0.503	—0.136	—0.144	—0.082	—0.045	—0.027	—0.018	—0.012	—0.008
	$3c$	4.254	1.104	0.103	—0.147	—0.163	—0.122	—0.081	—0.054	—0.037	—0.027
	$4c$	4.823	1.599	0.425	—0.007	—0.15	—0.17	—0.144	—0.11	—0.081	—0.059
	$5c$	5.266	2.007	0.741	0.194	—0.055	—0.152	—0.172	—0.157	—0.131	—0.104

Note. x is the distance from the centroid of the load to the point for which the settlement is being determined, that is, the distance between points, i and k; c is the spacing between the bars.

III.5. FOUNDATIONS FOR MACHINES INDUCING DYNAMIC LOADS

According to the character of the dynamic load they impose, machines may be classed into periodic- and nonperiodic-effect types.

The first group includes machines in which some parts rotate uniformly (turbogenerators, turbocompressors, turboblowers, turbopumps, electric generators, motor-generators, and electric motors) and machines in which uniform rotation is produced by a reciprocating mechanism (compressors, Diesel engines, and gang mills).

The second group includes machines in which some parts are in nonuniform rotation or in reciprocating motion (for example, the drive motors of rolling mills) and machines in which some parts are in reciprocating motion terminating in single or multiple impacts (free and die-forging hammers, and scrap choppers).

According to their construction, we may distinguish massive, wall-type and frame-type foundations (Fig. III.26).

Massive foundations are designed with recesses and holes to accommodate the machines they are to support, and pits for foundation bolts.

Wall-type foundations consist of longitudinal or transverse vertical walls rigidly connected to a bottom slab. The walls may be interconnected by beams or diaphragms.

Frame-type foundations are skeleton structures composed of girders and posts carried by a slab footing.

As with all other foundations, these should be adequately strong and stable.

The construction of the top part of a foundation depends on the overall dimensions of the machine. Also, it should provide easy access to the machine parts.

The foundation strength is checked by finding the soil bearing capacity multiplied by the reduction factor 0.8 for foundations intended to carry turbo-equipment, and 0.4 for forging-hammer foundations.

As a whole, a foundation should be proportioned so that the resultant of its weight and the weight of the machine passes through the centroid of the foundation bottom, or is applied with a small eccentricity (3 to 5%).

Foundations intended to support slow machines (those with a rotational speed of not over 1 000 rpm) are designed dynamically, which involves determining the peak amplitudes of forced vibrations in vertical and horizontal directions. The peak amplitudes thus found should not exceed the limits established by appropriate standards for the respective types of machines.

The peak amplitudes of vibrations are calculated with allowance for the elastic characteristics of the foundation material (subgrade).

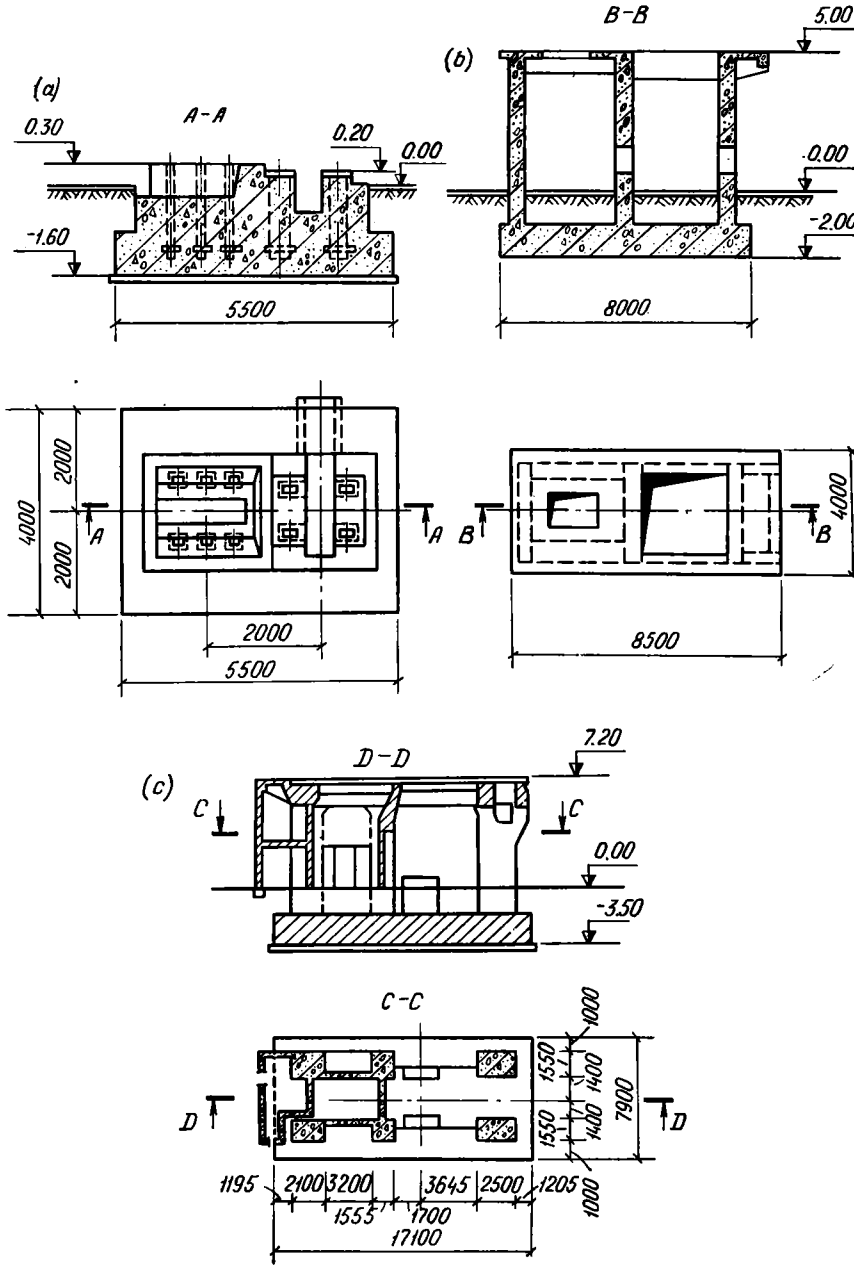


Fig. III.26. Foundations for machines inducing dynamic loads
(*a*) massive foundation; (*b*) wall-type foundation; (*c*) frame-type foundation

With frame-type foundations, account should also be taken of the elastic properties of the frames; massive and wall-type foundations are deemed to be rigid.

In terms of strength, the component parts of a foundation are designed as ordinary reinforced concrete members.

In more detail, the analysis, static and dynamic design of foundations for a particular type of machines are discussed in appropriate manuals.

In the USSR, the concrete used in foundations is at least M-150, and the reinforcing steel, class A-I through A-III.

Massive and wall-type foundations are only reinforced at their surface, at the places weakened by holes, and at the points where concentrated loads are applied. Frame-type foundations are reinforced as prescribed by relevant standards for the struts and girders of reinforced concrete frames.

Machine foundations are often cast in-situ and reinforced with welded-wire fabric and welded-bar mats. The formwork is composed of thin reinforced concrete slabs which are left in the foundation structure after the concrete has hardened.

CHAPTER IV

DESIGN OF ONE-STOREY INDUSTRIAL BUILDINGS

IV.1. CONSTRUCTION TYPES

1. Structural Elements

Buildings most commonly used in the metal-making, mechanical engineering, textile, food and other industries are of one-storey skeleton construction (Fig. IV.1*a*). A major distinction of such buil-

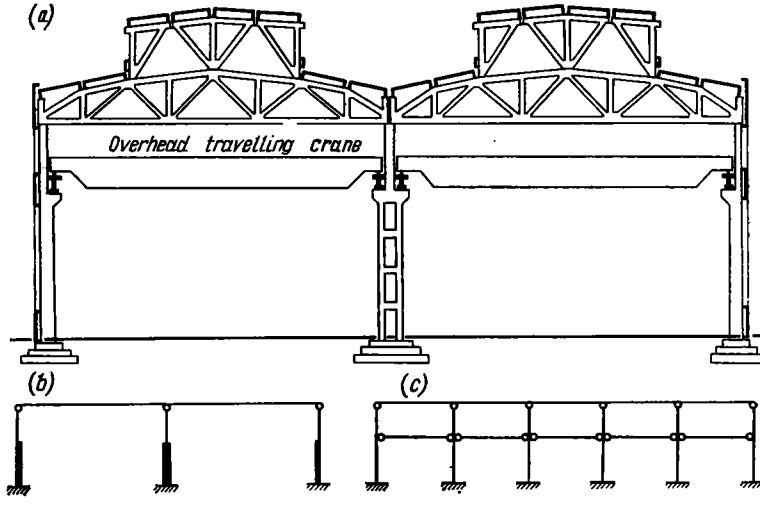


Fig. IV.1. One-storey crane-supporting industrial building
(*a*) structural cross section; (*b*) transverse frame; (*c*) longitudinal frame

dings is that they are equipped with overhead cranes travelling on runways supported on columns or carried by roof members. The roof of a one-storey industrial building may be composed of linear beams and slabs, or space shells.

The structural elements of a one-storey skeleton-construction building with a beam-and-slab roof include columns (or posts) built into their footings, roof girders (such as beams, trusses, and arches) supported on the columns, roof slabs placed on the girders, crane beams, and monitors intended to secure better light or venti-

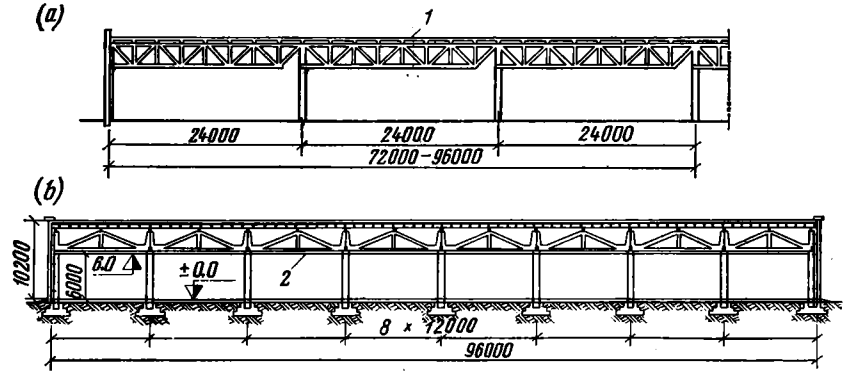


Fig. IV.2. Flat-roof one-storey industrial building
(*a*) cross section; (*b*) longitudinal section; *1*—flat roof; *2*—suspended ceiling

lation. The main structural element of the skeleton is the transverse frame formed by the columns and girders.

A one-storey skeleton-construction building owes its spatial rigidity and stability to the columns fixed in their footings, transverse frames, and longitudinal frames composed of the same columns, roof members, crane beams, and vertical braces (Fig. IV.1 *b* and *c*).

One-storey industrial buildings (without cranes or equipped with floor-type cranes) may have flat roofs with no monitors (Fig. IV.2).

2. Overhead Travelling Cranes

An overhead travelling crane is a lifting plant carried on a horizontal girder spanning between rails above window level at each side of a workshop. It consists of a hoisting crab with a hook on a flexible or rigid suspension, which can itself travel from end to end of the girder (Fig. IV.3). The whole area of the workshop between the rails can thus be traversed by the crab. In the longitudinal direction (along the building), the crane is moved by the girder which rides the rail track. For this purpose, the girder is equipped with wheels whose number for cranes with a lifting power of up to 50 tonnes is four (two wheels per rail).

Overhead travelling cranes are classed according to their duty and lifting power. Light-duty cranes are designed for irregular operation and slow motion (not more than 60 m/min); they are used in the machine rooms of thermal power stations, repair shops, etc. Medium-duty cranes are intended for active service and normal speed of motion (up to 100 m/min); they are used in machine shops, assembly departments, and moulding shops of reinforced concrete precasting plants. Heavy-duty cranes are designed for round-the-clock operation and high speed of motion (over 100 m/min); they are used in foundry shops, rolling-mill shops, forge shops, and the like. The lifting power of overhead travelling cranes may be 10 t, 20 t, 30 t, 50 t and higher.

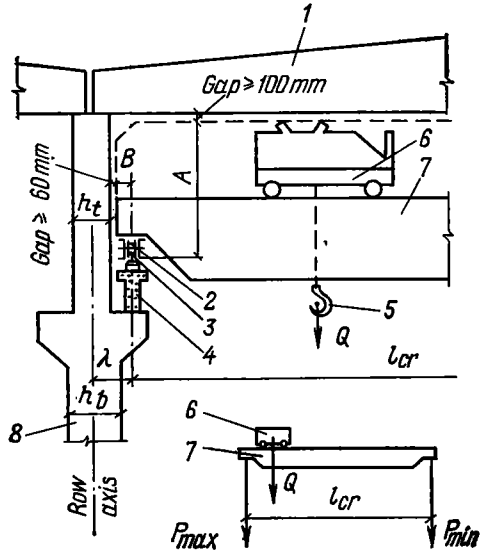


Fig. IV.3. Overhead travelling crane with a hoisting crab having a hook on a flexible suspension

1—girder; *2*—crane wheel; *3*—crane rail; *4*—crane beam; *5*—hook; *6*—hoisting crab; *7*—overhead travelling crane; *8*—column

An overhead travelling crane imposes vertical and horizontal loads on the skele ton of the building. The vertical loads are the self-weight of the girder, crab and load. They are transmitted through the crane wheels to the runways. The load imposed by a crane is a maximum when the crab carrying a load is in its extreme position at one end of the girder. In this case, the load exerted at the opposite end is a minimum.

The maximum basic vertical load, P^b_{max}, equal to the force exerted by a wheel on a rail, is established according to the lifting power of a given crane by relevant standards. The minimum basic vertical load, P^b_{min}, is found by analysing the crane girder as a beam simply supported at two ends (by four wheels):

$$P^b_{min} = (Q + Q_g + G)/2 - P^b_{max} \qquad (IV.1)$$

where Q is the lifting power of the crane; Q_g is the weight of the girder; and G is the weight of the crab.

The basic horizontal load directed across the runways and induced by crab braking is taken as follows:

— for cranes with flexible load suspension

$$T_{tr}^{b} = 0.05\ (Q + G) \tag{IV.2}$$

— for cranes with rigid load suspension

$$T_{tr}^{b} = 0.1\ (Q + G) \tag{IV.3}$$

The load T_{tr}^{b} may be directed inside or outside the span; it is transferred to one runway and is uniformly distributed between the two wheels.

The basic horizontal load applied along the rails and brought about by girder braking (by one brake wheel in cranes with a lifting power of up to 50 tonnes) is

$$T_{long}^{b} = 0.1\ P_{\max}^{b} \tag{IV.4}$$

In the design of structural elements for vertical and horizontal crane loads, the overload factor, n, is taken equal to 1.2.

When a crane is moving on, the crane beams are subjected to dynamic effects caused by sudden application of loads and shocks due to the unevenness of the runways, especially at rail joints. So, in the bearing-capacity analysis of crane beams, the design vertical and horizontal loads imposed by the crane are multiplied by the dynamic factor, $k_{dyn} = 1.1$. The dynamic effect of an overhead travelling crane is assumed not to be shared by other structural elements.

3. Building Proportioning

According to the manufacturing process involved, the grid size adopted for one-storey skeleton-type buildings equipped with overhead travelling cranes may be 12 × 18 m, 12 × 24 m and 12 × × 30 m, or 6 × 18 m, 6 × 24 m and 6 × 30 m. Most commonly, the bay length is 12 m. If, with such a bay, the wall panels are 6 m long, intermediate (trellis-work) columns are additionally installed along the sides. When the bay of a building is 12 m, the roof trusses may be placed at 6 m on centres, in which case, the necessary intermediate supports are provided by the secondary trusses (Fig. IV.4).

Experience shows that the most economical precast reinforced concrete roofs are those with a bay of 12 m and without secondary trusses.

In order that the same standard roof members may be used, the outer columns are disposed so that the layout axis of the row is within 250 mm of the outer face of a column (Fig. IV.5). For a bay of 6 m and cranes with a lifting power of not over 50 tonnes, the

outer column rows are arranged with a zero offset, so that the layout axis of the row coincides with the outer face of a column (Fig. IV.6*a*). At the ends of the building, the offset between the centre line of the

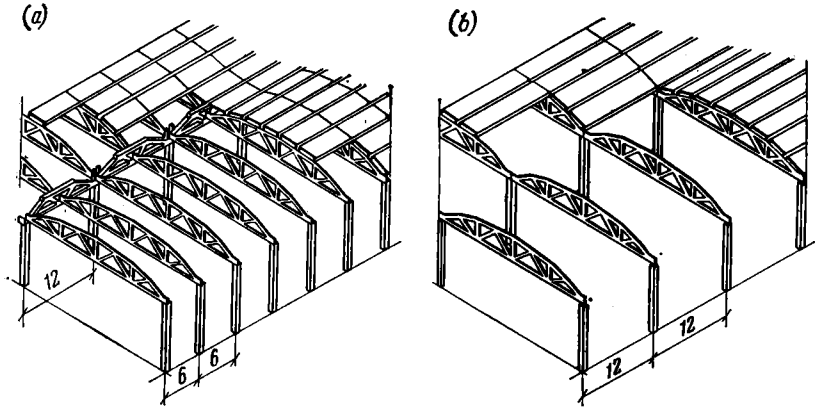


Fig. IV.4. Structural systems for a building
(*a*) with a bay of 6 m, with secondary trusses; (*b*) with a bay of 12 m, without secondary trusses

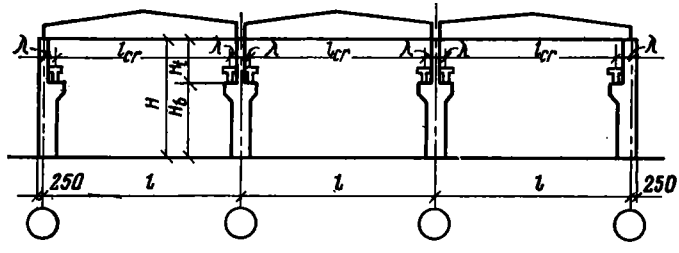


Fig. IV.5. Offset between structural members and layout axes on a cross section

column and the transverse layout axis is 500 mm (Fig. IV.6*b*). If it is wide and long, the building is divided by expansion joints into strain blocks.

Longitudinal expansion joints are usually installed at twin columns, using inserts (Fig. IV.6*c*). Here, the offset between the centre line of the columns and the longitudinal layout axes is 250 mm; if the bay of a building is 6 m, the offset is zero. Transverse expansion joints are likewise installed at twin columns, with a zero offset between the axis of the expansion joint and the transverse layout axis, and an offset of 500 mm between the centre line of the columns and the layout axis (Fig. IV.6*d*).

For overhead travelling cranes having a lifting power of not over 50 tonnes, the distance from the layout axis to the centre line of

the crane beam is $\lambda = 750$ mm (see Fig. IV.3). This distance is the sum of the girder cantilever, B, the side of the column cross section

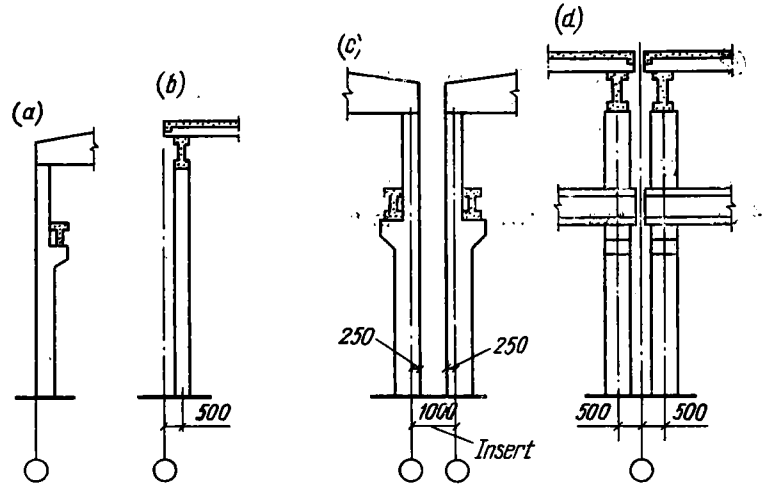


Fig. IV.6. Offset between the centre line of the columns and the layout axes (a) outer column row with a bay of 6 m; (b) at the end of the building; (c) at a longitudina expansion joint; (d) at a transverse expansion joint

above the crane, h_t, and the required clearance, C, between the crab and the column. For an outer column, $\lambda = B + h_t + C - 250$.

4. Transverse Frames

The girders of transverse frames may be with or without holes in the body. Their connection with the struts may be rigid or hinged. The shape of a girder, its construction, and type of connection with struts depend on the space to be spanned, roof type, and the manufacturing process and the erection technique adopted.

Rigid connection of the girder and columns of a frame reduces the bending moments, but such a frame must be designed individually, because the load applied to the columns gives rise to bending moments in the girder, and the load applied to the girder induces bending moments in the columns (Fig. IV.7*a*). With hinged joints, the girder and columns may be standardized separately because the loads applied to one member do not induce any bending moment in another (Fig. IV.7*b*). Hinged joints between girders and columns are easy to make; also, the component members are simple in shape and can readily be adapted to streamlined manufacture. Because of this, one-storey frames with hinged joints are considered to be

most economical and have been adopted as standard structures.

Physically, a hinge is produced by field-welding the supporting steel sheet of a girder to the embedded steel part at the end face of a column (Fig. IV.8).

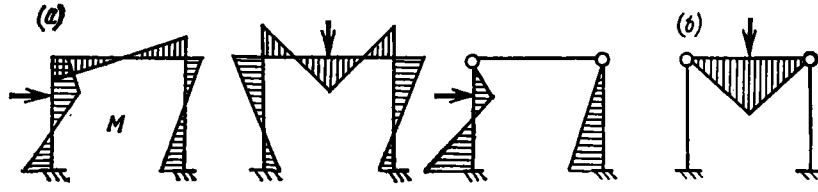


Fig. IV.7. To the selection of the most economical transverse frame structure; bending moment diagrams
(*a*) with rigid joints between the girder and columns; (*b*) with hinged joints

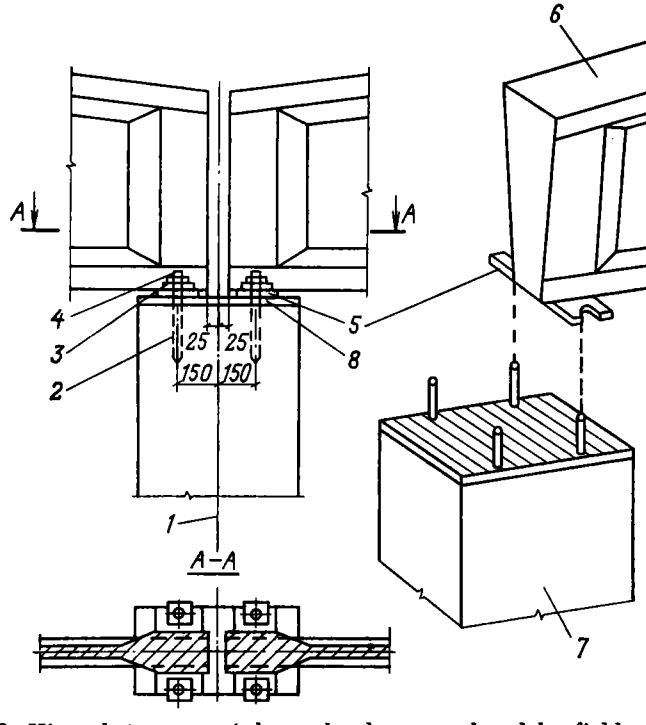


Fig. IV.8. Hinge between a girder and column produced by field welding and anchor bolts
1—row axis; *2*—anchors; *3*—washer; *4*—nut; *5*—steel plate, δ = 12 mm; *6*—girder; *7*—column; *8*—end-face steel plate

With spans of up to 18 m, girders are in the form of prestressed beams; with spans of 24 and 30 m, they are in the form of trusses.

The columns of a skeleton-construction building may be of the solid single-piece type rectangular in cross section, or of the open-work two-piece type (Fig. IV.9). When choosing a column type, it is necessary to take into account the lifting power of the overhead travelling crane to be used and the height of the building. Solid single-piece columns are used if the lifting power of the overhead

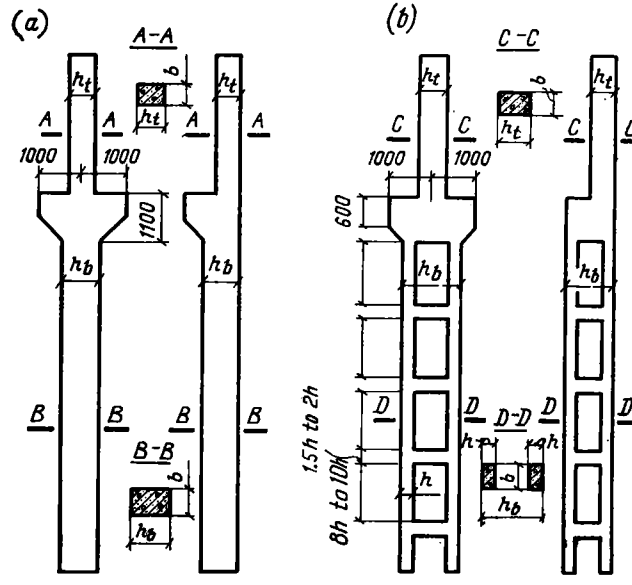


Fig. IV.9. Columns for one-storey buildings
(*a*) solid single-piece columns rectangular in cross section; (*b*) open-work two-piece columns

crane will not exceed 30 tonnes, and the building to be erected is relatively low. Open-work two-piece columns are intended for cranes with a lifting power of 30 tonnes or more, and for buildings more than 12 m high. The cross-sectional dimensions of a column above the crane beam it supports are assigned in view of the fact that the girders rest directly on the end face of the column without any special cantilevers. The depth of the column cross section is taken as $h_t = 500$ or 600 mm for intermediate columns, and $h_t =$ $= 380$ or 600 mm for marginal columns. The width of intermediate and marginal columns is adopted as $b = 400$ to 600 mm (the greater dimensions are specified for a bay of 12 m). The cross-sectional dimensions of solid single-piece columns below the crane beams they support are mainly assigned to ensure the required bearing capacity and adequate rigidity so that horizontal deflections of the columns in the plane of a transverse frame would not jam the

crane girder. Experience has shown that a column may be considered adequately rigid if the depth of its cross section is $h_b = 1/10H_b$ to $1/14H_b$.

Below the crane beams, open-work two-piece columns consist, as their name implies, of two pieces or side members joined by short spacers. In the bottom part of intermediate columns, the centre line of a side member may be set off from the axis of the crane beam. Here, the depth of the cross section for a two-piece column is taken as $h_b = 1\,400$ to $1\,600$ mm. In outer columns, $h_b = 1\,000$ to

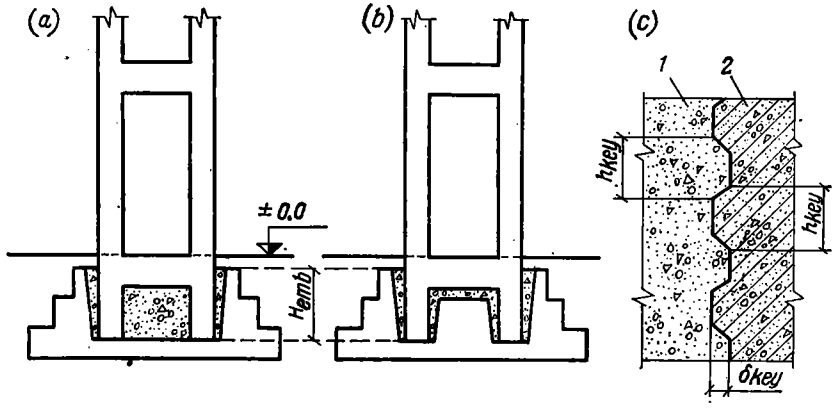


Fig. IV.10. Connection between a two-piece column and a footing
(a) single socket footing; (b) two independent footings; (c) concrete keys; 1—in-situ concrete; 2—column

1 300 mm. The depth of the cross section for a side member is taken as $h = 250$ or 300 mm, and the width as $b = 500$ or 600 mm.

The spacers in two-piece columns are set at $8h$ to $10h$ on centres. They are disposed so that the distance from floor level to the bottom of the first spacer should not be less than 1.8 m, and a convenient passage would be left between the side members. The bottom spacer is installed below floor level. The depth of the cross section for a spacer is taken as $1.5h$ to $2h$, and its width is assumed as that of a side member.

A two-piece column may be supported by a single socket footing, or by two independent footings. In the latter case, the concrete to be placed in-situ is reduced (Fig. IV.10). The depth (in metres) to which a column should be embedded in its socket is taken as

$$H_{emb} = 0.5 + 0.33h_b \text{ or } H_{emb} = 1.5b$$

whichever is the greater. In addition, the depth of embedment should be large enough to ensure the required anchorage length for the load-bearing reinforcement. If a tensile force is likely to arise in

one of the side members, concrete keys should be provided between the column and the concrete cast in-situ.

In length, single- and two-piece columns are usually manufactured-integral. Division into units to reduce the weight of the blocks

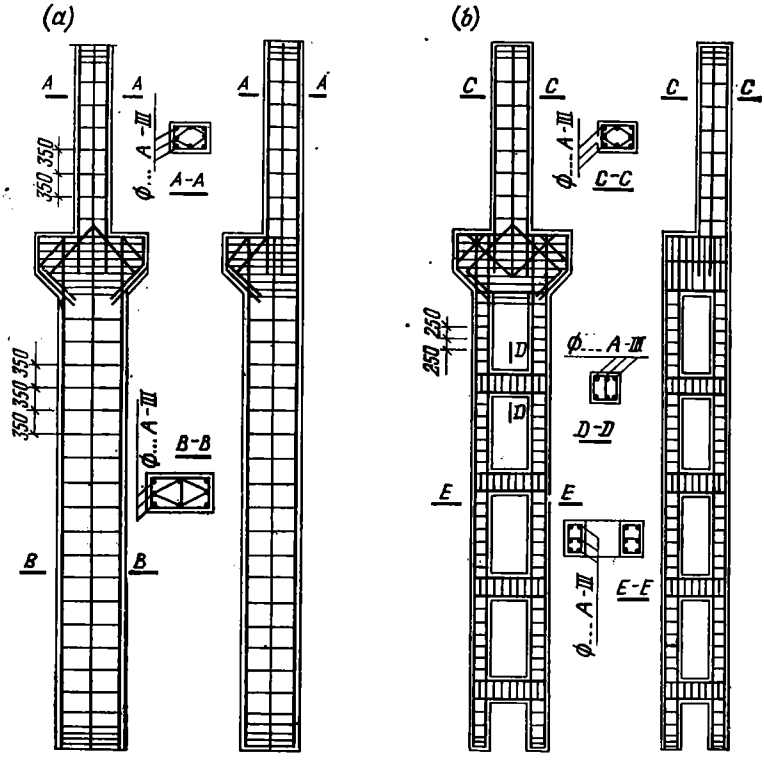


Fig. IV.11. Reinforcement of one-storey building columns
(a) single-piece rectangular columns; (b) open-work two-piece columns

to be handled is seldom done as this inevitably leads to problems in making the intermediate joints.

A likely reinforcement for single- and two-piece columns is shown in Fig. IV.11. Intermediate columns subjected to both positive and negative moments are reinforced symmetrically. The concrete brand used for columns may vary from M-200 to M-500.

5. Monitors

Monitors consist essentially of transverse trusses and struts carrying roof slabs, and supported on the girders of transverse frames. The space between the struts accommodates filler slabs. The width

of a monitor and the height of a clerestory should be large enough to secure adequate illumination of the workshop. As a rule, this is achieved with monitors $0.3l$ to $0.4l$ wide. In order that standard structural members may be used, monitors for spans of up to 18 m

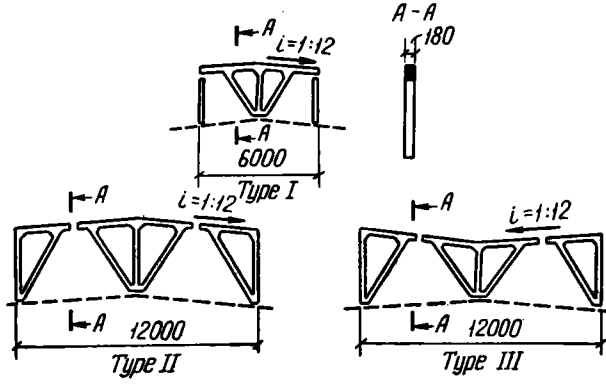


Fig. IV.12. Construction of monitor trusses

are taken to be 6 m wide, and those for spans of 24 and 30 m, 12 m wide (Fig. IV.12.). The load-bearing members of monitors are fastened to the girders of transverse frames by assembly bolts, and by welding together their embedded steel parts.

6. Bracing

The function of what we shall call vertical and lateral bracing* is to: (1) ensure the overall rigidity of the roof; (2) enhance the stability of the compressive chords of transverse frame girders; (3) resist wind forces against the ends of a building; and (4) take up the lateral thrust of overhead travelling cranes. Bracing interacts with the main elements of the building skeleton and contributes to its spatial rigidity.

Vertical Bracing. The forces induced by longitudinal horizontal loads (such as wind against the ends of a building, thrust of cranes, and so on) are resisted by the longitudinal frame of the building, with the roof acting as its girder. The roof slabs are connected to the columns via beams or trusses whose rigidity at right angles to their plane is small. Therefore, if no bracing were provided, a horizontal force applied to the roof might cause the girders to deform considerably at right angles to their plane (Fig. IV.13*a*),

* In the UK and US literature of the subject this is usually referred to as only lateral bracing.—*Translator's note.*

and a horizontal force applied to one of the columns and not shared by other columns might significantly deform it (Fig. IV.13*b*). Vertical bracing in the plane of the building columns is provided to make the roof act as a rigid structure which will not deform in the longitudinal direction.

Vertical lattice girders made of steel angles are installed between the columns forming the marginal sections of the building. They are

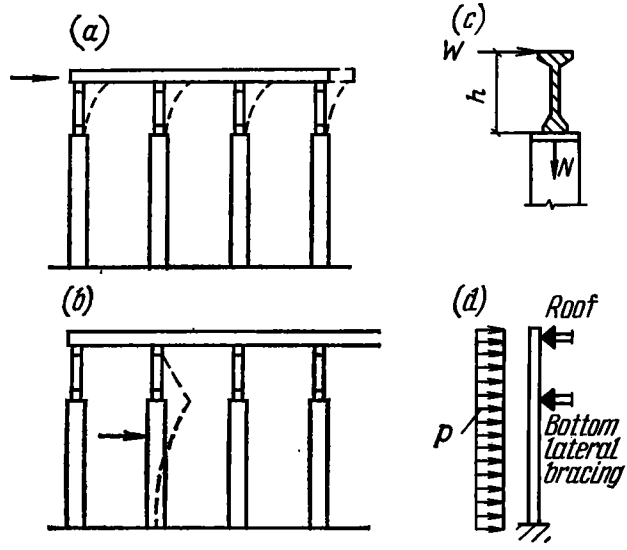


Fig. IV.13. Building skeleton deformations induced by lateral loads, and loading systems

combined by reinforced-concrete or steel-angle ties at the top of the columns (Fig. IV.14*a*). Lateral forces on the sides of the building are resisted by vertical cross bracing in the plane of the ends of the building. If the depth of the girder is small (under 800 mm), and the girder has a supporting rib able to take up a lateral force, it will be enough to brace the girders rigidly to the columns by knee braces. In this case, the embedded steel parts of the girders should be welded to those of the columns which should be designed to resist the moment $M = Wh$ and the vertical force, N (Fig. IV.13*c*). Vertical steel-angle braces between the columns are installed in each longitudinal row in the middle of each strain block. The braces are welded to the embedded steel parts of the columns.

Bottom Lateral Bracing. Wind forces against the ends of a building cause the columns of the end walls to bend. To reduce the effective length of these columns, the roof is used as a lateral support (Fig. IV.13*d*). In high and large-span buildings, it is advisable

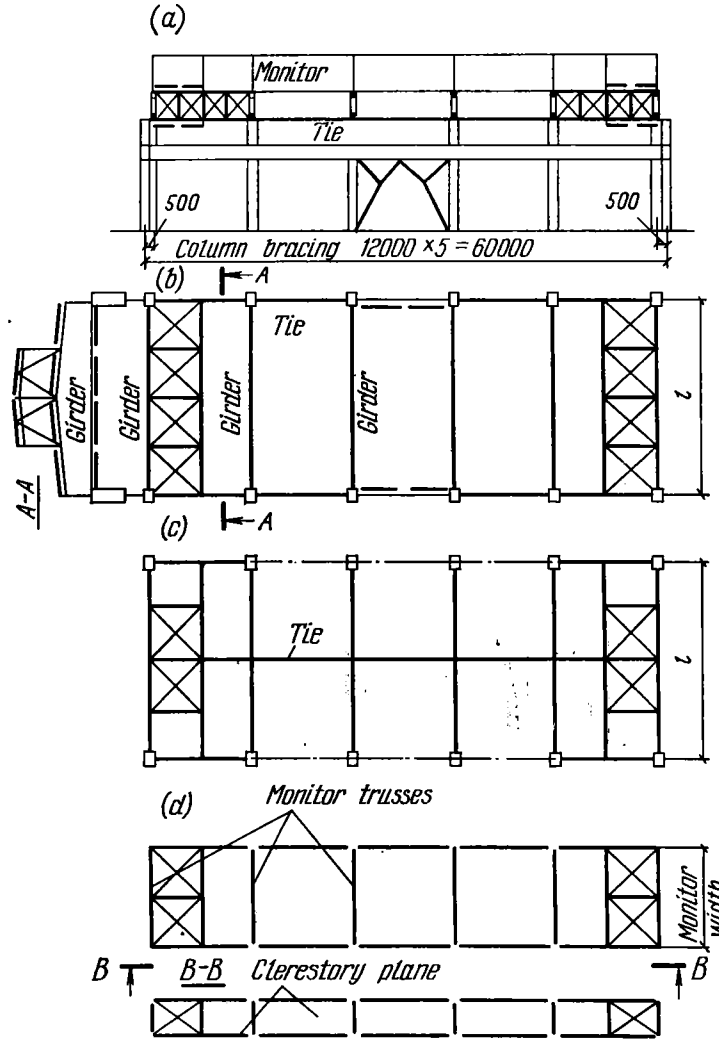


Fig. IV.14. Bracing
(*a*) vertical bracing; (*b*) bottom lateral bracing; (*c*) top lateral bracing; (*d*) monitor bracing

to make a lateral support for the end wall by providing cross bracing in the plane of the bottom chord of the girder (Fig. IV.14*b*). An additional support for the end wall may also be furnished by cross bracing in the top plane of the crane beams. Bottom lateral braces are made of cross steel angles which interact with the bottom chord

of the marginal girder. The forces due to the lateral bottom bracing are transmitted through the vertical braces to all columns of the strain block.

Top Lateral Bracing. To hold the tops of the girders in position, bracing is also provided in the plane of the top chord. For this purpose, the roof slabs are connected to the girders by welding together their embedded steel parts. In monitor-type buildings, the effective length of the compressive chord of a girder, which is likely to lose its position, is made equal to the width of the monitor. To reduce the effective length of the compressive chord of the girder, ties are installed along the axis of the monitor and fastened to the cross braces made of steel angles in the marginal bays of the strain block (Fig. IV. 14*c*). If the monitor does not extend the entire length of the strain block, no top lateral bracing is provided because, beyond the monitor, the reinforced concrete roof slabs themselves act as a rigid diaphragm. In this case, the ties are fastened to the roof members of the marginal bay.

Monitor Bracing. Monitor trusses are combined in a rigid space block by steel braces — vertical in the plane of the clerestory and lateral in the plane of the monitor roof (Fig. IV.14*d*).

7. Crane Beams

Since crane beams are subjected to dynamic loads imposed by overhead travelling cranes, it is recommended to confine prestressed reinforced concrete beams to light- and medium-duty cranes with a lifting power of not more than 30 tonnes. For heavy- and medium-duty cranes with a lifting power of 50 tonnes and more, steel crane beams are a better choice.

The best crane beams are those of I-section (Fig. IV.15). The wide top flange of the beam adds lateral stiffness, reduces the deflections in the case of lateral crane braking, and also improves the erection and service conditions for the runways and the crane. The bottom beam flange accommodates the prestressed reinforcement and provides additional strength when the prestress is transferred to the concrete. The design for vertical loads is carried out as for a T-section with its flange in compression, and that for horizontal loads is done as for a rectangular cross section taken as the top flange of the I-beam in question.

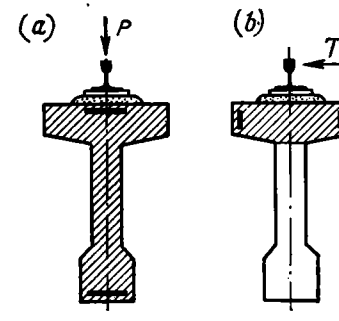


Fig. IV.15. Design sections of a crane beam
(*a*) for vertical load; (*b*) for lateral load

The depth of crane beams is taken as $h = 1/8l$ to $1/10l$, the depth of the top flange as $h'_f = 1/7h$ to $1/8h$, and the width of the top flange as $b'_f = 1/10l$ to $1/20l$. In order that runways may conveniently be fastened and straightened, the top flange should be not

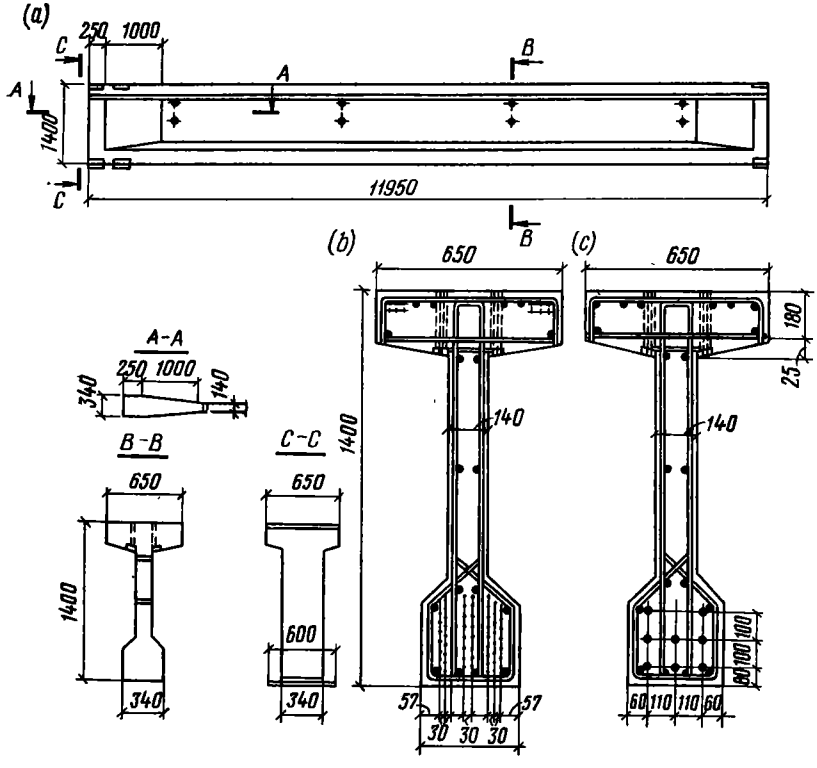


Fig. IV.16. 12-m prestressed crane beam
(a) general view; (b) prestressed wire reinforcement; (c) prestressed bar reinforcement

less than 500 to 650 mm wide. In the USSR, standard 6-m crane beams are 1 000 mm deep, and 12-m beams are 1 400 mm deep (Fig. IV.16).

To facilitate manufacture and erection, precast 6- and 12-m crane beams are cut into submultiple lengths subsequently joined together at the columns.

In the strength analysis of crane beams, the design loads imposed by overhead travelling cranes are multiplied by the overload factor, $n = 1.2$ and the dynamic factor, $k_{dyn} = 1.1$. The design vertical load thus found is

$$P_{max} = k_{dyn} n P^b_{max} \qquad \text{(IV.5)}$$

and the design horizontal load (per crane wheel) is

$$T_{tr} = 0.5\ k_{dyn} n T_{tr}^{b} \qquad \text{(IV.6)}$$

The horizontal load, T_{tr}, is actually applied at the head of the rail, but for simplicity and neglecting the small eccentricity, it is assumed to be applied at the middle of the flange depth.

Strength analysis is carried out for the design load due to two cranes of the same lifting power placed close to each other, and

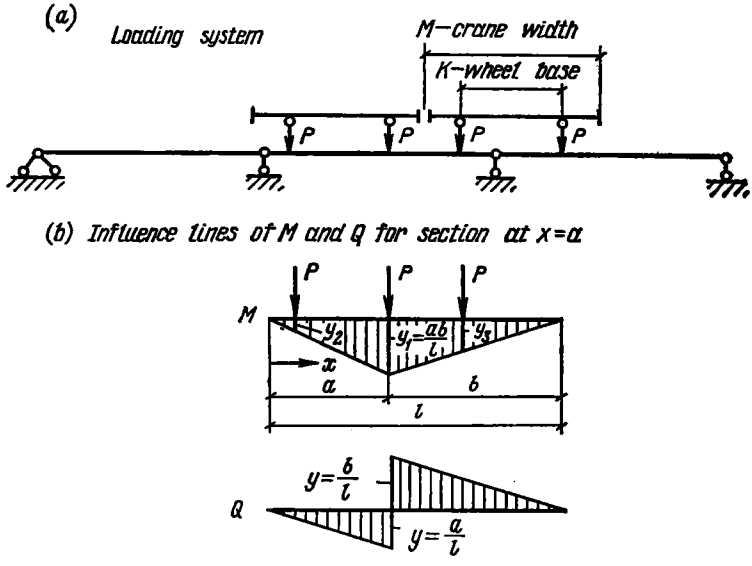


Fig. IV.17. To the design of a crane beam

multiplied by a load combination factor of 0.85 (for light- and medium-duty cranes). The live load due to the cranes is distributed across the crane beam in such a way that M and Q would be maximal at several sections within the span. The distances between the four forces transmitted through the wheels are adopted according to the overall width and the wheel base of the cranes (Fig. IV.17*a*). The design is carried out by plotting the influence lines of M and Q with one of the forces applied at the apex (Fig. IV.17*b*). The maximum forces are determined by adding together the products of the forces and the corresponding ordinates. For example, the maximum bending moment at a given section is

$$M = P_1 y_1 + P_2 y_2 + \ldots = \Sigma P y \qquad \text{(IV.7)}$$

Once the forces are found, the resultant diagrams for M and Q can readily be plotted. The ordinates of the resultant diagrams may be looked up in relevant tables given in handbooks.

Fatigue analysis is done for the design vertical load due to one overhead travelling crane. This load is found by multiplying the basic load by $n = 0.6$. The dynamic factor is not included in this analysis. The stresses in the concrete and the reinforcing steel are determined on the assumption that the transformed section behaves elastically under the external load and the prestress, N_0, taken with allowance for all losses. Here, the following conditions should be satisfied

$$\sigma_{c,\,max} \leqslant m_{c2} R_{pr} \quad \text{and} \quad \sigma_{s,\,max} \leqslant m_{s1} R_s$$

If the beam is to carry an alternating load, no tensile stresses are allowed in the concrete of the zone which is assumed to be in compression. The fatigue analysis of inclined sections is carried out on the assumption that the principal tensile forces acting at the centroid of the transformed section should be taken up by the stirrups in which the stresses are equal to the design strength, R_s, multiplied by the service factors, m_{s1} and m_{s2}.

Analysis for normal and diagonal incipient cracking in the beam is based on the following condition

$$\sigma_{c,\,ten,\,max} \leqslant R_{ten\,II} m_{c2}$$

The conditions for $\sigma_{prin,\,ten}$ and $\sigma_{prin,\,com}$ (see Chapter VII, Volume 1) should be re-written to include the stresses, $m_{c2}R_{ten}$ and $m_{c2}R_{pr}$.

The sag of the beam is determined with allowance for long- and short-time live loads taken with an overload factor equal to unity. The value of the sag should be $f \leqslant l/600$.

Prestressed crane beams are reinforced with high-strength wire, bars, or class K-7 wire strands. Since crane beams are designed to carry dynamic loads, their reinforcing cages should be tied, rather than welded. At the supports, crane beams are strengthened with ribs (by expanding the ends) and additional reinforcing bars, stirrups or wire fabric which contribute to the strength and crack-resistance of the beam ends when the prestress is transferred to the concrete. In the USSR, crane beams are made of M-400, M-500 or M-600 concrete, and the weight of a 12-m beam usually ranges between 10 and 12 tonnes.

Crane beams are fastened to the columns by welding together their embedded steel parts (Fig. IV.18*a*). To transmit horizontal braking forces, the joints are equipped with rib plates welded to the top embedded parts of the beams and special embedded parts in the columns. The impacts and shocks imposed on the beam by the travelling crane are damped by an elastic pad made of rubberized cloth 8 to 10 mm thick, which is inserted between the crane beam

and the rail. The pad also reduces wear of the rails. When placing the pad, it should be remembered that prestressed beams are cam-

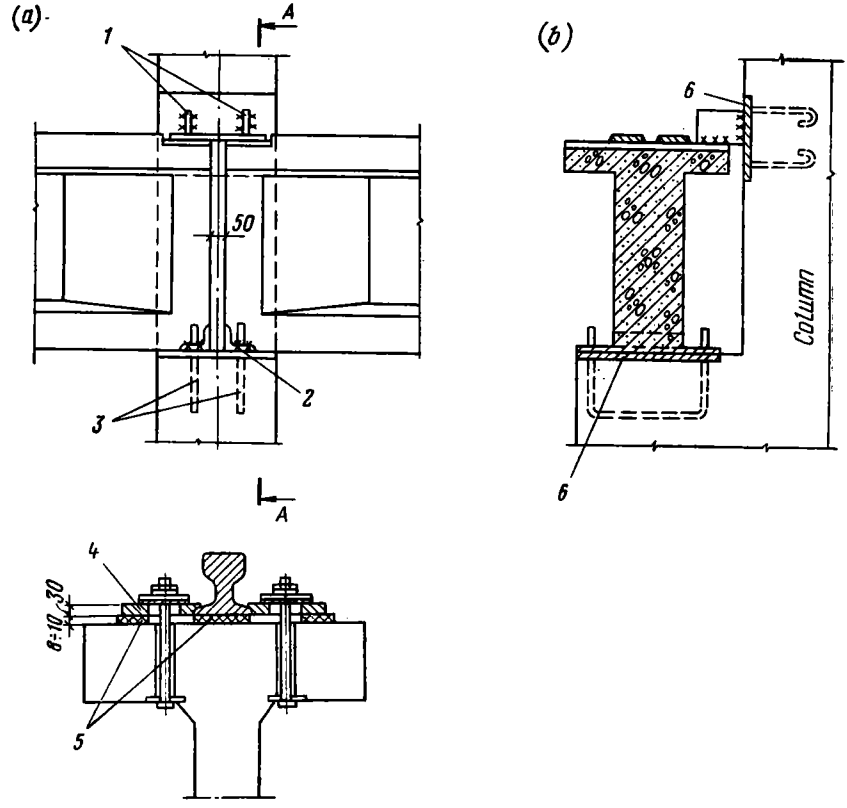


Fig. IV.18. (*a*) Fastening of a crane beam to a column; (*b*) fastening of a rail to a crane beam

1—100 × 12 mm rib plates; *2*—embedded part of the crane beam; *3*—anchors extending rom the column; *4*—clamp; *5*—elastic pad; *6*—embedded parts of the column, δ = 8 mm

bered, whereas the crane rails should be horizontal. After straightening, the rail is fastened to the beam by bolts and steel parts as shown in Fig. IV.18*b*.

IV.2. DESIGN OF TRANSVERSE FRAME

1. Loads and Their Disposition

The transverse frame of a one-storey building of skeleton construction is subjected to the dead load due to the weight of the roof and various live loads due to snow, vertical and lateral thrust of overhead

travelling cranes, positive and negative wind forces, etc. (Fig. IV.19*a*).

For design purposes, the connection between the girder and a column is regarded as a hinge, and that between a column and the footing is considered to be rigid. The length of a column is taken equal to the distance from the top of the footing to the bottom of

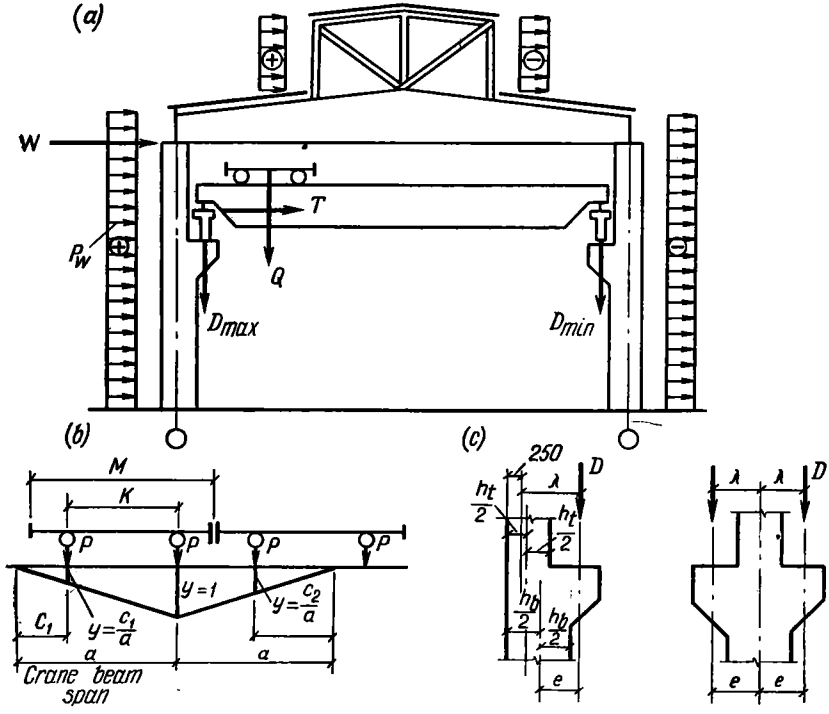


Fig. IV.19. Structural and loading system of a crane-supporting transverse frame

the girder. In designing the transverse frame, the objective is to determine the forces in the columns and to choose their cross-sectional area. The girder of the frame is independently designed as a single-span beam, truss or arch.

Dead Load due to Roof Weight. This is transferred to the column as a vertical force, N, induced by the girder. Its magnitude is calculated according to the loaded area of the roof. The vertical load is applied along the girder support axis, and is transmitted to the column, provided that the offset between the external face of the

column and the layout axis is 250 mm, with the eccentricity taken as

$e = 0.25/2 = 0.125$ m above the crane beam (in the case of a zero offset, $e = 0$);

$e = (h_b - h_t)/2 - 0.125$ below the crane beam (in the case of a zero offset, $e = (h_b - h_t)/2$).

The bending moments due to the eccentricity are equal to $M = Ne$.

Snow Load. The magnitude of this load is taken according to the climatic region and the roof configuration. The snow load is likewise transmitted to the column as a vertical force, N, and is calculated using the same loaded area as in the case of the roof load.

Live Load due to Overhead Travelling Cranes. The magnitude of the crane load is found for two cranes operating close to each other. The design vertical and horizontal crane loads are multiplied by the overload factor $n = 1.2$.

The vertical load imposed on a column is determined using the influence line of the support reaction for the crane beam. The concentrated force exerted by one crane wheel is placed directly at the support, and the remaining forces are disposed according to the standard distance between the crane wheels (Fig. IV.19*b*). The maximum force applied to the column is

$$D_{max} = P_{max} \Sigma y \qquad \text{(IV.8)}$$

Under the circumstances, the force applied to the opposite column of the frame is

$$D_{min} = P_{min} \Sigma y \qquad \text{(IV.9)}$$

The vertical force, D, is applied by the cranes through the crane beam with an eccentricity $e = 0.25 + \lambda - 0.5h_b$ to a marginal column (in the case of a zero offset, the eccentricity is $e = \lambda - 0.5\,h_b$) and $e = \lambda$ to an intermediate column (Fig. IV.19*c*).

The respective moments induced by the crane load are as follows

$$M_{max} = D_{max}\, e \quad \text{and} \quad M_{min} = D_{min}\, e$$

The horizontal load applied to the column due to the braking of two cranes operating close to each other is transmitted through the crane beam in the same manner as the vertical force. Accordingly, the lines of influence for this load are the same, and the load itself is found as

$$T = T_{tr} \Sigma y \qquad \text{(IV.10)}$$

Wind Load. The wind pressure per square metre of the wall and monitor surface is taken according to the locality and the height of the building. The wind pressure is considered to be positive from the weather side, and negative from the leeside. The wind

load is transmitted through the wall panels to the columns as a distributed load with an intensity of

$$p_w = q_w a$$

where a is the centre-to-centre distance between the columns.

Since the wind load is nonuniformly distributed over the height of the building, it is customary to simplify the matter by replacing it with a uniformly distributed load which produces the same bending moment at the bottom of the column.

The wind pressure applied to the monitor and the wall above the columns is replaced by a concentrated force, W, in the design loading system.

2. Spatial Behaviour of Building Subjected to Crane Load

A roof made of reinforced concrete slabs connected to one another by welding and concreting is a horizontal bracing diaphragm which

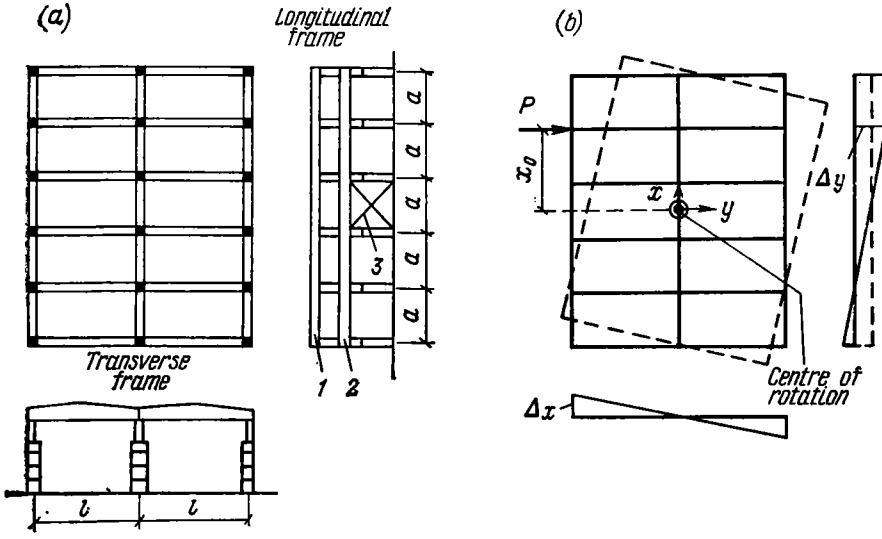


Fig. IV.20. Space block of a one-storey skeleton-type building
(*a*) block scheme; (*b*) block deflection; *1*—roof; *2*—crane beam; *3*—vertical braces between columns

is rigid in its plane. The building columns combined by the horizontal diaphragm into transverse and longitudinal frames act as a single space block. The size of such a block in plan depends on the distances between expansion joints (Fig. IV.20*a*).

Roof, snow and wind loads are simultaneously applied to all frames of a strain block. They, however, do not cause the skeleton to interact in space, so each frame may be designed as a separate structure. In contrast, the load imposed by overhead travelling cranes is only applied to two or three frames of the block, but, since the frames are combined by the horizontal diaphragm, the load is shared by the remaining frames, and the block behaves as an entity.

In a skeleton-construction building made of standard members with a regular spacing and constant rigidity, the centre of rigidity (that is, the point of application of the resultant of the reactive forces in the linear deflection of the block) coincides with its geometrical centre. Let us take this centre as the origin. We shall assume that x is the coordinate of a transverse frame, and y is the coordinate of a longitudinal frame (Fig. IV.20*b*). Let us apply a force, P, to the transverse frame with a coordinate, x_0, and determine the deflection of this frame. The force, P, will cause a linear deflection, and the moment $M = Px_0$ will cause an angular deflection of the block. On designating the reaction of the transverse frame to the unit deflection $\Delta = 1$ as r_{11x}, we get the linear deflection of the block

$$\Delta = P/nr_{11x} \tag{IV.11}$$

where n is the number of transverse frames in the block.

If the angular deflection of the horizontal diaphragm which is rigid in its plane is $\varphi = 1$, then the deflection of transverse frames is equal to $x\text{tg}\varphi$. Since, however, the finite angle, φ, is small and, as a consequence, $\text{tg}\,\varphi = \varphi = 1$, the deflection of the transverse frames is equal to their coordinate, x; the deflection of the longitudinal frames is y. The reactions arising in this case are:

— in the transverse frames

$$R_x = xr_{11x} \tag{IV.12}$$

— in the longitudinal frames

$$R_y = yr_{11y} \tag{IV.13}$$

where r_{11y} are the reactions of the longitudinal frames to the deflection $\Delta = 1$, determined with allowance for the resistance offered by the vertical bracing in the plane of the columns.

The torsion of the columns induced by the angular deflection of the horizontal diaphragm is negligibly small.

The angular stiffness of the block or the reactive moment of the block due to the unit angular deflection, $\varphi = 1$, of the diaphragm is

$$B_\varphi = M_{\varphi=1} = 2\sum_1^m xR_x + 2\sum_1^p yR_y \tag{IV.14}$$

where $m = n/2$ if the number of transverse frames, n, is even, or $m = (n - 1)/2$, if n is odd; $p = q/2$ if the number of longitudinal frames, q, is even, or $p = (q - 1)/2$, if q is odd.

According to Eqs. (IV.12) and (IV.13), the angular stiffness of the block with allowance for the reactions is

$$\left.\begin{aligned} B_\varphi &= 2r_{11x}\left(\sum_1^m x^2 + \beta \sum_1^p y^2\right) \\ \beta &= r_{11y}/r_{11x} \end{aligned}\right\} \qquad \text{(IV.15)}$$

The angular deflection of the block about the centre of rotation is

$$\varphi = M/B_\varphi = Px_0/2r_{11x}\left(\sum_1^m x^2 + \beta \sum_1^p y^2\right) \qquad \text{(IV.16)}$$

The deflection of the transverse frame with the coordinate, x_0, due to the force, P, can be found by adding together the linear and angular deflections of the block. Then,

$$\begin{aligned} \Delta &= P/nr_{11x} + x_0\varphi \\ &= P/nr_{11x} + Px_0^2/2r_{11x}\left(\sum_1^m x^2 + \beta \sum_1^p y^2\right) \end{aligned} \qquad \text{(IV.17)}$$

Now, let us find the reactive force due to the unit deflection, $\Delta = 1$, of the transverse frame by equating the deflection given by Eq. (VI.17) to unity. We get

$$P = c_{sp} r_{11x} \qquad \text{(IV.18)}$$

where

$$c_{sp} = 1/[1/n + x_0^2/2\left(\sum_1^m x^2 + \beta \sum_1^p y^2\right)] \qquad \text{(IV.19)}$$

The factor, c_{sp}, characterizes the spatial behaviour of the skeleton consisting of transverse and longitudinal frames. It should be kept in mind that the connections between the roof slabs are not perfectly rigid. According to experimental data, this can be taken care of by multiplying the factor, c_{sp}, by 0.7. Also, because the frames adjacent to that in question are likewise subjected to the crane load, we should multiply the expression for c_{sp} by 0.7 once more. So, in the end, we have

$$c_{sp} = 1/[2/h + x_0^2/\left(\sum_1^m x^2 + \beta \sum_1^p y^2\right)] \qquad \text{(IV.20)}$$

If we take into consideration the spatial behaviour of transverse frames only, then, with $\beta = 0$, Eq. (IV.19) yields the follow-

ing simplified solution

$$c_{sp} = 1/(1/n + x_0^2/2 \sum_1^m x^2) \qquad \text{(IV.21)}$$

For the second transverse frame (as reckoned from the end of the block) which is in the most unfavourable conditions as regards the contribution of adjacent frames, $c_{sp} = 3.4$ for a bay of 12 m, and $c_{sp} = 4$ for a bay of 6 m.

As is seen, transverse frames may be designed to resist the crane load with allowance for the spatial behaviour of the skeleton, using the slope-deflection method in which the reaction to the unit deflection of the transverse frame is modified by the factor, c_{sp} (Fig. IV.21).

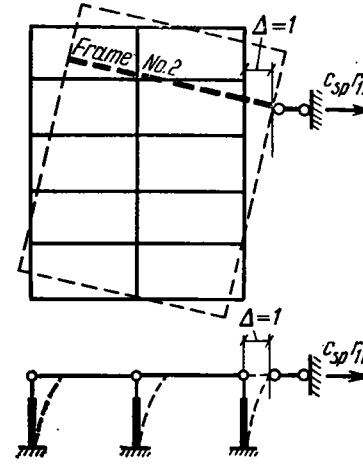


Fig. IV.21. To the spatial design of a one-storey skeleton-type building for crane loads

3. Determining Load-Induced Forces in Columns

A transverse frame can most conveniently be designed to resist various loads by the slope-deflection method which involves one unknown, Δ. This unknown is the horizontal deflection (the sway) of a plane loaded frame. Introducing an imaginary restraint in the direction of the unknown deflection, we get the basic load system for the frame (Fig. IV.22*a*). The basic system is subjected to the unit action of the unknown which gives rise to reactions, B_Δ, and bending moments in the columns (Fig. IV.22*b*). Then, the system is successively loaded by dead and live loads, N, M, T and p_w, which induce the respective reactions and bending moments in the columns (Fig. IV.22*c* through *e*). The reactions, B, in stepped columns of variable cross section and fixed at the top may be found using equations given in Appendix XII. In the equation

$$c_{sp} r_{11} \Delta + R_{1p} = 0 \qquad \text{(IV.22)}$$

r_{11} is the reaction of the transverse frame against the unit deflection, and $R_{1p} = \Sigma B$ is the sum of the load-induced reactions at the top of the columns considered to be positive if they point in the direction of the unknown deflection.

For all types of loads, except crane load, c_{sp} is unity.

We solve Eq. (IV.22) for the unknown, Δ, and find the elastic reaction

$$B_{el} = B + \Delta B_{\Delta} \qquad \text{(IV.23)}$$

In three-bay or wider frames, the top supports of the columns subjected to crane load are regarded as fixed, with $\Delta = 0$.

For frames in which the girders are located in two levels and the stiffness of the interior columns, B_{int}, is five times or more as great as that of the exterior columns, B_{ext}, the middle part may, for design

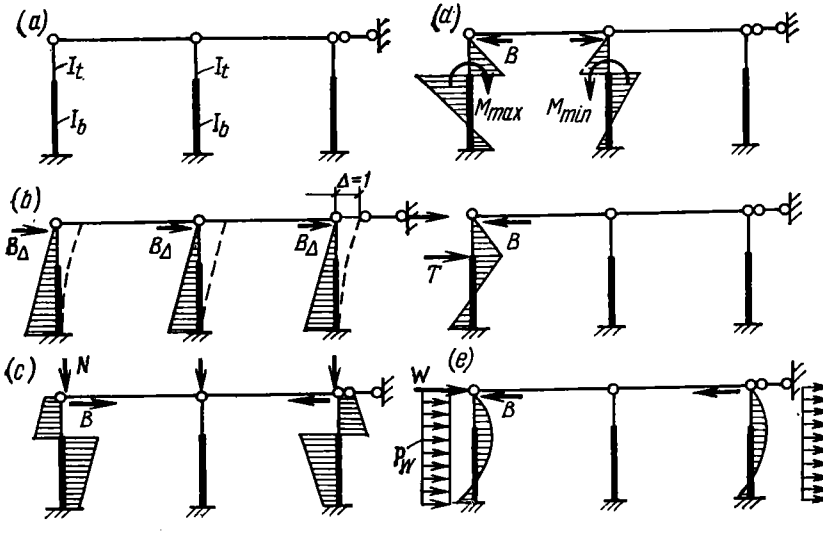


Fig. IV.22. Basic system for a transverse frame, and bending moment diagram

purposes, be treated as an independent single-bay frame (Fig. IV.23). This frame is likewise designed with allowance for the spatial interaction of the skeleton.

Bending moments and shearing forces in the cross sections of the columns are determined as in a cantilever beam subjected to an external load and reaction, B_{el}. As a rule, design calculations are done for three sections, namely section *1-0* above the crane cantilever, section *1-2* below the crane cantilever, and section *2-1* at the bottom of the column.

As a first step, moment diagrams are plotted for each type of load imposed on the frame, then a table is compiled for M, N and Q, and the design force or load combinations are chosen for the design sections of the column. Dead load is included in all combinations, and live loads in the most unfavourable ones.

According to relevant standards, live loads (such as snow, wind, and crane loads) imposed on a transverse frame are treated as short-

time loads. When a transverse frame is designed for the basic load combinations including one short-time live load, this load is taken without any reduction in magnitude. If the basic load combinations include two or three short-time live loads, the design magnitudes of these loads or respective forces are reduced, that is, multiplied by the combination factor 0.9. In such a case, one short-

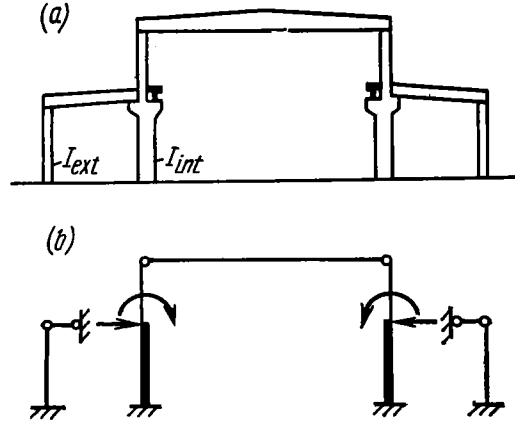


Fig. IV.23. To the design of a two-level transverse frame (a) structural system; (b) loading system

time live load is assumed to be the load imposed by two cranes on one runway, multiplied by 0.85 for light- and medium-duty cranes, or the load imposed by four cranes operating in different spans lying in the same plane, multiplied by 0.7.

The cross sections of transverse frame columns are designed with allowance for the effect that buckling has on the eccentricity of the longitudinal force. The columns are analysed for buckling as compression members. In addition, they are designed to resist loads arising in transit and during erection.

The effective length, l_0, of the top and bottom parts of prefabricated columns for buildings using overhead travelling cranes is different in and at right angles to the plane of the transverse frame, and is assigned according to Table IV.1.

The effective length of prefabricated reinforced concrete columns for buildings using no overhead travelling cranes is $l_0 = 1.5H$ for single-span buildings, and $l_0 = 1.2H$ for multi-span buildings.

The short crane-beam supporting cantilevers of columns (Fig. IV.24) are designed to resist shear subject to the condition

$$Q \leqslant k_3 k_4 R_{ten} b h^2{}_0 / a \qquad \text{(IV.24)}$$

TABLE IV.1. Effective Length l_0 of Prefabricated Reinforced Concrete Columns for Buildings Using Overhead Travelling Cranes

Load combination	Column part being designed	In plane of transverse frame	At right angles to transverse frame	
			with vertical braces	without vertical braces
Including crane load	Below crane beam	$1.5H_b$	$0.8H_b$	$1.2H_b$
	Above crane beam	$2H_t$	$1.5H_t$	$2H_t$
Excluding crane load	Below crane beam in:			
	single-span buildings	$1.5H$	$0.8H_b$	$1.2H$
	multi-span buildings	$1.2H$	$0.8H_b$	$1.2H$
	Above crane beam	$2.5H_t$	$1.5H_t$	$2H_t$

Here, the right-hand side of the expression should be not more than $2.5R_{ten}bh_0$; k_3 is taken as 1.2 for heavy concretes and 0.8 for porous-aggregate concretes; k_4 is taken as 1 for light- and medium-duty

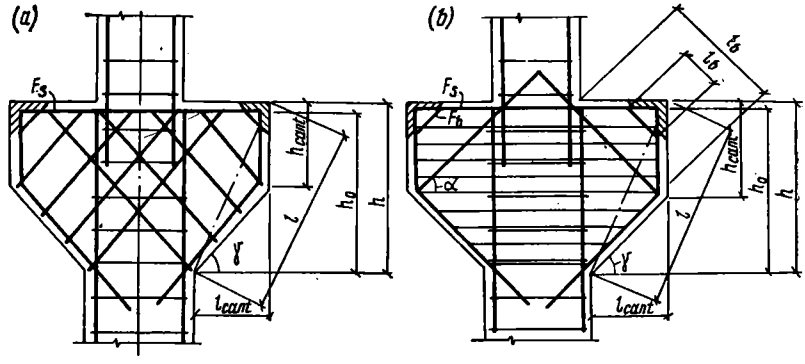


Fig. IV.24. Reinforcement of column cantilevers
(*a*) with inclined stirrups; (*b*) with horizontal stirrups and bent-up bars

cranes, 0.75 for heavy-duty cranes, and 0.5 for superheavy-duty cranes; h_0 is the effective depth of the cantilever at the support section; and a is the distance from the point of application of the shearing force, Q, to the bottom face of the column.

A cantilever is considered short if its arm is $l_{cant} \leqslant 0.9h_0$. The angle made by the compressive face of a cantilever with the horizontal should be $\gamma \leqslant 45°$, and the depth of the section at the free end should be $h_{cant} \geqslant h/2$ (where h is the depth of the support section).

When $h \leqslant 2.5a$, cantilevers are reinforced with diagonal stirrups; when $h > 2.5a$, with horizontal stirrups and bent-up bars. If $h > 3.5a$ and $Q \leqslant R_{ten} b h_0$, then bent-up bars may be omitted. In all cases, the distance between stirrups should not be more than 150 mm, nor more than $h/4$; the diameter of bent-up bars should be not more than 1/15 of the length of the diagonal part, nor more than 25 mm. The total cross-sectional area of the bent-up bars and diagonal stirrups crossing the top half of the line connecting the extreme points within the cantilever arm (see Fig. IV.24) should be not less than $0.002bh_0$ or

$$F_b + F_{st} = (Q - Q_c)/R_{s,\,st} \sin \alpha \qquad \text{(IV.25)}$$

whichever is the smaller. Here, determining Q_c, take $c = a + h_0/3$.

The cross-sectional area, F_s, of the longitudinal reinforcement in the cantilever is taken to carry the bending moment arising at the point where the cantilever joins the column, increased by 25%. The longitudinal reinforcing steel has anchors in the form of welded-on washers or angles.

4. Determining Forces in Two-Piece Columns

A transverse frame using open-work two-piece columns with allowance for the spatial interaction of the skeleton is designed in

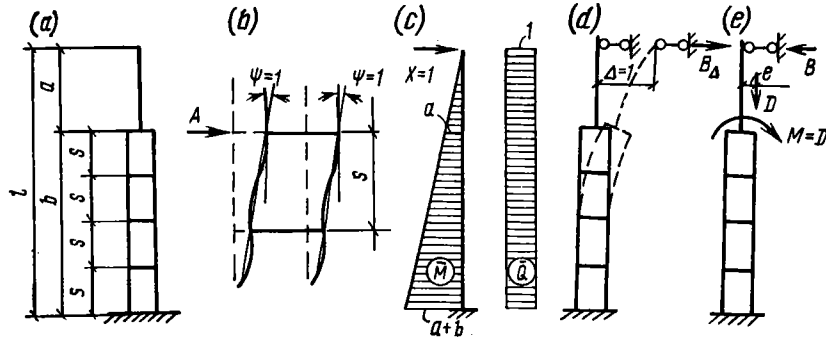


Fig. IV.25. Loading systems for two-piece columns

much the same way as that with solid single-piece columns. A two-piece column may be treated as a multi-storey single-bay frame (column frame), where the centre-to-centre distance between the side members is c, the centre-to-centre distance between the spacers is s, the number of panels is n, the length of the bottom frame part is b, the length of the top solid part is a, and the total length is l (Fig. IV.25*a*). Since the girders of the column frame are the short

rigid spacers, and the struts are the less rigid side members, the deformation of the girders may be neglected, and the girders themselves may quite accurately be deemed perfectly rigid. As has been proved experimentally, an alternative design system in which the girders are assumed to be elastic only slightly improves the accuracy of design. To determine the reactions, B, with the top of the column fixed, the two-piece column is treated as a rod of flexural rigidity, E_cI_b, and of finite shear rigidity, A. The shear rigidity of a two-piece column depends on the local bending of the side members. Numerically, it is equal to the force which causes a unit angular deflection of the columns (Fig. IV.25*b*):

$$A = 24E_cI/s^2 \tag{IV.26}$$

where I is the moment of inertia of a side member.

Let us apply a force, $X = 1$, to the upper end of the rod taken for the time being without its top support (Fig. IV.25*c*). The force produces the following deflection

$$\delta_{11} = \sum \int (\overline{M}^2/E_cI_i)\,dx + \int_0^b (\overline{Q}^2/A)\,dx = a^3/3E_cI_t + (l^3 - a^3)/3E_cI_b + ns^3/24E_cI \tag{IV.27}$$

where $I_i = I_b$ in the bottom part of the column, and $I_i = I_t$ in the top part of the column.

Hence, the reaction of the upper end of the column to the deflection, $\Delta = 1$ (Fig. IV.25*d*), is

$$B_\Delta = 1/\delta_{11} = 3E_cI_b/l^3\,(1 + k + k_1) \tag{IV.28}$$

where

$$k = \alpha^3\,(I_b/I_t - 1); \quad k_1 = (1 - \alpha)^3\,I_b/8n^2I; \quad \alpha = a/l \tag{IV.29}$$

I_t is the moment of inertia of the top part of the column; F is the cross-sectional area of the side member; and $I_b = Fc^2/2$ is the moment of inertia of the bottom part of the column (here, $2I$ is neglected as relatively small).

Assume that a two-piece column is subjected to a crane moment, M. The deflection caused by this moment is

$$\delta_{1p} = \int (\overline{M}M/E_cI_b)\,dx = Ml^2\,(1 - \alpha^2)/2E_cI_b$$

The reaction, B, with the top support of the two-piece column fixed (Fig. IV.25*e*) is

$$B = \delta_{1p}/\delta_{11} = 3M\,(1 - \alpha^2)/2l\,(1 + k + k_1) \tag{IV.30}$$

Here, the "minus" sign is dropped.

The reaction, B, of the two-piece column to the other loads given in Appendix XII can be found in a similar manner.

The expression for reactions, B, can be applied to stepped solid columns with $k_1 = 0$ and uniform-section solid columns with $k_1 = k = 0$, as well as to two-piece columns. When necessary, these expressions may be resolved for deflections, $\delta_{11} = l/B_\Delta$ and $\delta_{1p} = B/B_\Delta$. They may also be used in the design of a frame with allowance for elastic fixation of the column in the footing.

When a frame is designed for the effect of temperature variations, Δt, the allowance for the actual compliance of the column fixation in the footing (and also the actual rigidity of the column in cracked portions) results in a reduced bending moment. The reaction to the angular deflection of the column at the bottom section through $\varphi = 1$ is

$$B_\varphi = l/\delta_{11} = 3E_c I_b/l^2 (1 + k + k_1) \quad \text{(IV.31)}$$

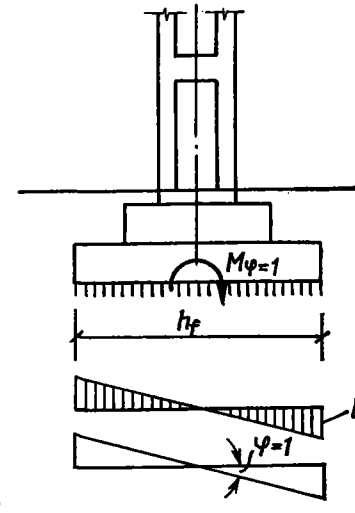


Fig. IV.26. To the design of the column fixation compliance

Let us determine the reactive moment brought about by the angular deflection of the footing through $\varphi=1$ (Fig. IV.26). For a footing with plan dimensions, h_f and b_f, the settlement at the edge is $y = 0.5h_f \operatorname{tg} \varphi = 0.5h_f$ (the deformation within the footing is neglected). The edge pressure of the footing is

$$p = C_\varphi y = 0.5C_\varphi h_f$$

where C_φ is the modulus of subgrade reaction for a nonuniformly compressed foundation material (see Chapter III).

The reactive moment brought about by the angular deflection of the footing is

$$M_{\varphi=1} = C_\varphi b_f h_f^3/12 = C_\varphi I_f \quad \text{(IV.32)}$$

where $C_\varphi I_f$ is the angular rigidity of the footing.

Example IV.1. *Given*: a two-piece column; $l = 17.6$ m; $a = 5.2$ m; $b = 12.4$ m; $s = 2.07$ m; $c = 1$ m; $n = 6$; $I_t = 13.8$; $I = 1$; and $F = 192$.
To find: The reaction, B_Δ.
Solution. Determine the following quantities

$I_b = Fc^3/2 = (192 \times 1^3)/2 = 96$

$\alpha = a/l = 5.2/17.6 = 0.296$

$k = \alpha^3 (I_b/I_t - 1) = 0.296^3 (96/13.8 - 1) = 0.16$

$k_1 = (1 - \alpha)^3 I_b/8n^2 I = [(1 - 0.296)^3 96]/(8 \times 6^2 \times 1) = 0.12$

Calculate the reaction

$$B_\Delta = 3E_c I_b/l^2 (1 + k + k_1)$$
$$= (3 \times 96E_c)/[17.6^3 (1 + 0.16 + 0.12)] = 4.1 \times 10^{-2} E_c$$

Example IV.2. *Given*: the two-piece column of Example IV.1; wind load intensity is equal to p.
To find: The reaction of the column.
Solution. Calculate the reaction

$$B = 3pl [1 + \alpha k + 1.33 (1 + \alpha) k_1]/8 (1 + k + k_1) = 6.4p$$

Example IV.3. *Given*: a stepped solid column; crane moment, $M = 500$ kN m; $l = 11.1$ m; $a = 3.85$ m; $b = 7.25$ m; $I_t = 1$; and $I_b = 8$.
To find: The reaction of the column.
Solution. Determine the following quantities: $\alpha = 0.35$ and $k = 0.3$. On putting $k_1 = 0$, calculate the reaction

$$B = 3M (1 - \alpha^2)/2l (1 + k)$$
$$= [3 \times 500 (1 - 0.35^2)]/[2 \times 11.1 (1 + 0.3)] = 45 \text{ kN}$$

Example IV.4. *Given*: a solid single-piece column of uniform cross section; $l = 11.1$ m; crane moment $M = 500$ kN m applied at a distance $a = 3.85$ m.
To find: The reaction of the column.
Solution. On finding $\alpha = 0.35$ and putting $k = k_1 = 0$, calculate the reaction

$$B = 3M (1 - \alpha^2)/2l = [3 \times 500 (1 - 0.35^2)]/(2 \times 11.1) = 58.5 \text{ kN}$$

Once the elastic reactions, B_{el}, are found, the forces, M, N and Q, in the design sections are computed about the geometrical axis of the two-piece column. The forces in the side members and spacers are found later, when choosing the required cross-sectional areas for these members.

The longitudinal forces in the side members are

$$N_{side} = N/2 \pm M\eta/c \qquad \text{(IV.33)}$$

where M and N are the design forces about the axis of the two-piece column; and $\eta = -1/(1 - N/N_{cr})$ is the buckling coefficient.

When finding the value of η, it is necessary to take into account the effect of the slenderness of the side members in the plane of flexure, treating the two-piece column as a composite section (Fig. IV.27*a*). The reduced radius of gyration, r_g^2, depends on the radius of gyration of the bottom part of the column, $r^2_b = c^2/4$, and the radius of gyration of the side member, $r^2 = h^2/12$.

The reduced slenderness of the column should satisfy the following condition

$$\lambda_g^2 = \lambda_b^2 + \lambda^2$$

or, with $l_0 = \psi H_b$,

$$l_0^2/r_g^2 = l_0^2/r_b^2 + s^2/r^2 \qquad \text{(IV.34)}$$

Substituting for r_b^2 and r^2, and dividing through the above equation by l_0^2 gives

$$1/r_g^2 = 4/c^2 + 12/\psi^2 n^2 h^2$$

Hence,

$$r_g^2 = c^2/4\,(1 + 3c^2/\psi^2 n^2 h^2) \qquad \text{(IV.35)}$$

where $n = H/s$ is the number of panels in the two-piece column.

In accordance with Eq. (IV.14) (see Volume 1), the assumed critical force may be written as

$$N_{cr} = 12.8\,E_c F\,(r_g/l_0)^2\,\{[0.11/(0.1 + t) + 0.1]/k_l + \mu_1 n\} \qquad \text{(IV.36)}$$

where F and μ_1 are the cross-sectional area and the reinforcement ratio of the side member, respectively.

The value of t is determined by Eqs. (IV.17*a*) and (IV.17*b*) (see Volume 1). The factor k_l is given by

$$k_l = 1 + \beta M_l/M$$

Here, the values of M and M_l are found about the axis of the side member.

The bending moment in the side members, with the zero-moment point lying in the middle of the panel height (Fig. IV.27*b*), is found as

$$M = Qs/4 \qquad \text{(IV.37)}$$

The bending moment in a spacer is found as the sum of the side-member moments at the joint

$$M_{sp} = Qs/2 \qquad \text{(IV.38)}$$

The shearing force in the spacer is

$$Q_{sp} = Qs/c \qquad \text{(IV.39)}$$

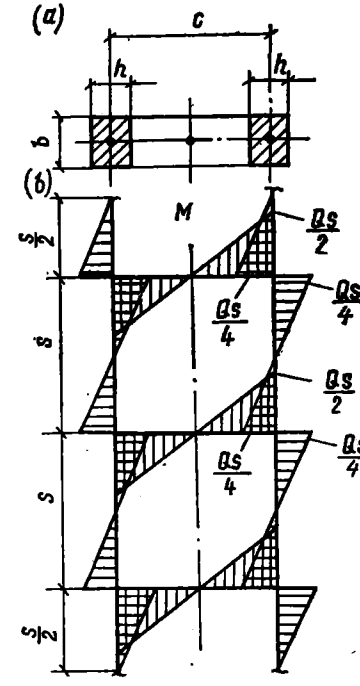


Fig. IV.27. To determining the forces in the side members and spacers of a two-piece column

If, when determining the longitudinal force by Eq. (IV.33), one of the side members is found to be in tension, the two-piece column should be designed with allowance for the reduced rigidity of this side member. In this case, the bending moments in the compressed side member and spacers are found on the assumption that all of the shearing force applied to the two-piece column is taken up by the side member in compression.

IV.3 ROOF CONSTRUCTION

1. Roof Slabs

No-purlin roofs use large ribbed slabs 3 × 12 m and 3 × 6 m in plan. They are directly supported on the girders of the transverse frames. Smaller slabs 1.5 × 12 m and 1.5 × 6 m in plan are used as supplementary members. These are installed near monitors to carry increased snow load, and also near the folds in the roof configuration. Roofs built with purlins use much smaller slabs (3 × × 0.5 m and 1.5 × 0.5 m in plan). In this case, slabs are supported by reinforced concrete purlins which, in turn, are carried by the girders of the transverse frames. No-purlin roofs are better adapted to the preassembly of members into larger elements, which helps to reduce the number of units to be handled during erection. Because of this, this type of roof is most commonly used for one-storey buildings of skeleton construction.

Ribbed slabs 3 × 12 m in plan, which have been adopted in the USSR as standard, have longitudinal ribs 100 × 450 mm in cross section, transverse ribs 40 × 150 mm in cross section, a flange 25 mm deep and angle brackets which strengthen the slabs against repetitive lateral crane thrust (Fig. IV.28). The longitudinal ribs are reinforced with prestressed bars or wire strands; the transverse ribs and flanges are reinforced with welded-bar mats and welded-wire fabric. The slabs are made of M-400 or M-500 concrete. Ribbed slabs 3 × 6 m in plan, which have also been adopted in the USSR as standard products, have longitudinal and transverse ribs and are reinforced with prestressed steel.

Double-T slabs 3 × 12 m and 3 × 6 m in plan have longitudinal ribs spaced 1.5 m apart and overhanging flanges (Fig. IV.29*a* and *b*). Since bending moments in such slabs are small, no transverse ribs are provided, which makes the slabs easier to manufacture. In 3 × 12 m slabs, the prestressed longitudinal ribs are fabricated first, and the flanges are concreted afterwards. The ribs are connected to the flange by reinforcement stick-outs and monolithic concrete. Because the components of double-T slabs are fabricated separately, the concrete brand number for the flanges may be reduced to M-200. Slabs 3 × 6 m in plan may be assembled from precast components or fabricated as one-piece units.

The design procedure for ribbed slabs has been discussed in Chapter II. The main parameters of roof slabs are given in Table IV.2.

Large-size slabs 3 × 18 m and 3 × 24 m in plan, supported on girders 6 or 12 m long, are used in ridge and low-pitched roofs (Fig. IV.30). Here, the double-T slabs consist of longitudinal ribs with a slope of 1:12 in the top chord, and a flange varying from 25 to 60 mm in depth. Large-size reinforced concrete barrel roof slabs

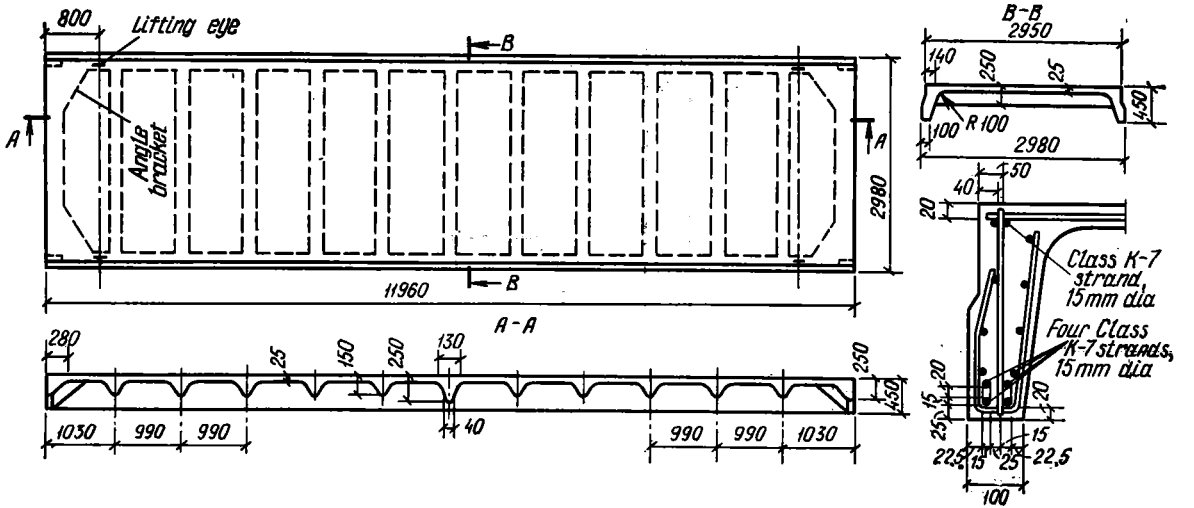


Fig. IV.28. Ribbed roof slab 3 × 12 m in plan

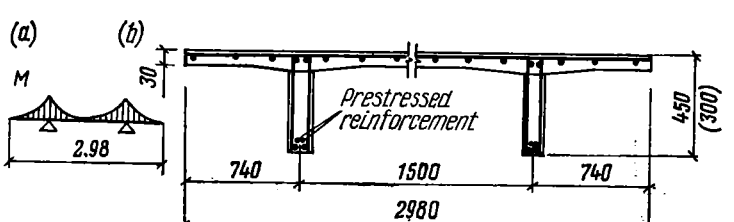


Fig. IV.29. Double-T roof slab

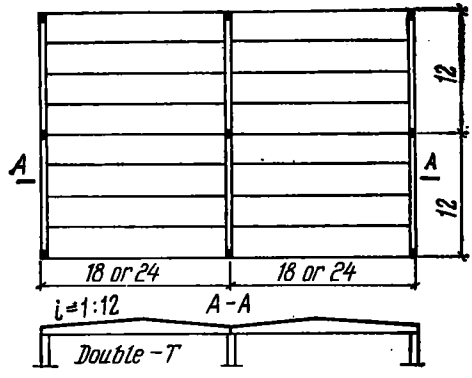


Fig. IV.30. Roof using ridged double-T slabs 3 × 18 m in plan

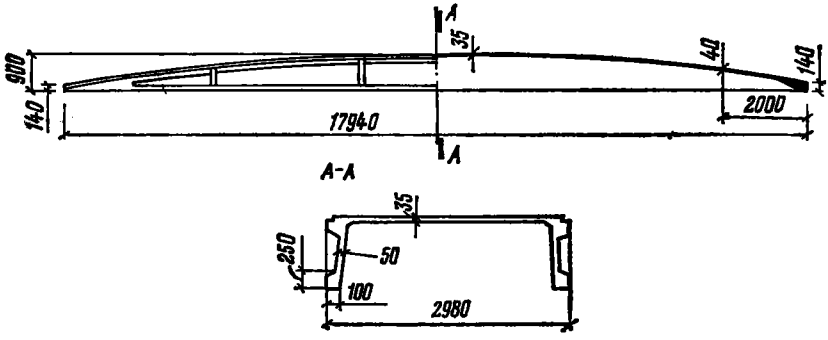


Fig. IV.31. Example of a large-size reinforced concrete barrel slab 3 × 18 m in plan

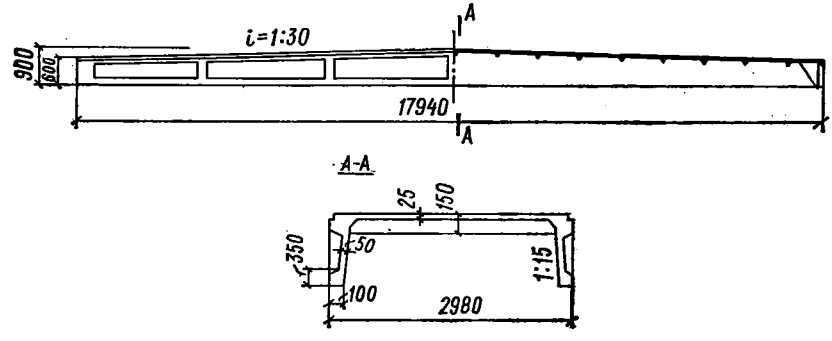


Fig. IV.32. Example of a low-pitched ribbed roof slab 3 × 18 m in plan

TABLE IV.2. Particulars of Roof Slabs

Slab type	Weight, tonnes	Concrete brand number	Reduced concrete depth, cm	Amount of steel per slab (in kg). Longitudinal ribs reinforced with	
				bars	wire strands or high-strength wire
Ribbed, 3×12 m	6.8	400, 500	7.65	265-391	205-288
Same, 3×6 m	2.38	300, 400	5.3	70-101	56-70
Double-T, 3×12 m	6.8	500 (200 for flange)	7.65	330	237
Same, 3×6 m	2.38	300	5.3	85	63
Ribbed, low-pitched, 3×18 m	12.2	500	8.98	—	581
Barrel, 3×18 m	10.9	500	8.03	—	431
Ridge, 3×18 m	15.1	500	11.2	—	382

have curved trapezoidal longitudinal ribs widened at the top and bottom, and a smooth flange 40 to 50 mm deep in the midspan and 140 to 160 mm deep at the supports (Fig. IV.31). Ribbed slabs intended for low-pitched roofs consist of trapezoidal longitudinal ribs with the upper chord sloping at 1:20 or 1:30, transverse ribs spaced 1 000 mm apart, and a flange 25 mm deep (Fig. IV.32).

Ribbed low-pitched slabs are somewhat inferior to large-size barrel slabs. Their advantage, however, is that the installation of low-pitched roofs can highly be mechanized, which is not the case with curved barrel roofs.

2. Roof Beams

In the USSR, the standard spans of roof beams are 12 and 18 m, although in some structures the figure may reach 24 m. In ridge roofs, the top chord of the beams may be constant-slope trapezoidal, broken or curved (Fig. IV.33*a* through *c*). The beams used in shed roofs have parallel chords or broken bottom chords, those used in flat roofs have parallel chords (Fig. IV.33*d* through *f*). Roof beams are available for bays of 6 or 12 m.

The most economical cross section of roof beams is the I-section. In the USSR, they are made with a web 60 to 100 mm wide. This width is chosen because it provides enough space for reinforcing cages and ensures adequate strength and crack resistance. At the supports, the web gradually widens to form stiffening ribs. In the midspan where shearing forces are small, the web may have round or polygonal holes made to reduce the amount of concrete, and to provide room for the wiring and other services.

In the midspan, the beam depth is taken as $1/10l$ to $1/15l$. The depth of a trapezoidal ridge beam in the midspan is governed by the slope of the top chord (1:12) and the standard depth at the support (800 or 900 mm). Owing to a somewhat greater slope of the top chord in the marginal quarter of the span, the depth of beams with

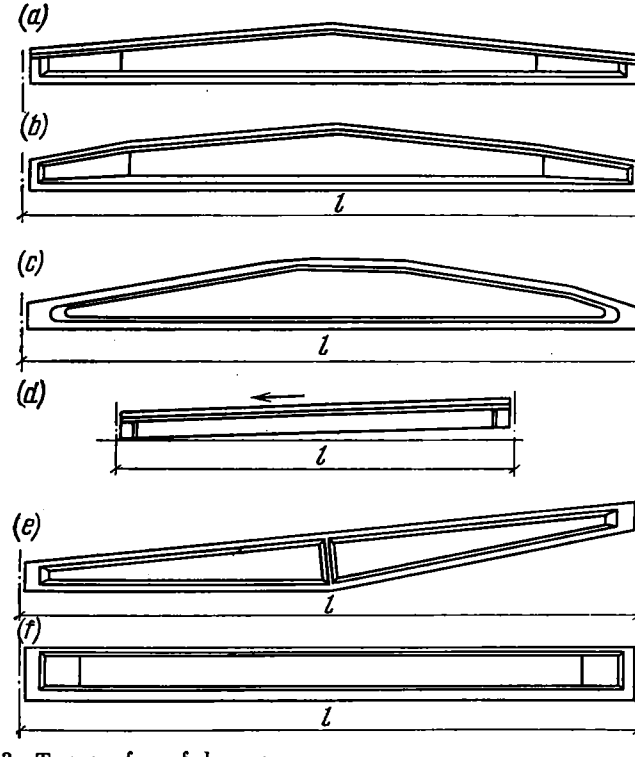


Fig. IV.33. Types of roof beams

a polygonal top chord is larger, with the depth at the support remaining standard. The configuration of roof beams with the curved top chord is close to that of the bending moment diagram; theoretically, such beams are more advantageous because they require less materials. This, however, is offset by the increased cost of manufacture due to the more complex shape.

To provide adequate stability in transit and during erection, the width of the top compression flange of a beam is taken as $1/50l$ to $1/60l$. The width of the bottom flange is taken as 250 to 300 mm for ease of installation of the longitudinal tensile steel.

Ridge beams are made of M-300 to M-500 concrete and reinforced with prestressed wire, bars, or wire strands (Fig. IV.34). When

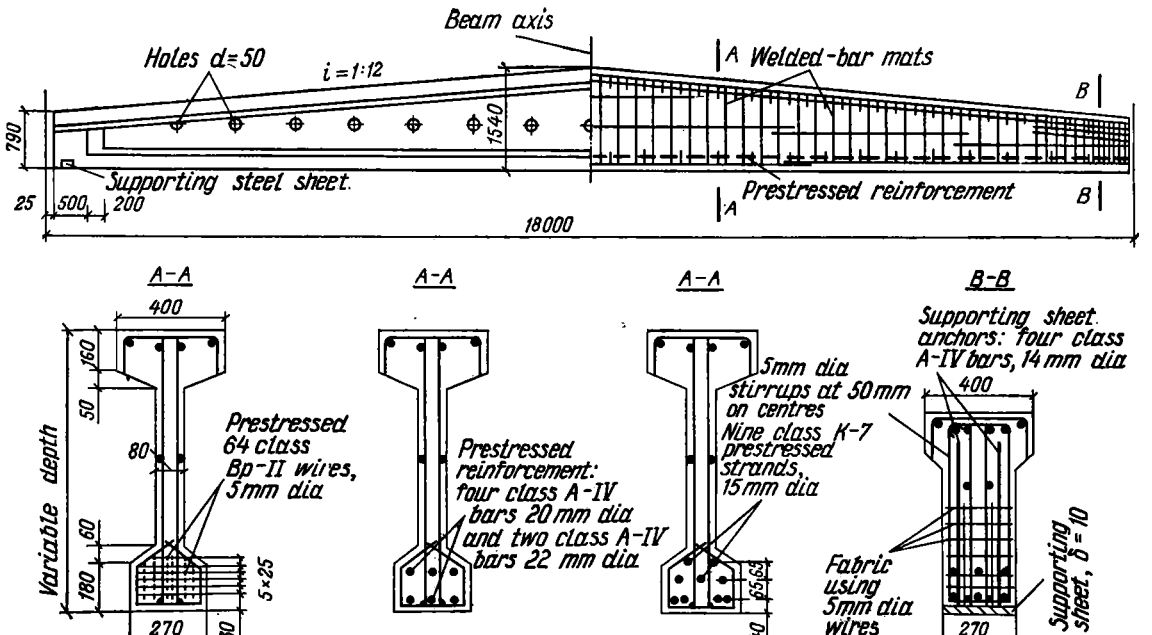


Fig. IV.34. 18-m I-section ridge roof beam

high-strength wire is used as reinforcement, it is distributed in groups of two wires along the depth of the beam, so that it is convenient to concrete the beam in a vertical position. The web reinforcement consists of welded-bar mats whose longitudinal bars serve as distribution reinforcement, whereas the transverse bars are intended to take up shearing forces. To prevent them from longitudinal cracking (or to limit the width of these cracks) when the prestress is transferred to the concrete, the beam portions at

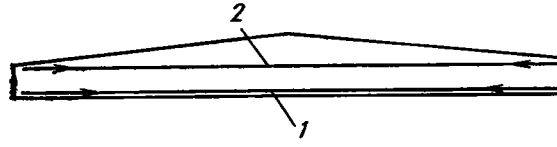


Fig. IV.35. Disposition of prestressed reinforcement in a ridge beam
1—bottom reinforcement; *2*—top reinforcement

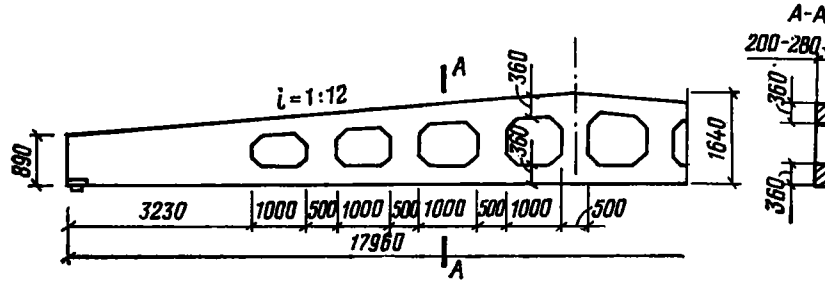


Fig. IV.36. 18-m rectangular lattice ridge beam

the supports are additionally reinforced with transverse bars welded to embedded steel parts. Crack resistance can also be enhanced by biaxial prestressing, in which case, the transverse bars are prestressed, too.

To limit the width of cracks occurring in the top zone of ridge I-beams when the prestress is transferred to the concrete, additional prestressed steel may be provided at the top of the support sections (Fig. IV.35). This reduces the eccentricity of the prestressing force and the preliminary tensile stresses in the concrete of the compression zone.

Rectangular ridge beams with closely spaced holes may arbitrarily be called lattice beams (Fig. IV.36). According to the design load to be carried, the width of rectangular lattice beams may be taken equal to 200, 240 or 280 mm. Beams of all types have steel parts embedded in their top chords for connection to roof slabs.

For design purposes, roof beams are treated as simply supported beams, with the load from the roof slabs transmitted through their ribs. If the number of concentrated forces is five or more, the load is assumed to be uniformly distributed over the beam. In ridge beams, the design section is that lying within a distance, x, of the support. For example, with the top chord sloping at 1:12 and a beam depth of $h = l : 12$ in the midspan, the beam depth at the support is $h_{sup} = l : 24$, and that within a distance, x, of the support is

$$h_x = (l + 2x)/24$$

Assume that the effective depth of the beam is $h_0 = \beta h_x$, and the bending moment due to the uniformly distributed load is

$$M_x = qx\,(l - x)/2$$

Then, the required cross-sectional area of the longitudinal reinforcement will be

$$F_{sx} = M_x/R_s\eta h_0 = 12qx\,(l - x)/R_s\eta\beta\,(l + 2x)$$

The design section for this beam will be that in which F_{sx} is a maximum. To find this section, we equate the derivative of F_{sx} to zero

$$dF_{sx}/dx = 0$$

Differentiating the above equation at $\eta\beta = \text{const}$ gives

$$2x^2 + 2xl - l^2 = 0$$

Solving this quadratic equation yields $x = 0.37l$. In the general case, the distance from the support to the design section is $x = 0.35l$ to $0.4l$.

In a monitor-type building, the design section may be chosen beneath a monitor strut.

The amount of transverse reinforcement to be used is found by inclined-section analysis which is then followed by analysis for cracking resistance and sagging, and also for strength and cracking resistance in transit and during erection. When determining sags in trapezoidal beams, it should be kept in mind that their stiffness varies with length.

At present, the optimum designs of roof beams are found by computers using suitable software.

The particulars of ridge roof beams according to their cross section and type of prestressed steel are given in Table IV.3.

Referring to the table, in I-beams, the amount of reinforcement is by about 15%, and the amount of concrete is by about 13% less than in lattice beams.

TABLE IV.3. Particulars of 18-m Ridge Roof Beams Spaced 6 m Apart and Subjected to Design Load of 3.5 to 5.5 kN/m²

Beam type	Weight, tonnes	Concrete brand number	Volume of concrete, m³	Total amount of steel per beam, kg
I-section with:				
prestressed bars	9.1	300, 400, 500	3.64	468-738
prestressed wire strands	9.1	400, 500	3.64	360-565
prestressed wire	9.1	300, 400, 500	3.64	359-552
Lattice with:				
prestressed bars	8.5-12.1	400, 500	3.4-4.84	530-875
prestressed wire strands	8.5-12.1	400, 500	3.4-4.84	418-662
prestressed wire	8.5-12.1	400, 500	3.4-4.84	397-644

If roof beams are to carry overhead cranes and loads, their amount of steel should be increased by 20 to 30%.

3. Trusses

Reinforced concrete trusses are used for spans of 18, 24 and 30 m, and for bays of 6 or 12 m. In contrast to steel trusses, the amount of steel required by reinforced concrete trusses is almost halved,

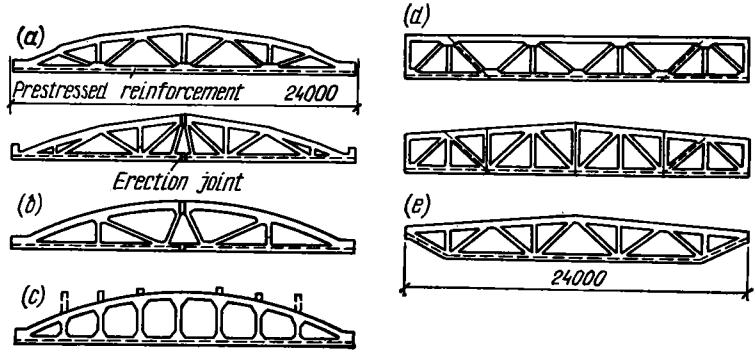


Fig. IV.37. Types of reinforced concrete trusses

but the labour consumption and the cost of manufacture are somewhat higher. For spans of 36 m and more, use is usually made of steel trusses. In practice, however, reinforced concrete trusses may be used for spans of about 60 m and more.

Ridge, low-pitched and flat roofs use reinforced concrete trusses differing in chord and lattice configuration. The main types of trusses are the camel-back truss with a polygonal top chord and linear portions between the joints (Fig. IV.37*a*); the bowstring truss

with widely spaced web members and a curved top chord (Fig. IV.37*b*); the bowstring truss without diagonal web members, with rigid joints between the chords and the struts, and a curved top chord (Fig. IV.37*c*); the Warren truss (Fig. IV.37*d*); the quadrangular truss (Fig. IV.37*d*); and the polygonal truss with a polygonal bottom chord (Fig. IV.37*e*).

Whatever the type of truss, the depth at the midspan is usually taken as 1/7 to 1/9 of their span. The top chord panels of all trusses, except the bowstring type with diagonal web members, are design-

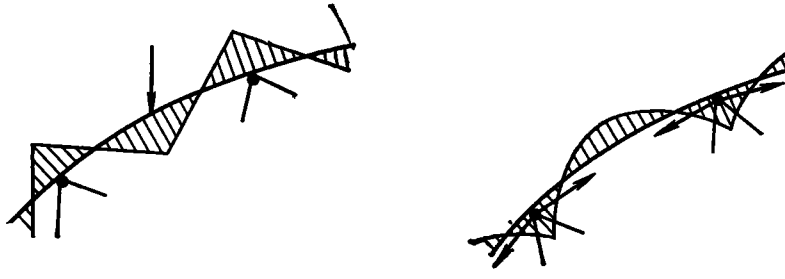


Fig. IV.38. Bending moment diagrams for the top chord of a bowstring truss

ed to be 3 m long in order that all of the roof slab load can be transmitted directly to the joints, thereby preventing the chord members from buckling. The bottom tensile chords of all trusses and the tensile diagonals of some trusses are prestressed, using the pretensioning technique.

As far as their statical behaviour is concerned, the camel-back and bowstring types are best because the shape of their top chord is almost a mirror-image of the bending moment diagram. The web members of such trusses carry insignificant loads, and the trusses are relatively shallow at the supports, which fact serves to reduce their weight and the height of the exterior walls. In bowstring trusses with web diagonals, the bending moments induced by the load applied to the top chord between the joints are reduced because of the eccentricity of the longitudinal force which gives rise to a reverse moment. Owing to this, the distance between the joints in the top chord may be increased, and the web members may be spaced wider apart (Fig. IV.38). In bowstring trusses without web diagonals, the bending moments arising in the struts and chord members are rather large. Because of this, they require additional

reinforcement to enhance their strength and crack-resistance. Nevertheless, such trusses are easier to fabricate and more convenient to install in buildings with low-pitched or flat roofs. Also, if additional struts are made above the top chord, they provide more room for services. Polygonal trusses with a broken bottom chord are more stable during erection and do not require additional bracing because their centre of gravity lies below the support level.

The cross-sectional area and the weight of all trusses may be reduced by increasing the concrete brand number (M-400 through M-600) and the percentage of chord reinforcement.

Trusses should preferably be fabricated in one piece, because their fabrication as half-trusses to be assembled later during erection increases the cost of construction. Trusses for a span of 18 m are fabricated in one piece, those for a span of 24 m may be made either in one piece or as two half-trusses, and those for a span of 30 m are manufactured as two half-trusses only. The web of a half-truss should be designed so that the bottom chord members could be connected between the joints (see Fig. IV.37*a*), as this makes the erection easier. To strengthen the bottom chord connection during erection, use is made of additional diagonals which are not taken into account in design calculations.

Trusses can be manufactured in two ways. The first method consists in that web members having reinforcement stick-outs are fabricated in advance and then embedded for a distance of 30 to 50 mm into the joints, and the chords are concreted. The second approach consists in that the chords and the web members are concreted simultaneously. The latter technique is used on a wider scale because in the former case the web members must be narrower than the chords; in the latter case, the chords and the web members may be of the same width.

For simplicity of manufacture, the top and bottom chords of a truss are made of the same width. The chord width is taken as 200 to 250 mm in trusses for a bay of 6 m, and 300 to 350 mm for a bay of 12 m.

The bottom tensile chord should be reinforced so as to provide enough space between the prestressed bars, wire strands and paired wires for the concrete mix to be poured and compacted. All of the tensile steel should be embraced by closed stirrups spaced 500 mm apart and installed as may be dictated by structural reasons.

The top compression chord and the web members are reinforced with nonprestressed welded-bar mats. If they are to resist great forces, the tensile web members are prestressed.

In order that forces can be transmitted from one member to another the joints of reinforced concrete trusses are angle bracketed, as this enhances the disposition and anchorage of the web reinforcement (Fig. IV.39). The truss joints are reinforced with closed bent bars

10 to 18 mm in diameter and vertical transverse bars 6 to 10 mm in diameter spaced 100 mm apart and combined into welded mats. The web reinforcement is extended into the joints, and the tensile bars are anchored by short bars, hooks, and button or rivet heads. The reinforcing steel is checked for security of anchorage by special analysis.

The joints at the supports are additionally reinforced with nonprestressed longitudinal steel and transverse bars which contribute to the anchorage of the tensile steel in the bottom chord and to the

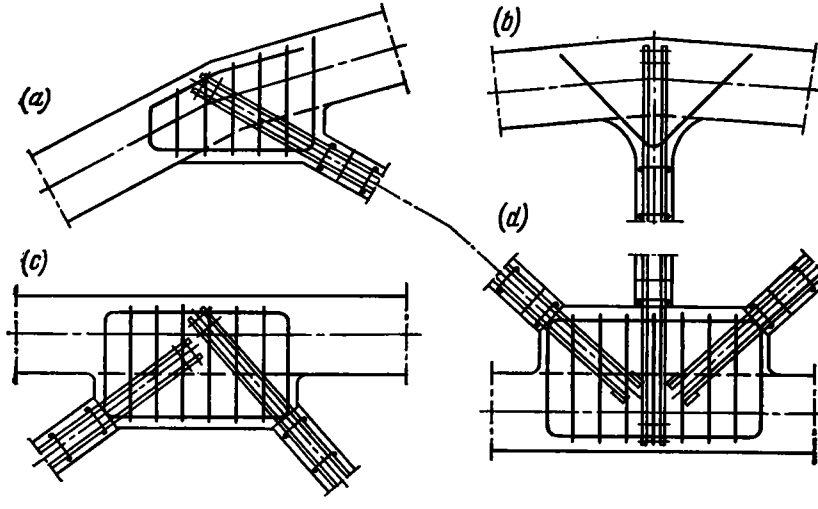


Fig. IV.39. Reinforcement of truss joints
(*a*), (*b*), (*c*) top chord; (*d*) bottom chord

shear strength of the joint. In addition, in order to prevent the chord from longitudinal cracking when the prestress is transferred to the concrete, use is made of transverse bars and welded-wire fabric welded to the steel parts embedded at the supports.

A likely reinforcement for a 24-m camel back truss is given in Fig. IV.40. Here, the prestressed reinforcement of the bottom chord consists of class K-7 wire strands, class A-IV bars and class Bp-II high-strength wire. Pretensioning is used throughout. The stirrups in the bottom chord are made of U-shaped fabric pieces embracing the prestressed steel. The joint at the support contains additional 12-mm-diameter nonprestressed longitudinal bars extending into the adjacent bottom chord panel, and transverse bars 10 mm in diameter.

The leading particulars of various truss types are given in Table IV.4.

TABLE IV.4. Leading Particulars of Roof Trusses Subjected to Design Load of 3.5 to 5.5 kN/m²

Type of truss	Truss weight, tonnes	Concrete brand number	Volume of concrete, m³	Amount of steel per truss (kg) with tensile chord reinforced with		
				bars	wire strands	high-strength wire
Camel-back, with diagonals, 18-m span:						
6-m bay	4.5-6	400	1.8-2.42	289-468	238-391	223-372
12-m bay	7.8-9.4	400-500	3.11-3.75	550-736	439-591	408-547
Camel-back, without diagonals, 18-m span:						
6-m bay	6.5	400-500	2.6	390-486	330-450	319-436
12-m bay	9.2-10.5	400-500	3.7-4.2	570-720	463-586	450-562
Camel-back, with diagonals, 24-m span:						
6-m bay	9.2	400-500	3.68	690-768	557-625	510-595
12-m bay	14.9-18.6	400	5.94-7.42	1 096-1 539	853-1 204	787-1 128
Camel-back, without diagonals, 24-m span:						
6-m bay	9.2-10.5	400-500	3.7-4.2	759-862	654-715	623-697
12-m bay	14.2-18.2	400	5.7-7.8	1 281-1 489	1 020-1 201	988-1 128

Referring to the table, camel-back trusses with diagonals take about 10% less steel and about 12% less concrete than camel-back trusses without diagonals.

If the trusses are designed to carry overhead travelling cranes, the amount of steel per truss should be increased by 20 to 30%.

Roof trusses are designed to resist dead loads (roof weight, truss weight and crane weight) and various live loads. The roof weight is assumed to be applied at the joints of the top chord, and the loads

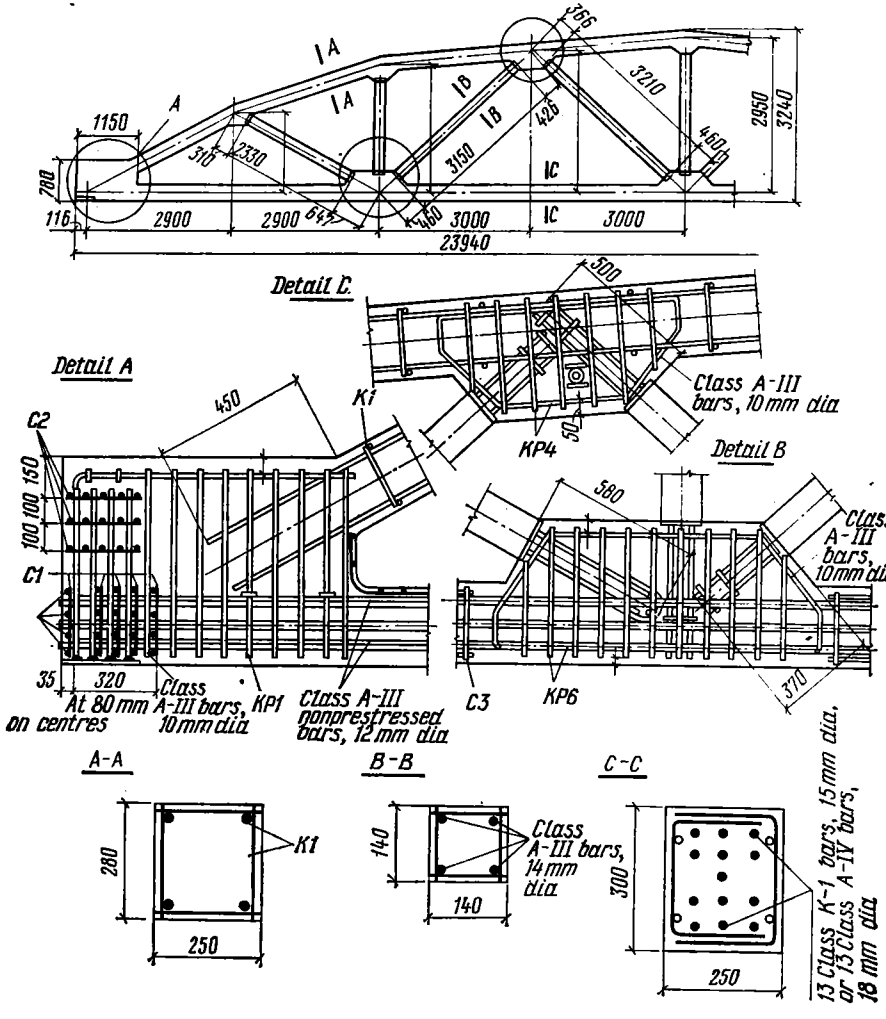

Detail C
Detail A
Detail B
Class A-III bars, 10 mm dia
Class A-III bars, 10 mm dia
At 80 mm on centres
Class A-III bars, 10 mm dia
Class A-III nonprestressed bars, 12 mm dia
A-A
B-B
C-C
Class A-III bars, 14 mm dia
13 Class K-1 bars, 15 mm dia, or 13 Class A-IV bars, 18 mm dia

Fig. IV.40. 24-m camel back reinforced concrete truss

due to overhead travelling cranes at the joints of the bottom chord. When designing a truss, it should be taken into account that snow load is nonuniformly distributed over the roof and near the monitors. Design should also be carried out for the case when one half of a

truss is subjected to snow and crane loads, which is unfavourable for the web members.

In the stress analysis of a truss, its chords and web members are assumed to be pin-connected at the joints. The rigidity of the joints has little effect on the forces in the truss members and may be neglected. When determining the bending moments due to a load applied between the joints, the top chord is treated as a continuous beam supported at the joints.

The strength analysis of cross sections for the chords and web members is carried out using the equations derived in Volume 1 for compression and tension members. It should be kept in mind that the effective length of compression members in and across the truss plane is different (Table IV.5).

TABLE IV.5. Effective Length l_0 of Compression Truss Members

Member	Effective length
Compression top chord in plane of truss:	
with $e_0 < (1/8)\, h_{ch}$	$0.9l$
with $e_0 \geqslant (1/8)\, h_{ch}$	$0.8l$
Compression top chord perpendicular to plane of truss:	
for portion under monitor 12 m or more wide	$0.8l$
in other cases	$0.9l$
Compression diagonals and struts in and perpendicular to plane of truss:	
with $b_{ch}/b_{st} < 1.5$	$0.9l$
with $b_{ch}/b_{st} \geqslant 1.5$	$0.8l$

Note. In the table, l is the centre-to-centre distance between adjacent joints; e_0 is the eccentricity of the longitudinal force; h_{ch} is the cross-sectional depth of the top chord; b_{ch} is the cross-sectional width of the top chord; and b_{st} is the cross-sectional width of the strut.

Experiments have shown that the reinforcement for a joint at a support may be designed using the loading system of Fig. IV.41*a*. It is assumed that the decrease in the design bearing capacity of the prestressed steel due to inadequate anchorage in the joint is made up for by the additional longitudinal nonprestressed reinforcement and transverse bars working in tension. The cross-sectional area of the longitudinal nonprestressed steel is given by

$$F_s = 0.2N/R_s \qquad \text{(IV.40)}$$

where N is the design force in the panel adjacent to the support.

Let us resolve the total force, N_{tr}, acting in the transverse bars normal to the axis within portion l_2 (from the face of the support to the internal face of the support joint) into a horizontal ($N_{tr}\text{ctg}\,\alpha$)

and a diagonal component (here, α is the angle which the horizontal makes with the line AB joining point A at the face of the support

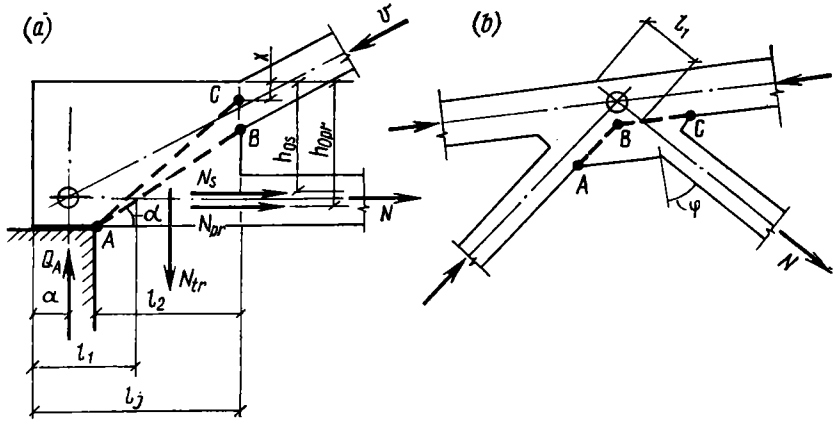


Fig. IV.41. To the design of truss joints
(*a*) support joint; (*b*) intermediate joint

to point B where the bottom face of the compression diagonal border on the joint). The condition for strength at the inclined section AB may be written as

$$N \leqslant N_{pr} + N_s + N_{tr}\text{ctg}\,\alpha \tag{IV.41}$$

Hence, the force is

$$N_{tr} = (N - N_{pr} - N_s)/\text{ctg}\,\alpha \tag{IV.42}$$

and the cross-sectional area of one transverse bar is

$$f_{tr} = N_{tr}/nR_{s,\,tr} \tag{IV.43}$$

In the above relations, N_{pr} is the design force in the prestressed longitudinal steel

$$N_{pr} = F_{pr}R_s l_1/l_{an} \tag{IV.44}$$

N_s is the design force in the nonprestressed longitudinal reinforcement

$$N_s = F_s R_s l_{1s}/l_{an,\,s} \tag{IV.45}$$

n is the number of transverse bars crossed by line AB (minus the bars placed within less than 10 cm of point A); l_1 and l_{1s} are the anchorage lengths in the support joint past line AB of the prestressed and nonprestressed reinforcement, respectively; and l_{an} and $l_{an,s}$ are the anchorage lengths at which the longitudinal prestressed and nonprestressed reinforcement acts at full strength, respectively.

For heavy concrete (M-400 and higher), l_{an} is taken as 1 500 mm for K-7 wire strands, 1 000 mm for Bp-II high-strength wire 5 mm in diameter, and 35 d for class A-IV reinforcing bars. For class A-III bars, $l_{an.s}$ is taken equal to 35 d.

The bending strength of the support joint is checked at the diagonal section AC (where AC is the line joining point A at the face of the support to point C at the bottom of the compression zone on the internal face of the joint) on the assumption that the bending moment due to external forces does not exceed the moment due to internal forces:

$$Q_A (l_j - a) \leqslant N_{tr} (l_2 - 10)/2 + N_s (h_{0s} - x/2) + N_{pr} (h_{0pr} - x/2) \quad \text{(IV.46)}$$

where Q_A is the support reaction of the truss; l_j is the length of the support joint; and a is the distance from the face to the centre of the support joint.

The depth of the compression zone in the inclined section is

$$x = (N_{pr} + N_s)/R_{pr}b \quad \text{(IV.47)}$$

The required cross-sectional area of reinforcement in an intermediate joint is determined using the loading system of Fig. IV.41*b*. Here, we likewise assume that the decrease in the design force of the reinforcing steel in the tensile diagonal along the anchorage length is compensated for by the tensile action of the transverse bars. The condition for strength in section ABC may be written as

$$N_{tr} \cos \varphi \leqslant N (k_2 l_1 + a)/k_1 l_{an} \quad \text{(IV.48)}$$

Using the above equation, we find N_{tr} and the cross-sectional area of one transverse bar

$$f_{tr} = N_{tr}/nR_{s,\,tr} \quad \text{(IV.49)}$$

where N is the design force in the tensile diagonal; φ is the angle made by the transverse bars and the tensile diagonal; n is the number of transverse bars crossed by ABC. minus the bars lying within less than 100 mm of points, A and C, and those anchored for a length less than 30 d (including the hooked portions of the transverse reinforcement) within the angle brackets; l_1 is the anchorage length of the steel of the tensile diagonal past line ABC; k_2 is the factor taking care of the behaviour of the joint combining the tensile and compressive diagonals: for top chord joints, $k_2 = 1$; for bottom chord joints, $k = 1.1$ if one of the members in the joint should satisfy the second category of cracking-resistance requirements, and the joint contains struts, or compression diagonals making an angle of more than 40° with the horizontal; in all other cases, $k_2 = 1.05$; a is the increase in the anchorage length of reinforcing steel, depending

on the type of anchorage: $a = 5\,d$ for two short bars, $a = 3\,d$ for one short bar and a hook, and $a = 2\,d$ for a button or rivet head; l_{an} is the anchorage length of the tensile diagonal at which the reinforcing steel acts at its full strength; for heavy concrete (brand M-400 and higher) and class A-III reinforcing steel, $l_{an} = 35\,d$; $k_1 = \sigma_s/3\,400$, where σ_s is the stress in the reinforcement of the tensile diagonal due to the design load.

The transverse bars of an intermediate joint combining two tensile web members are designed successively for each web member, using Eq. (IV.49) and assuming that the adjacent members are in compression.

The marginal reinforcement of an intermediate joint is designed using an assumed force

$$N_{ms} = 0.04\,(D_1 + 0.5D_2) \tag{IV.50}$$

whence

$$F_s = N_{ms}/n_2 R_{ms} \tag{IV.51}$$

Here, D_1 is the maximum force in the tensile diagonals meeting at the joint; D_2 is the force in the other tensile diagonal of this joint; n_2 is the number of the marginal bars in the joint; and $R_{ms} = 90$ MPa is the design strength of the marginal reinforcement, required to limit the crack width.

The crack-resistance analysis of the tensile chord of a truss with diagonals should be carried out taking into account bending moments arising due to the joint rigidity. In trusses where the forces resisted by the web members are small, these moments may quite accurately be determined by treating the bottom chord of the truss as a multi-span beam for specified settlements of the supports. The settlements are found using the deflection diagram of the truss members.

In addition, trusses should be designed to resist forces which are likely to arise in manufacture, in transit and during erection.

In the strength and crack-resistance analysis of trusses without diagonals, the chords and struts are assumed to be rigidly connected at joints. The forces, M, Q and N, are determined as for a closed-contour statically indeterminate system. Here, use may be made of both exact and approximate design methods.

Programs have been compiled for finding an economical design of a truss by computers.

4. Secondary Roof Structures

Secondary roof beams or trusses are used in one-storey industrial buildings with a roof truss bay of 6 m and a column bay of 12 m. They are made of M-400 or M-500 prestressed concrete, and are

reinforced with pretensioned wire strands, bars or high-strength wire. Roof trusses are welded to secondary structures during erection. A likely construction of a secondary roof truss is shown in Fig.

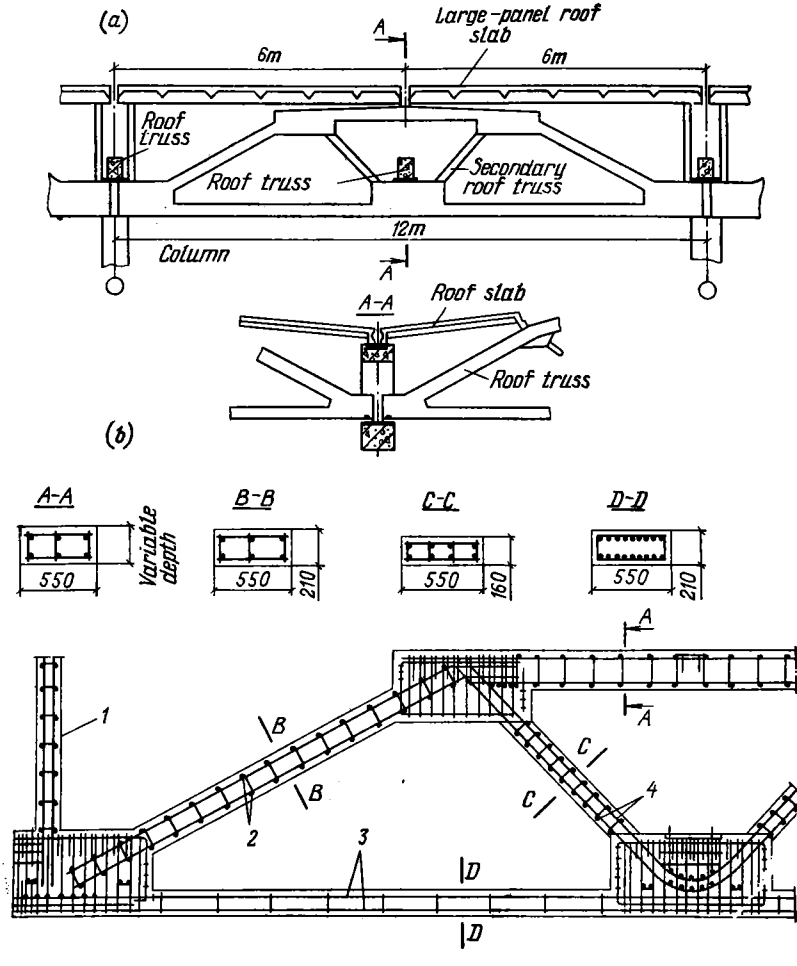


Fig. IV.42. (*a*) Construction and (*b*) reinforcement of a secondary truss
1—strut supporting a roof slab; *2*—reinforcing steel in a compressive web diagonal; *3*—prestressed steel in the bottom chord; *4*—prestressed steel in a tensile web diagonal

IV.42. The bottom chord of the truss is reinforced with prestressed wire strands, bars and high-strength wire. The amount of non-prestressed reinforcement in the tensile diagonals is determined by strength and crack-opening analysis.

From a roof truss the load is transmitted as a concentrated force applied to the bottom joint of the secondary truss in the midspan. The strength and crack-resistance analysis of secondary roof trusses is carried out with allowance for the rigidity of the joints.

5. Arches

For spans in excess of 30 m, reinforced concrete arches are more economical than trusses. Most commonly, use is made of two-hinged bowstring arches with a low rise ($f = l/6$ to $l/8$). The arch thrust is usually taken up by a string. It is more advantageous to use arches whose curve is almost a mirror-image of the bending moment diagram. The arch moment may be written as

$$M_{a,x} = M_{b,x} - Hy \qquad \text{(IV.52)}$$

where $M_{b,x}$ is the beam moment, and H is the arch thrust.

The shape of the bending moment diagram is determined on putting $M_{a,x} = 0$. Then,

$$y = M_{b,x}/H \qquad \text{(IV.53)}$$

With a uniformly distributed load and immovable supports, the bending moment diagram is a quadratic parabola

$$y = 4f\xi\,(1 - \xi) \qquad \text{(IV.54)}$$

where $\xi = x/l$.

In practice, however, the arch curve never has the shape of the bending moment diagram, because different combinations of live loads and also shrinkage and creep in the concrete give rise to additional bending moments. The effect of creep in the concrete is most pronounced in large-span arches. Because of this, the arch is given a shape such as will reduce the design forces to a minimum. To standardize the construction and simplify manufacture, low-rise two-hinged arches are given the shape of a circle.

The design procedure for arches is the same as for other compression members. Arches are made of rectangular or I-section, usually reinforced with symmetrical tensile and compressive steel, because both positive and negative moments are likely to arise. The string is made of prestressed concrete. To reduce the sag of the string, it is connected to the arch by reinforced concrete or steel ties spaced 5 or 6 m apart.

A likely reinforcement for a 36-m I-section two-hinged arch with a prestressed string is shown in Fig. IV.43. The arch is assembled of six blocks. The string is fabricated as a one-piece element with supporting blocks so as to increase its thrust strength. The prestressed reinforcement consists of pretensioned wire strands. During erection, the blocks are assembled by welding together the reinforce-

ment stick-outs or embedded steel parts, and the joints between the blocks are then concreted.

Large-span high-rise arches have a more complex curve and are usually made three-hinged. The arch thrust is transferred to the footings and the soil. In the case of soils having a low bearing capacity, the arch thrust is taken up by a tie rod placed below floor level.

Arches are designed to resist the dead load due to the weight of the roof and the self-weight of the arch, snow load distributed over the entire arch surface or applied at one side, and the concentrated load due to overhead travelling cranes. Large-span arches are also designed to resist shrinkage and creep of the concrete, and high-rise arches to resist wind load. For design purposes, the curve of a low-rise two-hinged arch is assumed to be a quadratic parabola (Fig. IV.44 *a*). As a first trial, the depth and width of the arch cross section are taken as

$$h = (1/30 \text{ to } 1/40)\, l \text{ and } b'_f = (0.4 \text{ to } 0.5)\, h$$

The trial cross-sectional area of the string reinforcement is determined according to the arch thrust

$$H = 0.9ql^2/8f \qquad \text{(IV.55)}$$

Two-hinged arches are designed as statically indeterminate systems with allowance for the deflections brought about by bending moments and normal forces. In the design of the prestressed string, allowance should be included for the transformed area of the concrete, $F_{tr} = F + nF_{pr}$. The prestress in the string, which cancels the strain in the reinforcing steel, reduces the mobility of the arch supports. As a result, the arch behaves under load approximately as an arch with fixed ends. This leads to an increased thrust, H, and a decreased bending moment in the arch.

Three-hinged arches are statically determinate. If the arch supports lie in the same level, the thrust is

$$H = M_0/f \qquad \text{(IV.56)}$$

where M_0 is the beam moment in the midspan of the arch.

The forces, M, N and Q, are determined for several sections along the arch length (Fig. IV.44*b*). Bending moments are found by Eq. (IV.52), longitudinal and shearing forces are given by

$$Q = Q_b \cos \varphi - H \sin \varphi \qquad \text{(IV.57)}$$

$$N = H \cos \varphi + Q_b \sin \varphi \qquad \text{(IV.58)}$$

where φ is the slope of the arch at a given section, and Q_b is the beam shearing force.

The forces in the sections, calculated for different load combinations, are tabulated, and the table is used to find the maximum and

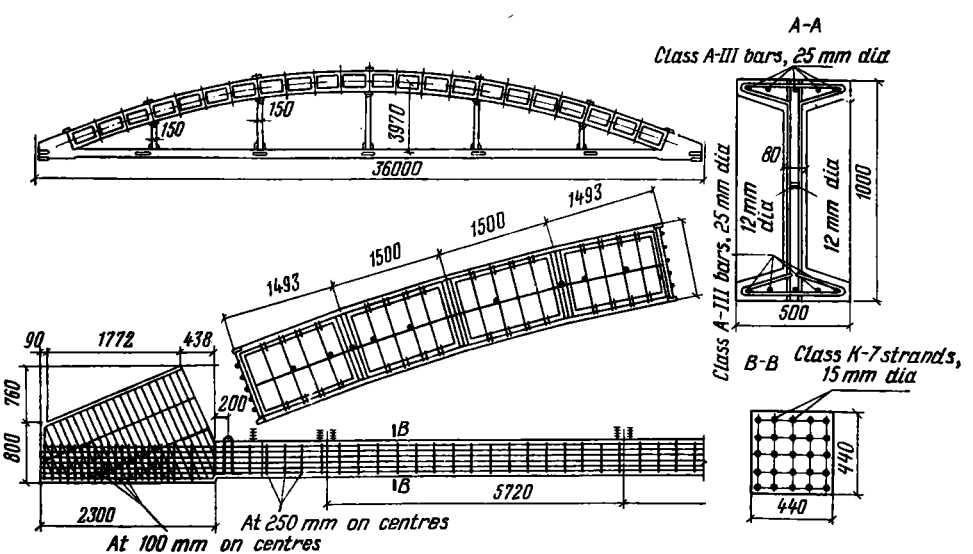


Fig. IV.43. 36-m r.c. bowstring arch

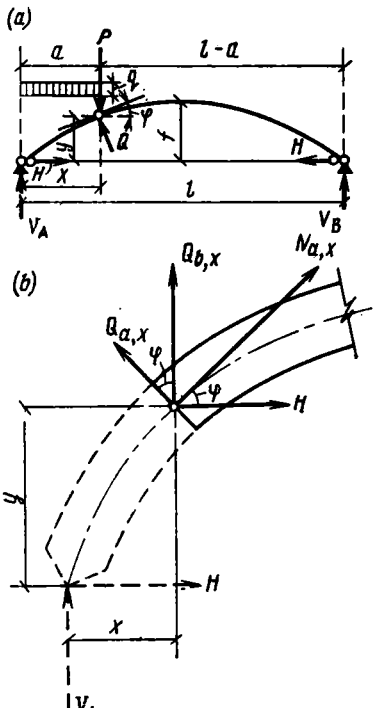


Fig. IV.44. To the design of arches

minimum design forces. The cross-sectional area of reinforcement is determined by the expressions derived for compression members. To take proper account of the buckling in the plane of the arch, the effective length of the arch is taken as 0.58 *s* for three-hinged arches, as 0.54 *s* for two-hinged arches, and as 0.36 *s* for no-hinge arches (where *s* is the length of the arch curve). Shearing forces in arches are small; the amount of transverse reinforcement is found by design and for structural reasons. The amount of reinforcement for the string is determined as for a tensile member, so as to ensure adequate strength and cracking-resistance.

IV.4. ONE-STOREY IN-SITU R.C. BUILDINGS OF SKELETON CONSTRUCTION

The main load-bearing structure in a one-storey in-situ r.c building of skeleton construction is a transverse frame. This is because the load due to the weight of the roof — beam-type or thin-walled space shell — is transmitted directly to transverse frames.

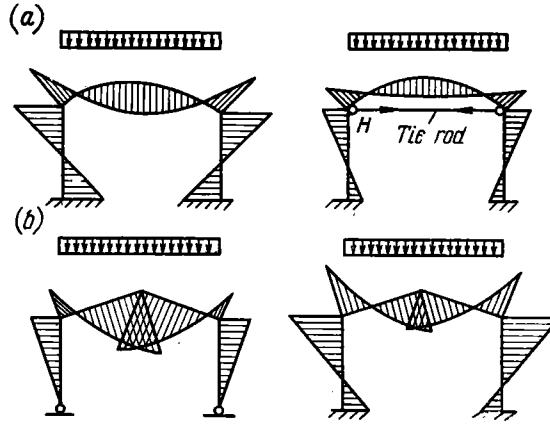


Fig. IV.45. To the selection of an economical in-situ r.c. frame
(*a*) bending moment diagrams for frames using a curved girder with and without a tie rod; (*b*) bending moment diagrams for frames using a broken-line girder and hinged or rigid joints between the struts and the footing

For spans from 12 to 15 m, the frames use straight girders and for spans from 15 to 18 m broken-line girders. For spans up to 18 m, the girders may be curved, without tie rods, whereas spans of 24 m and more require curved girders and tie rods. Curved-girder frames mainly serve as diaphragms in short-span shells which are a rather economical type of in-situ r.c. roofs. The tie rod resisting the horizontal deflections of the strut tops reduces bending moments and shearing forces in the struts and girders (Fig. IV.45*a*). In addition,

it reduces bending moments and shearing forces in the connections between the struts and footings, thereby making the footing construction simpler.

The joints between in-situ r.c. frames and footings may be either rigid or hinged. In rigid joints, the strut reinforcement is welded to the dowels of the footings; this type of joints is easy to manufacture and costs less. Hinged joints are used where considerable bending moments arise in the connection of a strut and a footing, and the underlying soil has a low bearing capacity, so that heavy footings

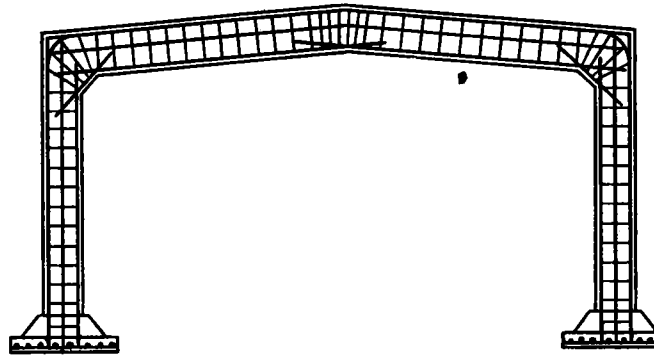


Fig. IV.46. Reinforcement of an in-situ r.c. frame

are required. At the same time, it should be remembered that hinged joints increase bending moments in the span, which necessitates the use of heavier girders (Fig. IV.45*b*).

A girder is reinforced as a beam with fixed ends; some of its longitudinal reinforcement is extended into the zone of negative moments at the support and terminated in the strut. The struts are reinforced as compression members, with some bars extending into the girder (Fig. IV.46).

When designing an in-situ r.c. frame, special attention should be given to the joints. The reinforcing steel in the joints should be distributed so as to take up the applied forces and to provide enough space for pouring and compacting the concrete mix. At the joint of a girder and a column, maximum tensile forces are likely to arise at some distance from the edge. Because of this, the tensile steel in the joint is rounded and extended into the adjacent member for a distance established by the bending moment diagram (Fig. IV.47*a*).

In the compression zone of the joint, considerable local stresses usually arise. To reduce these stresses, angle brackets are provided in the joints. The compressive steel of the girder and the column is extended into the joint, whereas the angle bracket is independent-

ly reinforced with longitudinal bars. Frames designed to resist relatively small forces have no angle brackets, which fact somewhat simplifies the construction.

At break-points (where a girder is bent), say, at a ridge joint, the resultant of the forces in the bottom tensile steel is directed along the bisector of the ridge angle, thereby tending to cause the

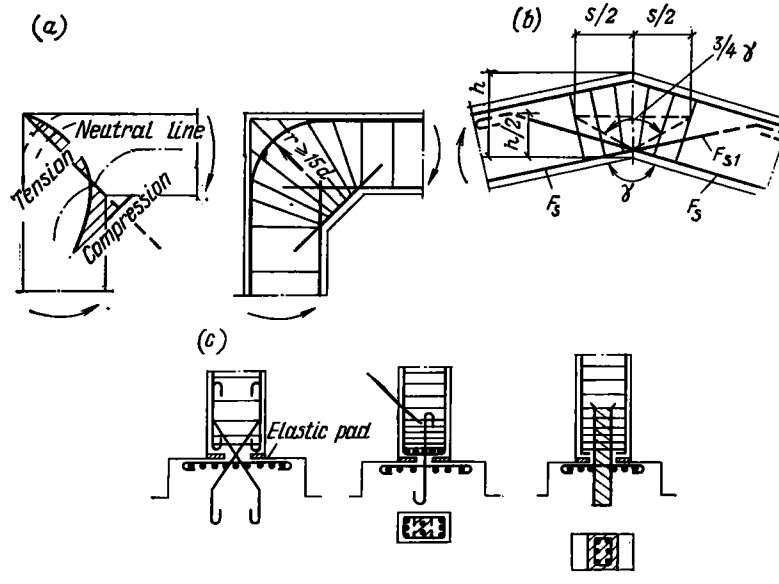


Fig. IV.47. Joint reinforcement of (*a*), (*b*) in-situ r.c. frames and (*c*) support hinges

steel to unbend and the concrete to spall (Fig. IV.47*b*). To avoid this, ridge joints are reinforced so that tensile bars overlap. Additional transverse bars may also be provided, whose cross-sectional area is found by calculation. The amount of transverse reinforcement should be sufficient to carry the tensile force equal to the vertical component of the forces in the longitudinal tensile bars which are not anchored in the compression zone:

$$P_1 = 2R_sF_{s1} \cos (\gamma/2) \qquad \text{(IV.59)}$$

or to take up 35% of the vertical component of the forces in all longitudinal tensile bars

$$P_2 = 0.7\, R_sF_s \cos (\gamma/2) \qquad \text{(IV.60)}$$

where F_{s1} is the cross-sectional area of the longitudinal tensile bars unanchored in the compression zone, and γ is the ridge angle in the tension zone.

The transverse reinforcement required by the design should extend for the following length

$$s = h \operatorname{tg} (3\gamma/8) \tag{IV.61}$$

The hinged joint of a frame strut and a footing is made to behave as a simplified (imperfect) hinge. To this end, the cross-sectional area of the strut is reduced to one half or one third of the basic area, the joint is reinforced with vertical or crossed bars, and the strut and footing portions making up the hinge are reinforced with transverse fabric (Fig. IV.47*c*). The longitudinal force of the strut is transmitted through the intact area of concrete and the reinforcing bars, and the lateral thrust is usually cancelled by friction.

CHAPTER V

THIN SHELL ROOFS

V.1. GENERAL

As their name implies, shell roofs consist essentially of thin shells (thin slabs) and edge structures (such as ring-beam supports and diaphragms in the shape of beams, trusses, arches, etc.). Shells are given the shape of a curve or a polyhedron.

Thin shell roofs are many and diverse (Fig. V.1*a* through *g*). They may use barrel (or cylindrical) shells and prismatic folded plates, shells of revolution with vertical axes of symmetry (such as domes), doubly-curved shells of positive and negative Gaussian curvatures usually rectangular in plan, and composite shells formed by intersecting curvilinear surfaces.

A special position is held by corrugated roofs, that is, roofs made of corrugated arches or multi-folded plates with a small corrugation-width-to-span ratio (Fig. V.1*h*), and also suspended roofs (held by guys) of various space and plan configurations (Fig. V.1*i* and *j*).

In practice, use is also made of many other thin shell roofs.

Thin shell roofs are especially advantageous in industrial and civil buildings where large unobstructed areas (about 30 $\times$ 30 m or more) are to be roofed over without intermediate supports. In fact, they are also widely used for smaller spans.

Owing to their action in two directions in plan, shell roofs are economical of material and have a smaller self-weight as compared with roofs made of plane structures (roof slabs, trusses, beams, arches, and secondary roof trusses). Shell roofs are very expressive architecturally.

Outside the USSR, thin shell roofs are commonly cast in-situ, which requires falsework and formwork.

In the Soviet Union, shell roofs are made of prefabricated members adapted to industrialized manufacture.

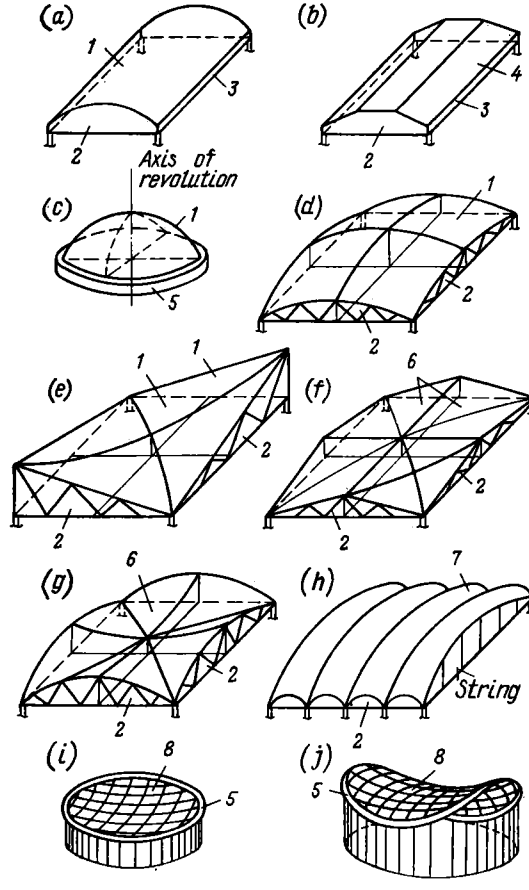


Fig. V.1. Most common thin shell roofs

(*a*) cylindrical (barrel) shell; (*b*) prismatic folded plate; (*c*) shell of revolution with a vertical axis (dome); (*d*) doubly-curved rectangular-plan shell of positive Gaussian curvature; (*e*) doubly-curved rectangular-plan shell of negative Gaussian curvature; (*f*) composite shell assembled from rectangular members; (*g*) composite shell assembled from triangular members; (*h*) corrugated vault; (*i*) positive-curvature suspended roof; (*j*) negative-curvature suspended roof; *1*—shell; *2*—diaphragm; *3*—edge member; *4*—face of a folded plate; *5*—ring-beam support; *6*—shell member; *7*—individual corrugation; *8*—suspended shell

The first thin shell roofs were erected in the 1920s. In the USSR, the early cylindrical shells were used to roof over a water tank (1925), a post-office building (1928), a motor depot (1929), and a machine-building plant (1931). The first reinforced concrete domes were used for a planetarium (1929), and theater buildings (1934 and 1939).

With progress in technology and experience, thin shell roofs have continually been improved.

Of late, a number of unique shell roofs made of precast members have been installed in some Soviet cities.

Nowadays, it has become standard practice to prestress contour structures and the corner zones of shells, ever wider use is made of lightweight concrete, whole-span shell-slabs (such as large-size reinforced concrete barrel slabs, hyperbolic shells, and the like), shells made of fine-mesh wire-fabric reinforced concrete, and also reinforced concrete shells combined with steel diaphragms.

Doubly-curved surfaces may be formed by the revolution or the translation method. In the former case, a surface is generated by revolving a plane curve (generatrix) about an axis lying in the same plane as the generatrix (Fig. V.1*c*). The latter technique consists in translating a plane generatrix along parallel guides (Fig. V.1*d*). A doubly-curved surface may also be formed by the translation of a plane curve (or a straight line, as a special case) along two nonparallel nonintersecting guides (Fig. V.1*e*).

Most commonly, shell roofs use shallow shells with a rise of not more than 1/5 or 1/6 of any base side.

In a surface of positive Gaussian curvature, the centres of curvature at any point of the surface lie on the same side of the surface. If the centres of curvature are located on either side of a surface, the surface is said to have a negative Gaussian curvature.

As has been proved experimentally, thin shell roofs, as well as other reinforced concrete structures, behave elastically in the initial loading stage (that is, prior to cracking in the concrete of the tension zone). After the onset of cracking, the rising loads and stresses in the concrete and reinforcing steel bring about plastic strains which grow until the materials attain the state of ultimate equilibrium. The elastic behaviour of shells has been investigated fairly well, whereas their behaviour in plastic state and the state of ultimate equilibrium has yet to be studied. The plastic behaviour of shells is expected to contribute to the stability of structures and to cut down material consumption.

In the general case, the normal sections of shells are subjected to normal forces, N_x and N_y, shearing forces, N_{xy} and N_{yx}, bending moments, M_x and M_y, shearing forces, Q_x and Q_y, and torques, H_x and H_y (Fig. V.2). The above forces are given per unit length of the section.

Thin shells have a low flexural rigidity as compared with the rigidity against the forces acting in the middle surface of the shell. As a result, external loads mostly give rise to the internal forces, N_x, N_y and N_{xy}. Because of this, in most shells subjected to loads uniformly distributed over their surface (such as self-weight and snow load), a moment-free state of stress exists over nearly all the area of the shell, whereas a state of total stress is developed in some limited zones where the middle surface experiences an ap-

preciable bending. Such bending shows up where the shell joins the edge structures, in regions of sharp or sudden change in the load magnitude, in regions of sharp or sudden change in the surface

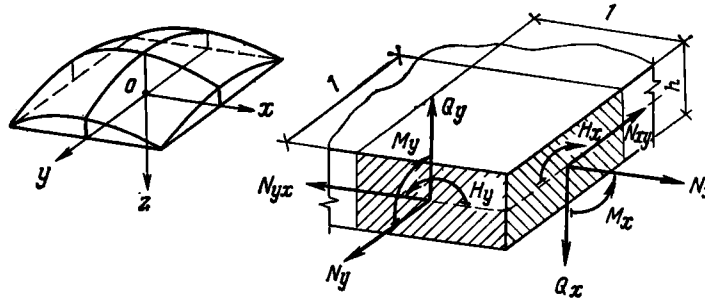


Fig. V.2. Forces acting in a shell

curvature, and also where local forces (that is, those concentrated in small areas) are applied.

The moment-free state of stress in thin shallow shells (see Fig. V.2) is described by the following equation

$$k_y \partial^2\varphi/\partial x^2 + k_x \partial^2\varphi/\partial y^2 - 2k_{xy}\partial^2\varphi/\partial x\,\partial y = -q \qquad \text{(V.I)}$$

where q is the load continuously distributed over the shell and normal to its surface.

The $\varphi\,(x, y)$ stress function is related to the internal forces of the shell as

$$N_x = \partial^2\varphi/\partial y^2, \quad N_y = \partial^2\varphi/\partial x^2 \text{ and } N_{xy} = -\partial^2\varphi/\partial x\,\partial y \qquad \text{(V.2)}$$

The surface curvatures, k_x and k_y, in the direction of the $0x$ and $0y$ axes and the torsional curvature of the surface, k_{xy}, are defined as

$$k_x = \partial^2 z/\partial x^2, \qquad k_y = \partial^2 z/\partial y^2 \quad \text{and} \quad k_{xy} = \partial^2 z/\partial x\,\partial y \qquad \text{(V.3)}$$

In most cases, the sag, w, of the middle surface in areas of local bending may be specified by giving only one coordinate, say, along the $0x$-axis. Then, the state of total stress can approximately be described by the following equation

$$-Dd^4w/dx^4 + k_xN_x + k_yN_y + 2k_{xy}N_{xy} = -q \qquad \text{(V.4)}$$

Here, D is the cylindrical flexural rigidity of the shell, defined as

$$D = EI/(1 - \nu^2) = \text{(approx.)}\ Eh^3/12 \qquad \text{(V.5)}$$

where h is the thickness of the shell, and ν is Poisson's ratio which for concrete is 1/6.

V.2. CONSTRUCTIONAL FEATURES OF THIN SHELL ROOFS

The structure of a thin shell roof is chosen according to the purpose of the building, its architectural proportions, size, and erection method. Roofs made of prefabricated members should be designed so as to minimize labour consumption in the manufacture of precast members, to keep to a minimum the number of member types and sizes used, to simplify erection and jointing, and to minimize the number of reusable supporting fixtures. In-situ r.c. roofs should be adapted to the use of travelling or portable reusable formwork.

To ensure the necessary strength and rigidity of precast members in manufacture, in transit and during erection, they are usually strengthened around the periphery. The result is ribbed shells. Instead of ribs, use may be made of removable steel spreaders installed at the edges of members for transit and erection, and left in place until the concrete in the joints hardens.

The type of joint between precast shell members is chosen according to the nature and magnitude of the forces likely to occur in the joint.

In all cases, joints must be concreted. For better concreting, the width of a joint should be not less than 30 mm if the depth of the members at the joint does not exceed 100 mm, nor less than 50 mm if the depth of the member exceeds 100 mm.

If a joint is designed to transmit axial or eccentrical compressive forces (in the latter case, the eccentricity should not exceed the kern size) and small shearing forces, it may be reinforced with overlapping reinforcement stick-outs to suit structural considerations.

If joints are spaced not more than 1.5 m apart, tensile and shearing forces transmitted through a joint may be taken up by the reinforcement provided in transverse joints. In the joints, the reinforcement stick-outs of adjacent members are welded to one another.

The reinforcement of precast shell members may also be connected by embedded steel parts welded to it. During erection, the connection between the embedded steel parts is provided by welded fishplates. The cross-sectional area of the fishplates and the length of the welds are found by calculation.

If a joint is designed to transmit considerable shearing forces, the faces of the members to be joined should be shaped so as to form concrete keys preventing the members from mutual displacement.

It is good practice to use prestressed edge structures in shell roofs. For one thing, this contributes to the cracking-resistance of tension areas; for another, prestressed edge structures help to combine the precast roof members in a single entity.

Shell areas subjected to biaxial compression should be checked for stability.

At the stage of ultimate equilibrium, the load-bearing capacity of structures is practically independent of intermediate states of stress. As a consequence, the strength-analysis procedure for precast; in-situ, precast/in-situ, prestressed and nonprestressed shell roofs is the same. In the design in terms of other limit states, special attention should be given to the forces due to the self-weight of precast members, arising prior to joint concreting.

In addition, prestressed shell roofs should be checked for cracking-resistance under service loads, and also for strength in the course of prestressing.

Prefabricated members should be checked for strength under loads likely to occur in manufacture and transit.

The amount of reinforcement and proportions of thin shell roofs are found according to the internal normal and shearing forces and bending moments.

The maximum principal compressive stresses should not exceed R_{pr}.

In the compression zones and zones where the principal tensile stresses are smaller than R_{ten}, the reinforcement is distributed for structural reasons, with the cross-sectional area of reinforcing steel being not less than 0.2% of the concrete, and with the bars being spaced 20 to 25 cm apart. For shell slabs more than 8 cm thick, double fabric is recommended.

Where the principal tensile stresses exceed R_{ten}, all of the internal forces should be taken up by the reinforcing steel in the form of bars placed so as to match closely the direction of the principal tensile stresses, or in the form of fabric made up of longitudinal and transverse bars.

The amount of reinforcement carrying bending moments in smooth shells is determined as that in slabs. The reinforcing steel is disposed in the tension zone to fit the bending moment diagram, with a minimum concrete cover.

At the joints between edge members and diaphragms, shells are reinforced with double mats made of bars 6 to 10 mm in diameter and spaced not wider than 20 cm apart.

In ribbed structures, the amount of rib load-bearing reinforcement is designed according to the bending moments arising in manufacture and service. Ribs are reinforced with welded-bar mats in which transverse bars 5 to 6 mm in diameter are spaced 20 to 25 cm apart.

Edge structures are designed according to the general rules of structural mechanics to resist the forces transferred by the shells, and erection loads.

Small openings and holes in shells are hemmed with flanges. The cross-sectional area of the flanges in the compression zones of shells is taken equal to that of the removed concrete. Oblong openings are strengthened with intermediate spacers. The flanges

of openings cut out in the tension zones of shells are reinforced so that the steel carries all of the forces which would be applied to the removed part.

V.3. ROOFS MADE OF CYLINDRICAL SHELLS AND PRISMATIC FOLDED PLATES

Cylindrical shell roofs (see Fig. V.1*a*) consist essentially of thin cylindrical slabs, edge members and end-face diaphragms. The whole structure is supported on columns placed at the corners.

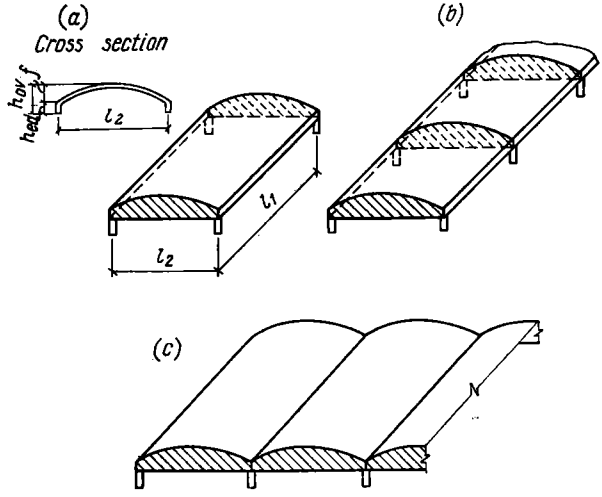


Fig. V.3. Types of cylindrical shells
(*a*) single-span shell; (*b*) multi-span shell; (*c*) multi-barrel shell

The main parameters of a shell (Fig. V.3*a*) are the longitudinal span, l_1 (that is, the centre-to-centre distance between diaphragms), the arch span, l_2 (that is, the distance between edge members), and the rise, f.

Shells may be round, elliptic or parabolic in cross section. Owing to the ease of manufacture, the first type is used most frequently.

According to their construction, cylindrical shells (Fig. V.3) may be classed into single-span shells supported on two diaphragms placed along their generatrices, multi-span shells supported on more than two diaphragms, single-barrel shells, multi-barrel shells consisting of several single-barrel shells, plain shells, and ribbed shells.

The stress-strain state of a shell depends on the relative magnitudes of l_1 and l_2. Shells with $l_1/l_2 > 1$ or 2 are rated as long, and with $l_1/l_2 < 1$, as short.

The overall thickness, h_{ov}, of a nonprestressed shell including the thickness, h_{ed}, of its edge member is usually not less than $l_1/15$ to $l_1/10$. The thickness of prestressed shells may be smaller. Also, the value of h_{ov} should not be less than $l_2/6$. The thickness, h, of

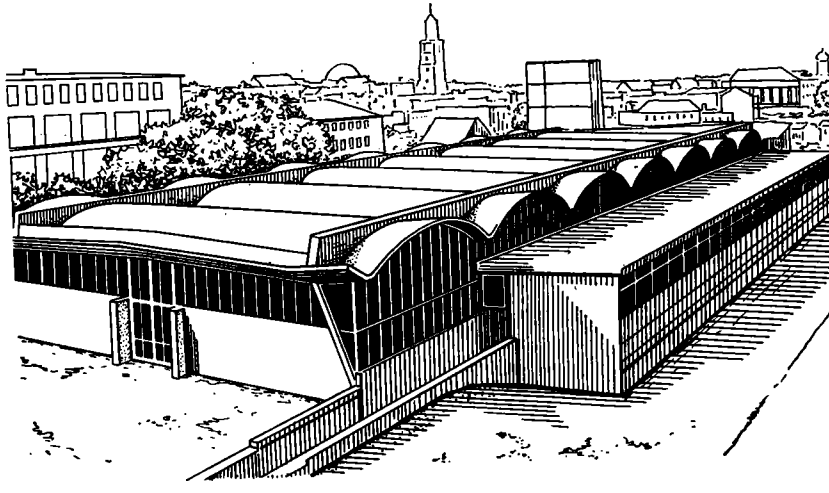


Fig. V.4. In-situ r.c. multi-barrel shell roof of the Water Supply Institute of the Technical Higher School in Darmschtadt (FRG)

an in-situ shell is taken as $l_2/200$ to $l_2/300$ but not less than 5 or 6 cm. Prefabricated ribbed shells should be at least 3 cm deep (which is governed by the conditions of manufacture).

Figure V.4. shows a multi-barrel shell roof.

1. Long Shells

Under certain conditions, a long cylindrical shell resisting the self-weight and snow load deforms as a beam of span, l_1, width, l_2, and of an odd-shaped cross section of depth, h_{ov}, including the edge members (see Fig. V.3). The bottom part of the shell cross section is in tension, and the top part in compression.

The function of the edge members is to contribute to the strength and rigidity of the roof cross section, to accommodate the tensile load-bearing reinforcement, and to strengthen the straight-line edges of the shell against local loads. The shape and size of the edge members depend on the construction of the shell and are subject to design calculation.

In-situ r.c. shells are usually made plain. If it is to carry suspended concentrated loads, a shell is made ribbed. Prefabricated shells

generally have longitudinal and transverse ribs intended to provide additional strength in manufacture, in transit and during erection.

The most popular diaphragms are solid beams, trusses, bowstring arches and frames (Fig. V.5). To secure better light and ventilation, cylindrical shells may be made of the shed type (Fig. V.6*a*), or have

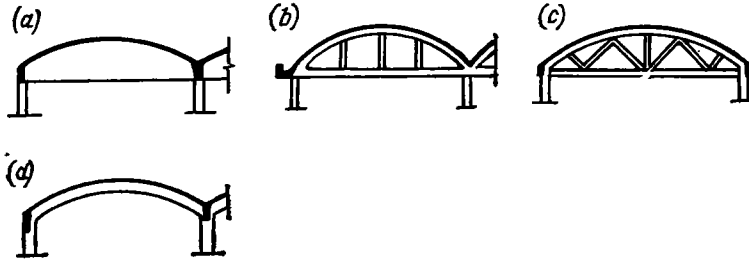


Fig. V.5. Types of diaphragms
(*a*) solid beam; (*b*) bowstring arch; (*c*) truss; (*d*) frame

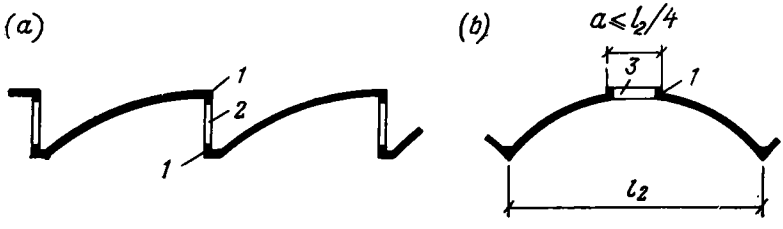


Fig. V.6. Shells with skylight openings
(*a*) shed-type shell; (*b*) barrel shell; *1*—longitudinal rib; *2*—strut in the plane of the clerestory; *3*—spacer

skylight opening at the apex (Fig. V.6*b*). The openings are hemmed with longitudinal ribs; long openings are strengthened with spacers.

The internal forces in shells may be determined by accurate and approximate design methods. The latter are preferable for the structural analysis, and the former for the design of a shell; although, in most cases, approximate methods yield quite satisfactory results.

When $l_1/l_2 > 3$ (or $l_1/l_2 \geqslant 2$ for intermediate barrels), single- and multi-barrel symmetrical shells with the edge members unsupported in the longitudinal direction may be designed as tray-like beams for strength, rigidity and crack-resistance against vertical uniformly distributed symmetrical loads imposed by the self-weight (see Fig. V.3). The strength of shells is determined for design loads, and rigidity and crack-resistance for basic loads. For design purposes, a unilateral uniformly distributed snow load which does not

exceed 1/4 of the total snow load may be replaced by a symmetrical load of the same magnitude. Light live loads imposed by telphers with a lifting power of up to 1 tonne, suspended to the edge members, may be regarded as a symmetrical load applied simultaneously to both edge members.

All ribbed long cylindrical shells having at least three transverse ribs not less than $l_2/25$ in depth may be treated as reinforced concrete beams.

In other cases, long shells are analysed as elastic systems deformable in space.

The butterfly edge of a multi-barrel shell with the edge members unsupported in the longitudinal direction may approximately be designed as a symmetrical single-barrel shell, and the intermediate barrels as single-barrel shells but with allowance for the fixation of the longitudinal edges against displacement in the lateral direction (Fig. V.7).

In the case of a multi-barrel shell subjected to a uniformly distributed load (Fig. V.8), it is sufficient to limit the analysis to a single-barrel shell on hinged immovable supports, of span, l_0, equal to the distance from the edge member to the zero-point on the bending moment diagram plotted for the corresponding multi-span beam, and then to assume that the internal forces in the shell vary in the same manner as the ordinates of the bending moment diagram of the multi-span beam.

Let us analyse a long cylindrical shell of symmetrical circular cross section subjected to a vertical symmetrical load in the state of ultimate equilibrium as a reinforced concrete beam. Figure V.9 shows a diagram representing the state of stress in the shell cross section. Here, F_s is the total cross-sectional area of the tensile steel; θ_1 is one half of the shell arch central angle; θ_p is one half of the central angle of the compression zone arch; c is the distance from the resultant of the forces in the tensile steel to the centre of curvature of the circular part of the shell; h and R_y are the depth and the radius of the cylindrical part of the shell, respectively; and d_1 is the distance from the resultant of the forces in the tensile steel to the top of the edge member.

The condition for strength in terms of the internal force moments taken about the centre of the circular part of the shell is

$$M_{\max} \leqslant 0.8\ (2R_{pr}hR_y^2 \sin \theta_p - cR_sF_s) \qquad \text{(V.6)}$$

where $M_{\max}$ is the moment due to external forces, calculated about the same point as in a beam, and 0.8 is an empirical service factor.

The position of the neutral axis may be found from the following equation

$$2R_{pr}\theta_pR_yh = R_sF_s \qquad \text{(V.7)}$$

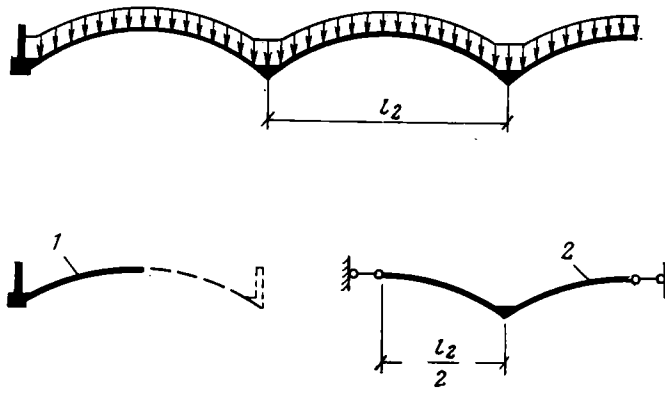
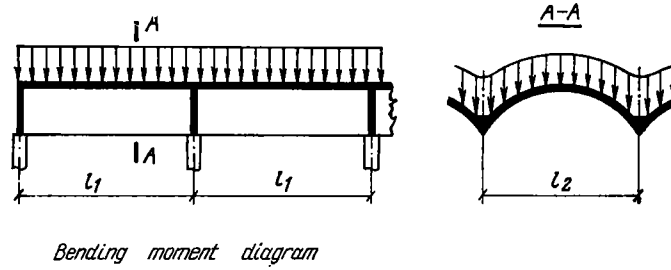


Fig. V.7. Loading systems for a multi-barrel shell
(*1*) butterfly edge; (*2*) intermediate barrel



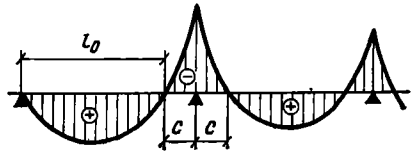


Fig. V.8. Loading system for a multi-barrel shell

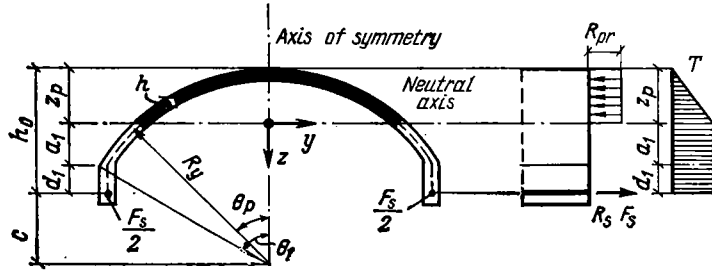


Fig. V.9. To the design of a cylindrical shell in the state of ultimate equilibrium as an r.c. beam

When checking the shell for strength, the value of θ_p found from Eq. (V.7) is substituted into Eq. (V.6). When calculating the amount of reinforcement required, both expressions are combined as

$$\sin \theta_p - c\theta_p/R_y - M/2R_{pr}hR_y^2 = 0 \qquad \text{(V.8)}$$

The above equation may be solved by successive approximation by putting $\sin \theta_p = \theta_p$ and finding F_s from Eq. (V.7).

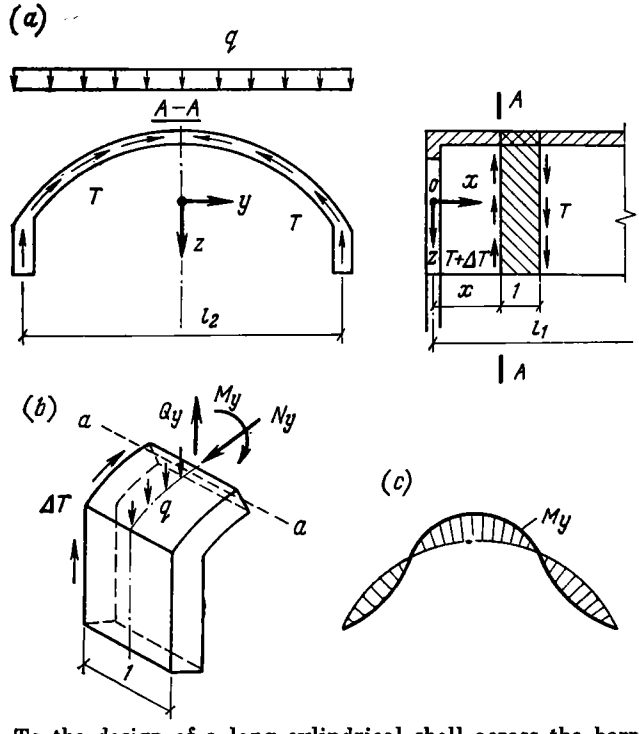


Fig. V.10. To the design of a long cylindrical shell across the barrel
(a) barrel strip of unit length with the tangential forces; (b) part of the barrel strip with the load and internal forces; (c) diagram for the bending moments acting across the barrel

The tangential forces in the shell reach their maximum at the support; they can be computed using the expression from a course on the strength of materials

$$T = Q_0 S/I \qquad \text{(V.9)}$$

where Q_0 is the shearing force at the support, calculated as for a beam.

In order to determine bending moments, M_y, acting across the barrel, it is necessary to consider a transverse strip of unit length (Fig. V.10*a*). It is subjected to an external vertical distributed load, q, applied to its surface, and tangential forces, T and $T + \Delta T$,

acting in the planes of the sections bounding the strip. In the strip, the load, q, the differential tangential force, ΔT, and the forces, M_y, Q_y and N_y, are in equilibrium (Fig. V.10*b*). Consequently, the bending moment, M_y, in any section of the strip is determined as the sum of the moments due to q and ΔT about the axis, a-a, in the section in question (Fig. V.10*b*).

The diagram for M_y is shown in Fig. V.10*c*.

The internal forces in a long cylindrical shell (Fig. V.11) regarded as an elastic space system may approximately be found from the moment-free state of stress. For a shallow shell, we may use Eq. (V.1) where k_x is assumed to be zero. Hence,

$$k_y = 1/R_y; \quad k_{xy} = 0; \quad D = 0; \quad \partial^2\varphi/\partial x^2 = -qR_y \tag{V.10}$$

Assume that the shell loaded by q cannot be deformed at the level of the edge member and diaphragm tops along the edges (state I). In this case, the φ stress function (as a first approximation) may be taken as

$$\varphi^{\mathrm{I}} = (1/60)\, qR_y\, (x^4 - 6x^2a^2 + 5a^4)\, (y^4 - 6y^2b^2 + 5b^4)/a^2b^4 \tag{V.11}$$

and the forces N_x^{I} and N_{xy}^{I} are given by

$$N_x^{\mathrm{I}} = \partial^2\varphi/\partial y^2 = (1/5)\, qR_y\, (x^4 - 6x^2a^2 + 5a^4)\, (y^2 - b^2)/a^2b^4 \tag{V.12}$$

$$N_{xy}^{\mathrm{I}} = -\partial^2\varphi/\partial x\, \partial y = -(4/15)\, qR_y\, (x^3 - 3xa^2)\, (y^3 - 3yb^2)/a^2b^4 \tag{V.13}$$

The diagram for N_x^{I} at the section, $x = 0$, is shown in Fig. V.11*a*. The ordinates of N_x^{I} and N_{xy}^{I} at individual points are

$$\left.\begin{aligned} N_{x1}^{\mathrm{I}} &= -qR_y\, (a/b)^2 \\ N_{x4}^{\mathrm{I}} &= (4/3)\, qR_y a^2/bh_{ed} \\ S_{q5}^{\mathrm{I}} &= N_{xy}^{\mathrm{I}}\, (x = a;\ y = b) = -(16/15)\, qR_y\, (a/b) \end{aligned}\right\} \tag{V.14}$$

Now, let us detach the shell from the edge members (state II). In the shell, the state of stress remains the same; in the edge members, the shearing forces, $S_{ed}^{\mathrm{I}} = -S_{sh}^{\mathrm{I}}$, acting along the lines of contact with the shell give rise to new longitudinal forces. At the top and bottom of the edge members (Fig. V.11*b*), they are

$$\left.\begin{aligned} N_{x3}^{\mathrm{II}} &= (8/3)\, qR_y a^2/bh_{ed} \\ N_{x4}^{\mathrm{II}} &= -0.5N_{x3}^{\mathrm{II}} = -(4/3)\, qR_y a^2/bh_{ed} \end{aligned}\right\} \tag{V.15}$$

The actual zero level of N_x does not lie in the top face of the edge members, because there are additional shearing forces, S^{II}, acting between the shell and the edge members. Treating these forces as a load (state III), we get the stress function as

$$\varphi^{\mathrm{III}} = -\bar{S}_0\, (x^6 - 3x^4a^2 + 3x^2a^4 - a^6)\, (y^4 - y^2b^2)/3.372a^5b^3 \tag{V.16}$$

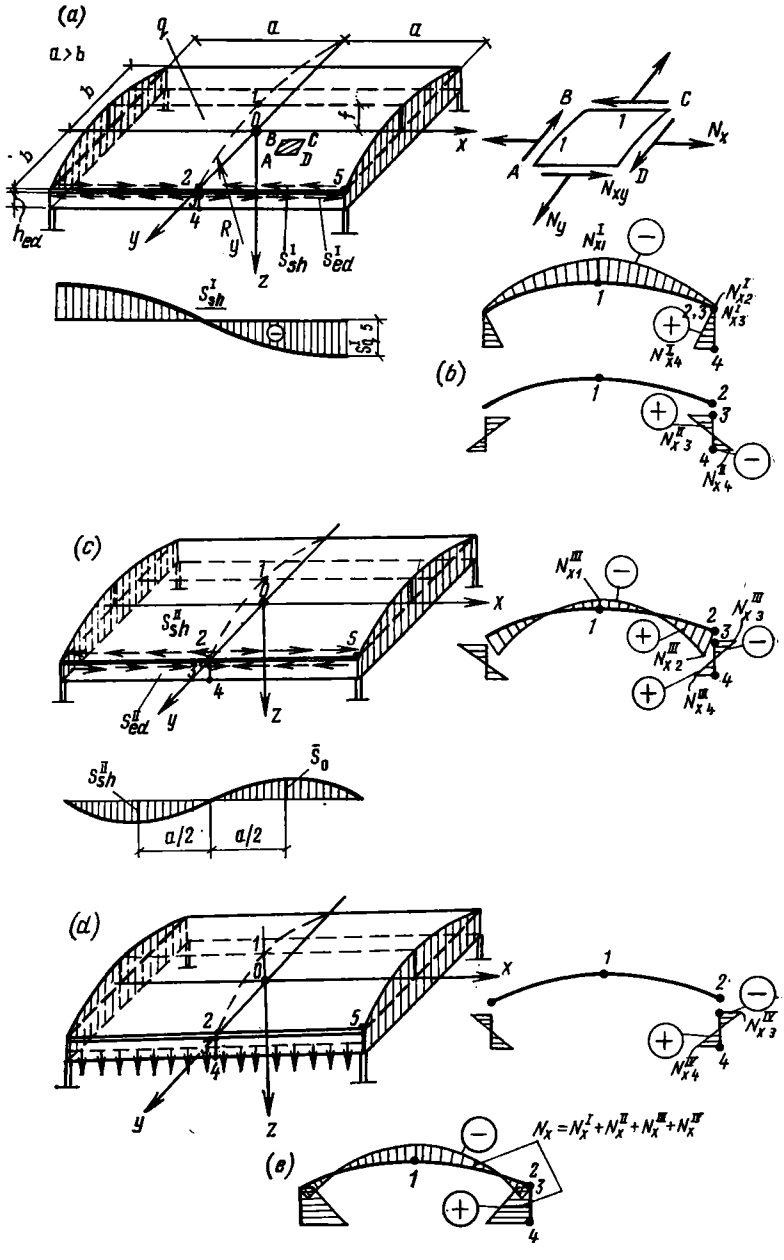


Fig. V.11. To the design of a long cylindrical shell as a space system

(*a*) shell on a nondeformable edge, loaded by q; (*b*) forces, N_x, in the edge members detached from the shell; (*c*) forces, N_x, induced by the additional shearing forces at the joint between the shell and edge members; (*d*) forces, N_x, induced by the load applied to the edge members; (*e*) resultant diagram for N_x

The internal forces are given by

$$N_x^{III} = \partial^2\varphi/\partial y^2 = -\bar{S}_0 \times 12\,(x^6 - 3x^4a^2 + 3x^2a^4 - a^6) \times (6y^2 - b^2)/3.372a^5b^3 \quad (V.17)$$

$$N_{xy}^{III} = -\partial^2\varphi/\partial x\,\partial y = +\bar{S}_0 \times 12\,(x^5 - 2x^3a^2 + xa^4) \times (2y^3 - yb^2)/3.372a^5b^3 \quad (V.18)$$

The diagrams for N_x^{III} at the section, $x = 0$, are given in Fig. V.11*c*; the ordinates at individual points are as follows

$$\left.\begin{aligned} N_{x1}^{III} &= -0.593\bar{S}_0 a/b \\ N_{x2}^{III} &= 2.97\bar{S}_0 a/b \\ N_{x3}^{III} &= -2.37\bar{S}_0 a/h_{ed} \\ N_{x4}^{III} &= (1.185\bar{S}_0/1.685)\,(a/h_{ed}) \end{aligned}\right\} \quad (V.19)$$

In addition, it is necessary to take into account the load applied directly to the edge members, in particular, their self-weight, q_{ed} (state IV, Fig. V.11*d*). Then, the forces at the faces of an edge member are given by

$$N_{x4}^{IV} = -N_{x3}^{IV} = 3q_{ed}a^2/h_{ed}^2 \quad (V.20)$$

Along the lines of contact between the shell and the edge members, their total longitudinal strains (and also the total longitudinal stresses, if their moduli of elasticity are equal) should be equal

$$\left.\begin{aligned} &\Sigma\varepsilon_{sh} = \Sigma\varepsilon_{ed};\quad \Sigma N_x^{sh}/hE = \Sigma N_x^{ed}/b_{ed}E \\ &\Sigma N_x^{sh}/h = \Sigma N_x^{ed}/b_{ed} \end{aligned}\right\} \quad (V.21)$$

Using the above expressions, we may re-write the equality of stresses as follows

$$(2.97\bar{S}_0/h)\,(a/b) = (8/3)\,qR_y a^2/bF_{ed} - 2.37\bar{S}_0 a/F_{ed} - q_{ed}a^2/2W_{ed} \quad (V.22)$$

On solving the above equation for $\bar{S}_0$, we add together the solutions for all four states. Figure V.11*e* shows the resultant diagram for N_x.

This solution makes it possible to calculate the forces, N_{xy}, at any point of the roof. In long cylindrical shells, the force N_y, is of minor importance.

The moments, M_y, acting across the barrel may be calculated using the above method (see Fig. V.10).

Near the diaphragms, the shell is subjected to local moments, M_x.

Recalling that

$$k_x = 0; \qquad k_{xy} = 0; \qquad k_y = 1/R_y$$

and $N_y = EF\varepsilon_y = E\,(1 \times h)\,(\partial v/\partial y - w/R_y)$

$$= (\text{approx.}) - Ehw/R_y$$

we may re-write Eq. (V.4) as

$$(s^4/4)\,(d^4w/dx^4) + w = R_y^2 q/Eh \tag{V.23}$$

Recalling that

$$-D\,d^2w/dx^2 = M_x$$

the solution of this equation (with the origin placed at the diaphragm) is

$$M_x = C_1 e^{-\varphi}\cos\varphi + C_2 e^{-\varphi}\sin\varphi \tag{V.24}$$

where C_1 and C_2 are the integration constants:

$$\varphi = x/s \quad \text{and} \quad s = 0.76\sqrt{R_y h}$$

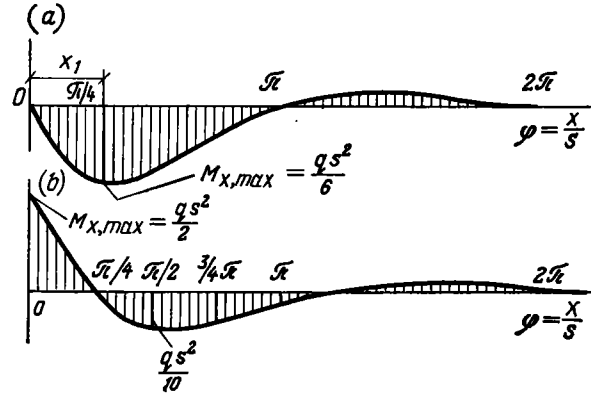


Fig. V.12. Bending moment diagrams
(*a*) with the shell hinged to the diaphragm; (*b*) with the shell fixed at the diaphragm

In prefabricated structures where shells are hinged to diaphragms

$$M_x = 0.5s^2 qe^{-\psi}\sin\varphi = 0.289\ qR_y he^{-\psi}\sin\varphi \tag{V.25}$$

The maximum bending moment

$$M_{x,\,max} = qs^2/6 = 0.0937qR_y h \tag{V.26}$$

is at the distance

$$x_1 = \pi s/4 = 0.597\sqrt{R_y h} \tag{V.27}$$

In multi-barrel shells where the single-barrel shells and the diaphragms are rigidly connected to one another, the bending moment is

$$M_x = 0.5qs^2 e^{-\varphi}\,(-\cos\varphi + \sin\varphi) \tag{V.28}$$

The bending moment reaches its maximum above the intermediate diaphragm

$$M_{x,\max} = -qs^2/2 = -0.289qR_y h \qquad (V.29)$$

The diagrams for these moments are shown in Fig. V.12.

Example V.1. *Given*: a roof using a long cylindrical shell; $l_1 = 2a = 30$ m; $l_2 = 2b = 18$ m; $R_y = 17.45$ m; $f = 2.5$ m; $h = 8$ cm; $h_{ed} = 1.5$ m; $b_{ed} = 15$ cm, load applied to the cylindrical part of the shell, $q = 3\,670$ N/m²; load due to the self-weight of the edge member, $q_{ed} = 5\,400$ N/m².

To find: The longitudinal stresses, σ_x.

Solution. Equation (V.22) gives

$$(2.97 \times \overline{S}_0 \times 1\,500)/(8 \times 900) = (8 \times 0.367 \times 1\,745 \times 1\,500^2)/(3 \times 900 \times 15 \times 150) - (2.37 \times \overline{S}_0 \times 1\,500)/(15 \times 150) - (54 \times 1\,500^2 \times 6)/(2 \times 15 \times 150^2)$$

Hence, $\overline{S}_0 = 373$ N/cm.

Calculate the stresses, taking into account the thickness of the structure: at point *1*, Eqs. (V.14) and (V.19) yield

$$\sigma_{x1} = -qR_y\,(a/b)^2\,(1/h) - 0.593\overline{S}_0\,(a/b)\,(1/h)$$
$$= -0.367 \times 1\,745 \times (1\,500/900)^2/8 - 0.593 \times 373 \times (1\,500/900)/8 = -222 - 45 = -267 \text{ N/cm}^2$$

at point *2*, Eq. (V.19) gives

$$\sigma_{x2} = 2.97\overline{S}_0\,(a/b)\,(1/h) = 2.97 \times 373\,(1\,500/900)/8 = 231 \text{ N/cm}^2$$

at point *4*, Eqs. (V.14), (V.15), (V.19) and (V.20) yield

$$\sigma_{x4} = 1.185\overline{S}_0 a/1.685h_{ed}b_{ed} + 3q_{ed}a^2/h^2{}_{ed}b_{ed}$$
$$= (1.185 \times 373 \times 1\,500)/(1.685 \times 150 \times 15) + (3 \times 54 \times 1\,500)/(150^2 \times 15) = 175 + 1\,080 = 1\,255 \text{ N/cm}^2$$

These results are close to those obtained by the design of the roof as an inscribed folded system with allowance for moments across the barrel (known as the Vlasov method*).

Long cylindrical shells strained under load are considered to be stable if the normal stresses, $\sigma_x = N_x/h$, and the tangential stresses, $\tau = N_{xy}/h$, determined for the elastic state of the structure do not exceed the following values

$$\sigma = E_c h/16R_y \quad \text{and} \quad \tau = 0.075E_c\,(h/R_y)^{3/2} \qquad (V.30)$$

where h is the thickness of a plain shell.

For ribbed shells, the above expressions use assumed ("fictitious") quantities

$$h_f = \sqrt{12I/F} \quad \text{and} \quad E_{c.f} = E_c F/bh_f \qquad (V.31)$$

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* P. L. Pasternak et al., Special Course on Reinforced Concrete Structures. Moscow, Stroiizdat, 1961 (*in Russian*).

where b is the centre-to-centre distance between the ribs, and F and I are the area and the moment of inertia of a T-section consisting of a rib and a flange of width b, respectively.

The results of the static analysis are used to determine the cross-sectional area of the shell reinforcement (Fig. V.13). The cross-

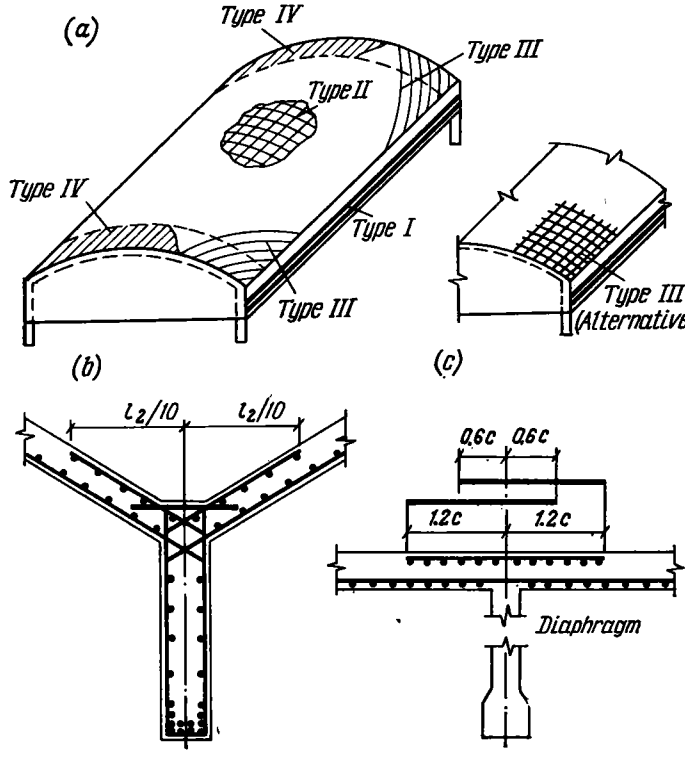


Fig. V.13. Reinforcement of a long shell
(a) shell; (b) shell reinforcement near an intermediate edge member; (c) same, above an intermediate diaphragm

sectional area of the longitudinal tensile steel of type I (if the shell is analysed as a space system) is computed by the following formula

$$F_s = Z_{max} / 1.1 R_s \qquad (V.32)$$

Here, Z_{max} is the amount of tensile forces from the diagram for N_x and 1.1 is the factor taking care of the increase in the arm of the internal couple due to the concentration of this reinforcement at the bottom of the edge members.

In roofs using vertical edge members, 80% of F_s is disposed within the edge member, of which 60% is concentrated at the bottom. In shells with other types of edge members, 40 to 60% of F_s is distributed over the bottom part of the edge member, and the rest in the tension zone of the shell where the steel is disposed linearly. The cross-sectional area of steel in the tension zone should be at least 0.2% of that of the concrete.

Along the barrel the cross-sectional area of the longitudinal reinforcement of type I may be reduced in proportion to N_x, but at least 30% of the reinforcement should extend as far as the support. The cross-sectional area of the longitudinal reinforcement is reduced by using bars of a smaller diameter and by welding at the joints, rather than by cutting off the bars.

In the longitudinal direction, the compression zone of the shell is reinforced for structural reasons with bars 5 to 6 mm in diameter spaced 20 to 25 cm apart. The cross-sectional area of the reinforcement is found as for a slab in proportion to the maximum ordinates of the M_y diagram (see Fig. V.10*c*). Across the barrel, the reinforcing bars are placed according to the sign of the moment diagram.

The above bars are combined into a type II fabric which is distributed over the entire shell (Fig. V.13*a*).

Near diaphragms, the normal forces, N_x, are approximately equal to zero, whereas the shearing forces, N_{xy}, reach their maximum and give rise to the principal tensile forces equal to N_{xy} in magnitude and making an angle of 45° with the horizontal generatrix. In the regions where the principal tensile stresses exceed R_{ten}, they are taken up solely by the reinforcement. If the cross-sectional area of type II fabric proves to be insufficient, use is made of additional type III reinforcement (diagonal bars or square-mesh fabric) anchored in the edge members or diaphragms.

The regions where the shell adjoins the diaphragms are congested with type IV reinforcement which is designed according to the diagram for M_x (see Fig. V.12).

In multi-barrel shells, the regions near intermediate edge members are additionally reinforced with transverse bars 6 to 10 mm in diameter and spaced 10 to 20 cm apart (Fig. V.13*b*), which are intended to resist the moments, M_y, at the supports.

Within the regions of length c near supports in multi-barrel shells (see Fig. V.8), the forces, N_x, are assumed to be varying longitudinally in proportion to the bending moments, and the forces, N_{xy}, in proportion to the shearing forces of a similar multispan beam. Within regions c, the bending moments, M_y, are assumed to be distributed uniformly.

The cross-sectional area of the tensile steel above the intermediate diaphragms is also found by Eq. (V.32), but with the factor 1.1 dropped. In the middle half of the barrel arch, the tensile steel may

be distributed uniformly, whereas in the side quarters of the barrel arch, it should be disposed according to a triangular diagram. Along the shell, all of the steel is extended on either side of the diaphragms for at least $0.6c$ (Fig. V.13*c*), and half of the steel is extended as far as $1.2c$ reckoning from the intermediate diaphragm. Here, c is the distance from the support to the zero-moment section (see Fig. V.8).

The diaphragms resist the shearing forces acting in the middle surface of and transmitted from the shell (Fig. V.14). The static

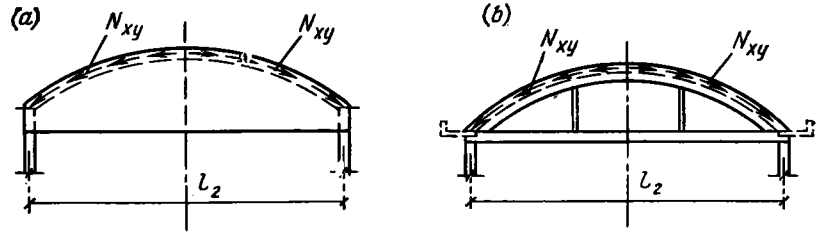


Fig. V.14. Transfer of forces from the shell to
(*a*) beam diaphragm; (*b*) arch diaphragm

analysis of diaphragms consists in determining the internal forces, M, N and Q, brought about by N_{xy}, taking into account the structural features of the diaphragms.

In truss diaphragms, the forces, N_{xy}, are transferred from the middle surface of the shell to the top chord axis with an appropriate bending moment applied, and are applied as concentrated loads at the truss joints. The longitudinal forces in the truss members are found and the members are proportioned in compliance with the general rules.

Bowstring arch diaphragms (Fig. V.14*b*) are similar to ordinary arches in construction. Under the action of N_{xy}, the middle part of an arch is in eccentrical tension, the regions at the supports are in eccentrical compression, and the string is in tension.

In practice, the component members of long precast cylindrical shell roofs can be fabricated with or without edge members.

In the former case (Fig. V.15*a*), all prefabricated members are assembled into a single structure by means of prestressed reinforcement passed through longitudinal ducts. Such structures, however, are very complex in shape, require very accurate alignment of reinforcement ducts, and are very expensive to erect because they need falsework.

In the latter case (Fig. V.15*b*), prefabricated members have a simpler shape and can be erected without falsework (shell panels are laid on edge members which are additionally supported during erec-

tion). The joints between shell panels and edge members, however, are rather complex because concrete keys are required for better transmission of shearing forces. In view of this, the quality of such joints is rather difficult to control.

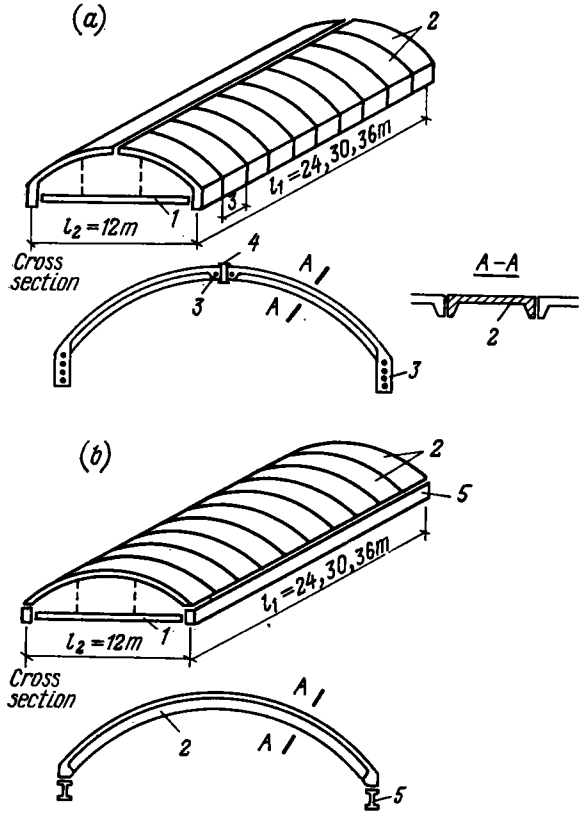


Fig. V.15. In-situ r.c. roofs using long cylindrical shells
(*a*) shell monolithically connected to edge members; (*b*) shell detached from edge members; *1*—diaphragm string; *2*—precast shell panel; *3*—prestressed reinforcement; *4*—fishplate; *5*—edge member

There also exist other ways of dividing cylindrical shells into precast component members. For example, good designs have been developed for shallow shell roofs. Outside the USSR, roofs built of long cylindrical shells are very popular. They are mostly cast in-situ as this meets the established local reinforced concrete practice.

2. Short Shells

As already mentioned, shells are rated as short if $l_1/l_2 < 1$ (Fig. V.16).

The practical recommendations for the design of in-situ r.c. short shells of $l_2 = 12$ to 30 m, $l_1 = 6$ to 12 m and $f \geqslant l_2/7$ are based on large-scale experiments. For structural reasons, the thickness

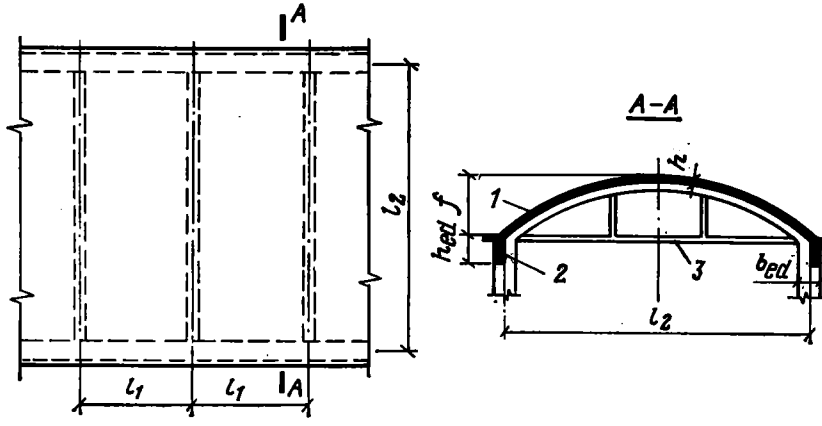


Fig. V.16. In-situ r.c. short cylindrical shell
1—barrel slab; *2*—edge member; *3*—diaphragm

of shells is taken as 5 or 6 cm for $l_1 = 6$ m, and 7 or 8 cm for $l_1 = 9$ to 12 m. The depth of edge members is assumed as $h_{ed} = l_1/10$ to $l_1/15$, and the width as $b_{ed} = 0.2\ h_{ed}$ to $0.4\ h_{ed}$. Shells are reinforced from structural considerations with fabric made of bars 5 to 6 mm in diameter and spaced 10 to 20 cm apart.

In the direction of l_1, a shell is analysed as a beam. At the midspan of a single-span single-barrel shell, the bending moment is given by

$$M = ql_2l_1^2/8 \tag{V.33}$$

The required cross-sectional area of the tensile steel is determined as

$$F_s = M/R_s z = ql_2l_1^2/4.5R_s\,(f + h_{ed}) \tag{V.34}$$

where z is the arm of the internal couple; according to calculations and tests, $z = 0.55\,(f + h_{ed})$.

The tensile steel thus determined is placed in the edge members. If we have a single-span multi-barrel shell, the cross-sectional area

of the reinforcement in intermediate edge members is taken equal to F_s, whereas in the marginal members it is taken as $F_s/2$. The cross-sectional area of the steel in the middle spans of a multi-span shell is taken as one half of that in the marginal spans.

The longitudinal bars of the edge member reinforcement are combined into welded mats, in which the transverse bars are disposed to suit structural requirements. Near the edge members, the shell is reinforced with additional fabric pieces (Fig. V.17*a*). Also, additional reinforcement is provided above the diaphragms extending

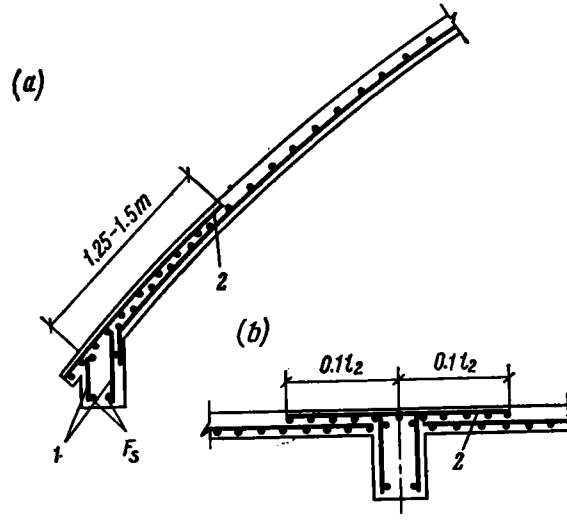


Fig. V.17. Reinforcement of in-situ r.c. short shells
(*a*) at the edge members; (*b*) above intermediate diaphragms; *1*—welded-bar mats; *2*—additional welded-bar mats

for $0.1l_1$ on either side of the diaphragm (Fig. V.17*b*). In either case, additional reinforcement is of the same type as the main fabric.

In the direction of l_2, the diaphragm is analysed in interaction with the shell (Fig. V.18).

Experiments and calculations have shown that, under load, a statically determinate diaphragm (an arch with a cut string) is in compression, and the maximum compressive force $\bar{N}_{y,\max} = -qRl_1$ (where R is the radius of curvature of the shell) is at the apex. Along the arch, the compressive force varies as a quadratic parabola

$$\bar{N}_y = -4qRl_1 y\,(l_2 - y)/l_2^2 \qquad \text{(V.35)}$$

From the condition for equilibrium, we may conclude that the diaphragm resists the forces of the same magnitude but opposite in sense. So the internal forces in statically determinate diaphragms should be determined as

$$\left.\begin{aligned} M_y &= M_y^0 - \bar{N}_y c_y; \quad Q = Q_y^0 \\ N_y &= N_y^0 + \bar{N}_y \end{aligned}\right\} \tag{V.36}$$

where M_y^0, Q_y^0 and N_y^0 are the moment and the forces calculated about the diaphragm axis as in a statically determinate system under total vertical load; and c_y is the distance from the diaphragm axis to the middle surface of the shell.

Once the forces in a statically determinate diaphragm are found, the effect of the string is taken account of by the usual methods of

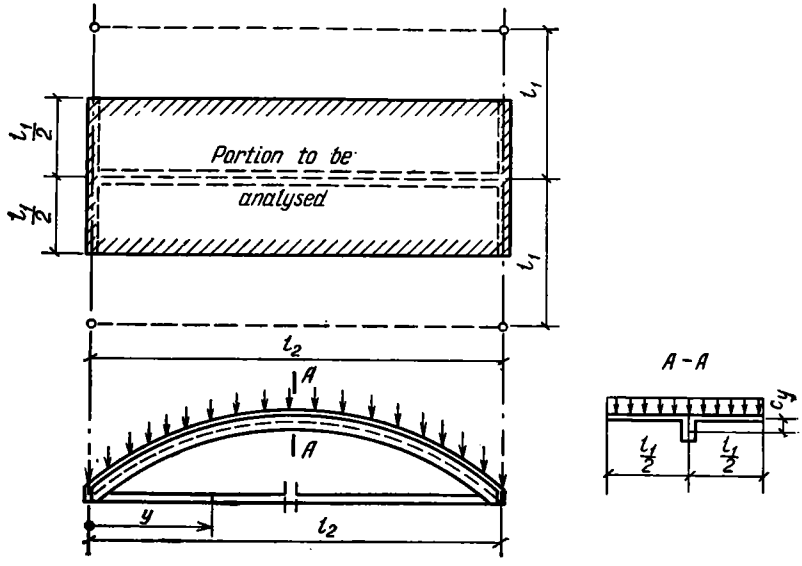


Fig. V.18. Loading system for the diaphragm of a short shell

analysis applicable to statically indeterminate systems. Here, the shell parts adjoining the diaphragm are not taken into consideration, and the forces due to redundant unknowns are resisted solely by the diaphragm sections.

Prefabricated roofs using short cylindrical shells consist of diaphragms, roof channel slabs, and edge members (Fig. V.19*a*). The joints

between the prefabricated slabs should properly be concreted and crossed by anchors. The joints between the slabs and diaphragms are strengthened by concrete keys.

Among the advantages of prefabricated roofs are the ease of member manufacture and erection, and the increased overall rigidity of the system. This, however, is somewhat offset by the complex construction of the joints between the roof slabs and trusses.

Another type of short cylindrical shell roofs uses large-size reinforced concrete barrel slabs 3 m wide roofing over areas 12 to 24 m in span (Fig. V.19*b*). A large-size reinforced concrete barrel slab

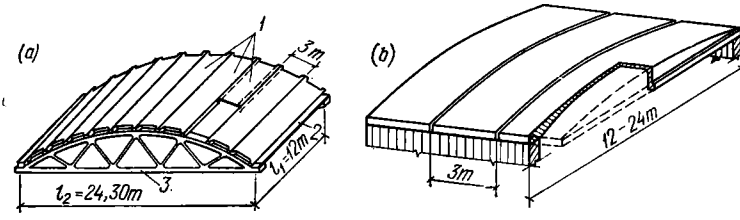


Fig. V.19. Precast roofs using short cylindrical shells
(*a*) trusses and roof slabs; (*b*) large-size r.c. barrel slabs; *1*—precast slab; *2*—edge member; *3*—truss diaphragm

is a shallow thin cylindrical shell curving in the longitudinal direction. It is strengthened with two longitudinal variable-section diaphragms serving as ribs, and transverse edge members. The main prestressed load-bearing reinforcement is disposed in the ribs. Large-size r.c. barrel slabs have found wide application in building construction.

3. Prismatic Folded-Plate System

A folded-plate roof consists essentially of flat slabs (monolithically connected to one another at their longer sides), edge members, and diaphragms (Fig. V.20*a*).

Folded-plate roofs may bridge one or several longitudinal and transverse spans. In the longitudinal direction, the design of folded plates is based on the same assumptions as that of long cylindrical shells.

In the transverse direction, folded-plate roofs behave in bending as multi-span beam slabs with a broken axis (the ribs are treated as supports) (Fig. V.20*b*). The width of the side faces usually ranges between 3 and 3.5 m. The transverse span l_2 of three-face folded plates is from 9 to 12 m. The longitudinal span l_1 usually exceeds l_2, and the thickness of folded plates is taken as $l_1/7$ to $l_1/10$.

In the transverse direction, folded plates are reinforced as multi-span slabs, to fit the bending moment diagram. The longitudinal compressive reinforcement (in the direction of l_1) is placed for structural reasons; it consists of bars 5 to 8 mm in diameter spaced 20

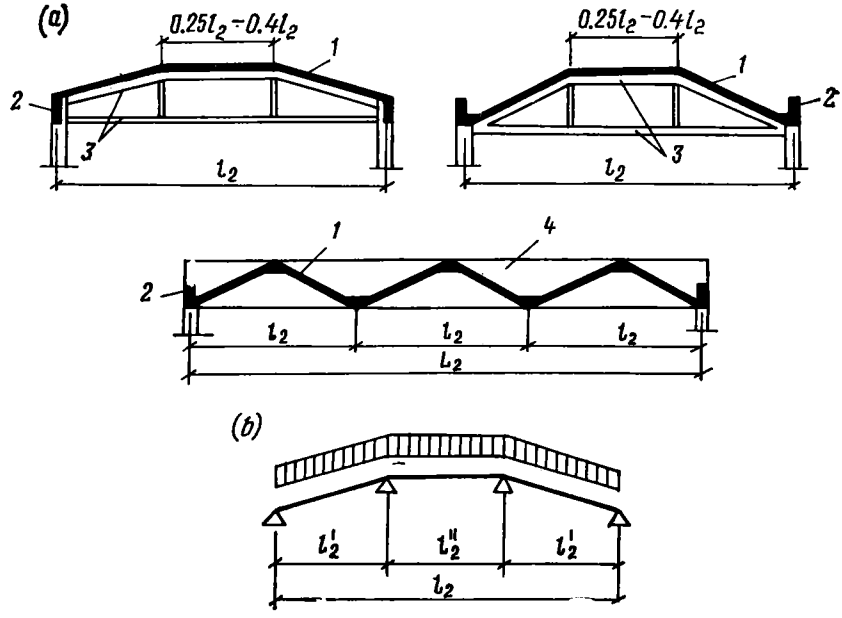


Fig. V.20. Prismatic folded-plate roofs
(a) types of cross sections and diaphragms; (b) loading system for a folded plate in the transverse direction; 1—folded plate; 2—edge member; 3—queenpost diaphragm; 4—beam diaphragm

to 25 cm apart. The amount of longitudinal tensile reinforcement in the direction of l_1 is found by calculations; this reinforcement is placed in the edge members.

In other respects, folded-plate roofs are analysed and designed as roofs built of long cylindrical shells.

POSITIVE GAUSSIAN CURVATURE
V.4. RECTANGULAR-PLAN ROOFS MADE OF SHELLS OF

Rectangular-plan reinforced concrete roofs using shells of positive Gaussian curvature use 25 to 30% less material than roofs built of cylindrical shells. In such structures, the supports can be spaced wider apart, which is valuable for industrial and civil buildings.

This type of roof consists essentially of a doubly-curved thin shell and edge diaphragms monolithically connected with the shell

(see Figs. V.1*e* and V.21*a*). The roof may be supported on columns placed at the corners or around the periphery.

Among the multiplicity of curvilinear surfaces, the most attractive ones are the elliptic paraboloid and the translational circular surface.

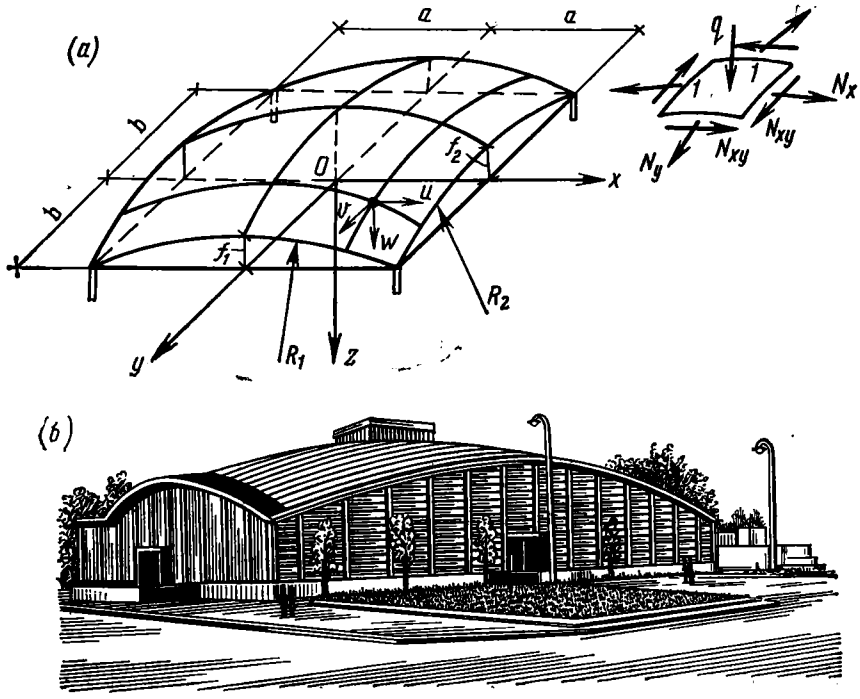


Fig. V.21. Rectangular-plan roof made of shells of positive Gaussian curvature (*a*) design system; (*b*) unit-size shell element; (*c*) roof of a supermarket in Moscow (made of prefab members)

Doubly-curved shells are mostly made shallow, with the ratio of the rise to any dimension in plan being not over 1 : 5.

In shallow shells, the surface of the elliptic paraboloid and the translational circular surface having the same base and rise almost coincide. So, in practice, they are interchangeable.

As already noted, thin shells have poor flexural rigidity, and may be analysed as moment-free structures, that is, in terms of N_x, N_y and N_{xy} only (Fig. V.21*b*). Bending moments arising only in the regions of local bending may be determined by individual analysis.

Flat diaphragms supporting a single shell are regarded as absolutely flexible at right angles to their plane. In contrast, in their plane, most diaphragms (such as beams and some trusses) may be

deemed nondeformable. In view of this, when $x = \pm a$ and $y = \pm b$, we should have $N_x = N_y = 0$.

If the coordinate axes in the translational shell of Fig. V.21*a* coincide with the main curvatures, the torsional curvature is $k_{xy} = 0$, and the stress function satisfying the boundary conditions may be taken as

$$\begin{aligned}\varphi(xy) = {} & a_1 (x^4 - 6x^2a^2 + 5a^4)(y^4 - 6y^2b^2 + 5b^4) \\ & + a_2 [x^8 - (22/9)\, x^6a^2 + (13/9)\, x^4a^4](y^4 - 6y^2b^2 + 5b^4) \\ & + a_3 (x^4 - 6x^2a^2 + 5a^4)[y^8 - (22/9)\, y^6b^2 + (13/9)\, y^4b^4] \\ & + a_4 [x^8 - (22/9)\, x^6a^2 + (13/9)\, x^4a^4] \\ & \times [y^8 - (22/9)\, y^6b^2 + (13/9)\, y^4b^4] \end{aligned} \tag{V.37}$$

where a_1, a_2, a_3 and a_4 are the constants.

According to Eqs. (V.2), the respective forces are

$$\begin{aligned} N_x = \partial^2\varphi/\partial y^2 = {} & 12a_1 (x^4 - 6x^2a^2 + 5a^4)(y^2 - b^2) \\ & + 12a_2 [x^8 - (22/9)\, x^6a^2 + (13/9)\, x^4a^4](y^2 - b^2) \\ & + 4a_3 (x^4 - 6x^2a^2 + 5a^4)[14y^6 - (55/3)\, y^4b^2 + (13/3)\, y^2b^4] \\ & + 4a_4 [x^8 - (22/9)\, x^6a^2 + (13/9)\, x^4a^4] \\ & \times [14y^6 - (55/3)\, y^4b^2 + (13/3)\, y^2b^4] \end{aligned} \tag{V.38}$$

$$\begin{aligned} N_{xy} = -\partial^2\varphi/\partial x\, \partial y = {} & -16a_1 (x^3 - 3xa^2)(y^3 - 3yb^2) \\ & - 16a_2 [2x^7 - (11/3)\, x^5a^2 + (13/9)\, x^3a^4](y^3 - 3yb^2) \\ & - 16a_3 (x^3 - 3xa^2)[2y^7 - (11/3)\, y^5b^2 + (13/9)\, y^3b^4] \\ & - 16a_4 [2x^7 - (11/3)\, x^5a^2 + (13/9)\, x^3a^4] \\ & \times [2y^7 - (11/3)\, y^5b^2 + (13/9)\, y^3b^4] \end{aligned} \tag{V.39}$$

The constants for the roofs of square buildings (which are very popular in practice) with $a = b$, $R_x = R_y = R$ and $q = \text{const}$ are as follows*:

$$a_1 = 0.00833qR/a^6; \quad a_2 = a_3 = 0.0365qR/a^{10} \text{ and}$$
$$a_4 = 0.353qR/a^{14} \tag{V.40}$$

After the forces, N_x, N_y and N_{xy}, have been found, the principal forces and the angles that they make with the x-axis are determined by the following expressions

$$\left.\begin{aligned} N_{prin\,I} \\ N_{prin\,II} \end{aligned}\right\} = (N_x + N_y)/2 \pm \sqrt{[(N_x - N_y)/2]^2 + N_{xy}^2}$$
$$\left.\begin{aligned} \operatorname{tg} 2\alpha_1 \\ \operatorname{tg} 2\alpha_2 \end{aligned}\right\} = \mp 2N_{xy}/(N_x - N_y) \tag{V.41}$$

* This is an approximate solution yielded by the method of collocations, using Eqs. (V.1).

The force diagrams for a square-plan shell having $a = b$ and $R_x = R_y = R$, and subjected to a uniformly distributed load are shown in Fig. V.22. Here, the magnitudes of the forces are given for individual points.

Referring to the diagrams, almost all of the shell resists biaxial compression, and only the regions at the corners carry compressive forces in one direction and tensile forces in the other (Fig. V.22*c*).

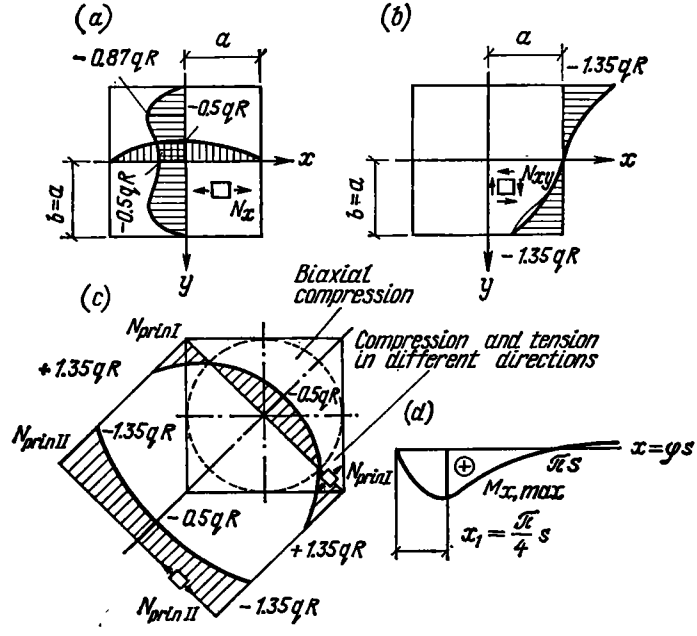


Fig. V.22. Internal force diagrams for a rectangular-plan shell
(*a*) N_x at $x = 0$ and $y = 0$; (*b*) N_{xy} at $x = a$; (*c*) principal forces at $x = y$; (*d*) bending moments, M_x, in area of local bending at $x = -a$

However small, the bending moments arising near the supports must be taken into consideration.

If the joint between the shell and the diaphragm acts as a hinge (with respect to bending moments), the solution of Eq. (V.4) for the region, say, along the edge with $x = -a$ gives Eqs. (V.25) through (V.27).

The diaphragms are designed to resist the shearing forces, N_{xy}, transferred from the shell, and the self-weight.

Example V.2. *Given*: a low-pitched shell; $2a = 2b = 40$ m; $f = 6$ m; $R_x = R_y = R = 68.2$ m; $h = 7$ cm; and $q = 5\,000$ N/m^2.
To find: The internal forces.

Solution. Find the necessary forces using the data of Fig. V.22.

Calculate the maximum compressive force in the central part of the shell (in kN and m)

$$N_{x,\max} = N_{y,\max} = -0.5qR = -0.5 \times 5 \times 68.2 = -170.5 \text{ kN/m}$$

Compute the maximum compressive force outside the central part

$$N_{x,\max} = -0.87qR = -0.87 \times 5 \times 68.2 = -297 \text{ kN/m}$$

Determine the maximum principal compressive, tensile and shearing forces in the corners of the shell

$$N_{prin\ I} = -N_{prin\ II} = N_{xy,\max} = 1.35qR$$
$$= 1.35 \times 5 \times 58.2 = 460 \text{ kN/m}$$

Using Eq. (V.26), find the maximum bending moment with the origin located at the diaphragm

$$M_{x,\max} = 0.0937Rhq = 0.0937 \times 68.2 \times 0.07 \times 5 = 2.24 \text{ kN m}$$

which is applied at the distance given by Eq. (V.27)

$$x_1 = 0.597\sqrt{Rh} = 0.597\sqrt{68.2 \times 0.07} = 1.3$$

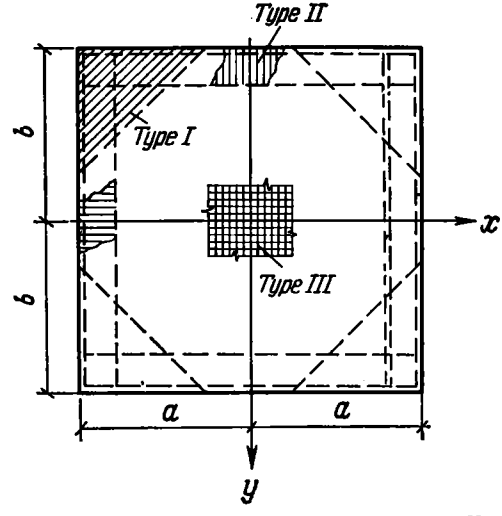


Fig. V.23. Reinforcement of a shallow rectangular-plan shell of positive Gaussian curvature

The shells are reinforced according to the internal forces arising under external load (Fig. V.23).

The shell corners are reinforced with type I diagonal steel intended to take up the principal tensile forces; near the edges, use is made of type II reinforcement intended to resist local bending moments; and the bulk of the shell is reinforced with type III steel placed for structural reasons. It is advisable to prestress the type I reinforcement.

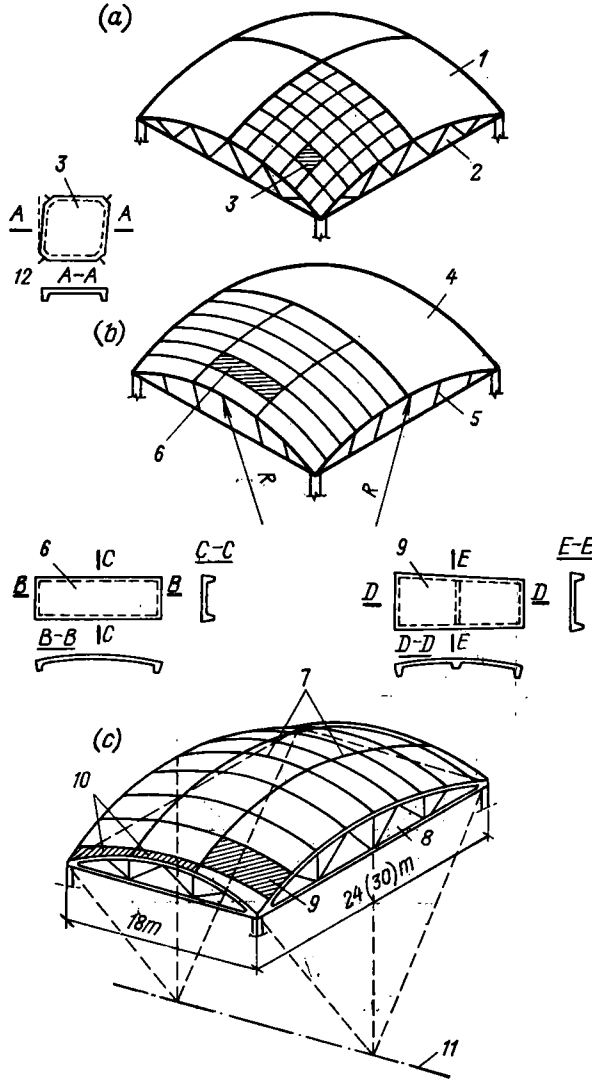


Fig. V.24. Roofs using shallow shells of positive Gaussian curvature

(*a*) translational shell (made of flat slabs 3 m square); (*b*) spherical shell (made of cylindrical panels 3 × 12 m in the shell surface); (*c*) shell of revolution (assembled from cylindrical members); *1*—translational surface; *2*—vertical diaphragm; *3*—prefab member of a shell; *4*—spherical surface; *5*—inclined edge structure; *6*—prefab member of a shell; *7*—surface of revolution with a horizontal axis; *8*—camel-back truss; *9*—type-design prefab member of the side zone; *10*—supplementary members; *11*—axis of revolution; *12*—reinforcement stick-outs

The connection between the shell and the diaphragm is designed in terms of the shearing forces, N_{xy}. The most commonly used diaphragms are beams, trusses and bowstring arches. The strings and the bottom chords of trusses are prestressed.

At the corners, the shell resists the maximum compressive forces acting in the diagonal direction. Here, the shell thickness is customarily increased.

The stability of these shells is analysed using the same equations as for shells of revolution (see Sec. V.6).

In Soviet practice, roofs using prefabricated shallow shells of positive Gaussian curvature can be laid out in one of three ways. In the first (Fig. V.24*a*), a translational shell is divided into slabs having the same nominal dimensions in plan (3 m square). The slabs are made flat and strengthened with ribs around the periphery. In the central part of the shell, the slabs are square, and near the shell edges, diamond-shaped. The corner slabs have diagonal ribs with longitudinal ducts intended for prestressed corner reinforcement. Where the shell is designed to resist large shearing forces, the joints between the slabs are made with concrete keys. Among the drawbacks of this system are the relatively small size of the prefabricated members, the high cost of and labour consumption in erection as it requires elaborate jigs and conductors, and a large number of connections and welded joints.

In the second (Fig. V.24*b*), a spherical shell is divided into cylindrical panels with nominal dimensions of 3×12 m in the shell surface. This type is free from the drawbacks characteristic of the previous one, but the cylindrical panels are difficult to manufacture and transport, and inclined planes in the edge structures are undesirable from the architectural point of view.

In the third (a type design for column grids 18×24 m and 18×30 m, see Fig. V.24*c*), a shell of revolution (with a horizontal axis) is divided into three zones, namely a middle zone consisting of identical rectangular-plan cylindrical ribbed slabs with nominal plan dimensions of 3×6 m, and two side zones made of identical trapezoidal-plan cylindrical slabs. Also supplementary members are installed near the edge members. The edge structure is a camelback truss with a triangular web. This design may be used in buildings having several spans in both directions.

V.5. RECTANGULAR-PLAN ROOFS MADE OF SHELLS OF NEGATIVE GAUSSIAN CURVATURE

We shall discuss rectangular-plan roofs using shells of negative Gaussian curvature with curvilinear surfaces of the second order. Two types of shells of this configuration are in general use. In the first type, the lines of principal curvatures are parallel to the base

sides (Fig. V.25*a*); in the second type, the lines of principal curvatures are directed along the base diagonals (Fig. V.25*b*).

Shells of the first type can be regarded as translational shells, and designed as discussed above. Also, it is necessary to remember that the forces, N_x, are tensile, because the curvature of the surface in the direction of the $0x$-axis is negative. In the direction of the

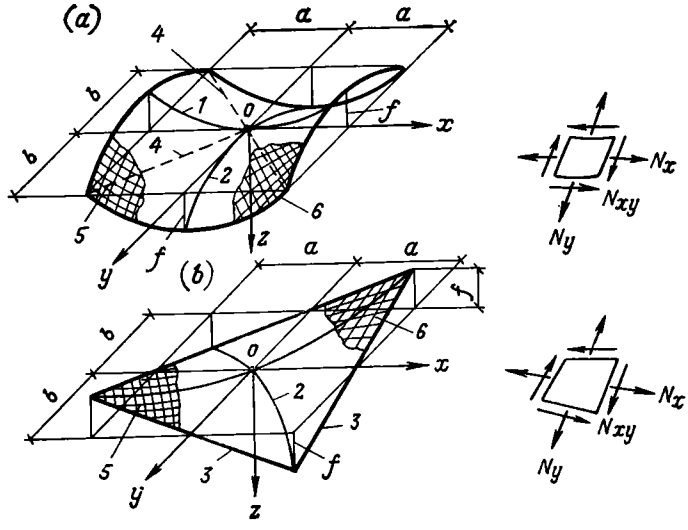


Fig. V.25. Rectangular-plan shells of negative Gaussian curvature
(*a*) principal curvature lines parallel to the base sides; (*b*) rectilinear generatrices parallel to the base sides; *1*—line of principal negative curvature; *2*—line of principal positive curvature; *3*—rectilinear generatrix; *4*—straight lines in the surface; *5*—curvilinear-bar reinforcement; *6*—straight bar reinforcement

positive curvature, the shell remains in compression. All of the tensile forces, N_x, should be taken up by the load-bearing reinforcement which should be prestressed.

The surface of the second type is described by the following equation

$$z = (f/ab)\, xy \tag{V.42}$$

Its curvatures are

$$\left.\begin{aligned} k_x &= \partial^2 z/\partial x^2 = 0; \quad k_y = \partial^2 z/\partial y^2 = 0 \\ k_{xy} &= \partial^2 z/\partial x\, \partial y = f/ab \end{aligned}\right\} \tag{V.43}$$

In this case, the equilibrium equation (V.1) reduces to

$$+2\,(f/ab)\,(\partial^2\varphi/\partial x\, \partial y) = +q \tag{V.44}$$

Let us consider a shell subjected to a uniformly distributed load, q.

The stress function is

$$\varphi = (qab/2f)\, xy \tag{V.45}$$

It satisfies the boundary conditions: $N_x = 0$ at $x = \pm a$, and $N_y = 0$ at $y = \pm b$ (owing to the absolute flexibility of the edge structures at right angles to their plane), and also the equilibrium equation (V.44).

According to Eqs. (V.2),

$$\left.\begin{aligned} N_x &= \partial^2\varphi/\partial y^2 = 0; \quad N_y = \partial^2\varphi/\partial x^2 = 0 \\ N_{xy} &= -\partial^2\varphi/\partial x\, \partial y = -qab/2f \end{aligned}\right\} \tag{V.46}$$

Thus, N_x and N_y are equal to zero not only at the shell boundaries, but also over the entire shell; the shearing forces, N_{xy}, are constant and oppose the forces acting in the shell element of Fig. V.25*b*. As a whole, the shell is subjected to pure shear.

In practice, most convenient surfaces are those having $a = b$, such as equilateral hyperbolic paraboloids. In such shells, the principal forces (those directed along the principal curvatures) and their directions are given by Eqs. (V.41):

$$\left.\begin{aligned} N_{prin\,I} &= -N_{prin\,II} = qab/2f \\ \alpha_{prin\,I} &= -\alpha_{prin\,II} = -45^\circ \end{aligned}\right\} \tag{V.47}$$

Referring to the above equations, the shell resists constant tensile forces along the line of the negative principal curvature, and compressive forces along the line of the positive principal curvature.

All of the principal tensile forces should be taken up by the load-bearing reinforcement of one direction diagonal in plan (that is, by the curvilinear reinforcement) or by the load-bearing reinforcement of two directions along the edges (that is, by the rectilinear reinforcement) as shown in Fig. V.25*b*.

The shearing forces are transferred from the shell to the edge structures. If the edge structures are rigid walls (Fig. V.26*a*), they are capable of resisting shear without additional supports; in the case of trusses (Fig. V.26*b*), additional supports must be installed in order to resist the lateral truss deflections brought about by the shearing forces, N_{xy}, transferred from the shell (position *3* in Fig. V.26*b* and *c*), or a diagonal tie rod.

From the equilibrium of forces in the truss (zero total moment about support, B, with N_{xy} applied along the top chord axis) (see Fig. V.26*b*), we find that the reaction of support, A, is zero. So, the uniformly distributed load in the system of Fig. V.26*b* is taken up only by two supports, B and D, whereas supports, A and C, are unloaded. In this case, no stresses develop in the bottom chord and web members of the truss.

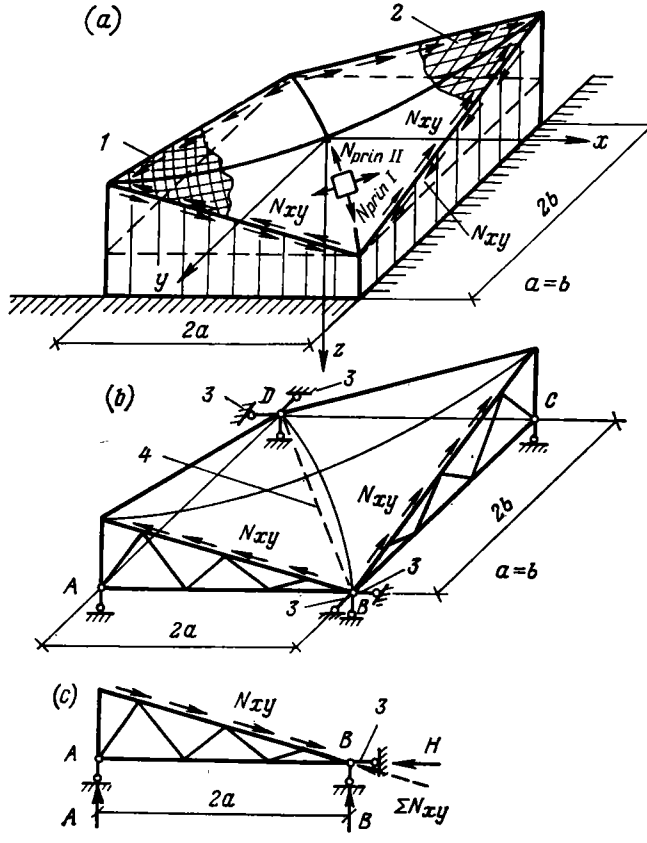


Fig. V.26. Shells of negative Gaussian curvature

(*a*) shells supported on a wall around the periphery; (*b*) same, on trusses; (*c*) truss subjected to the shearing forces transferred from the shell; *1*—curvilinear load-bearing reinforcement; *2*—straight bar reinforcement alternative; *3*—horizontal truss supports; *4*—tie rod substituting horizontal supports

The division of shells of negative Gaussian curvature into members for the purpose of prefabrication is similar to that shown in Fig. V.24.

V.6. DOMES

Domes display an especially good spatial behaviour. Also, they are more economical of materials than other space structures. A dome roof consists essentially of a shell and a ring-beam support (see Figs. V.1*c* and V.29*a*). In domes with a central aperture, a top ring beam to hem the aperture is installed.

A dome resting on a continuous hinged movable support coinciding with the tangent to the shell is a statically determinate structure (Fig. V.27*a*). Like other space structures, thin domes may be analysed by the moment-free theory.

A dome element bounded by two meridians and two parallels is subjected to the meridian force, N_1, parallel force, N_2, and shearing

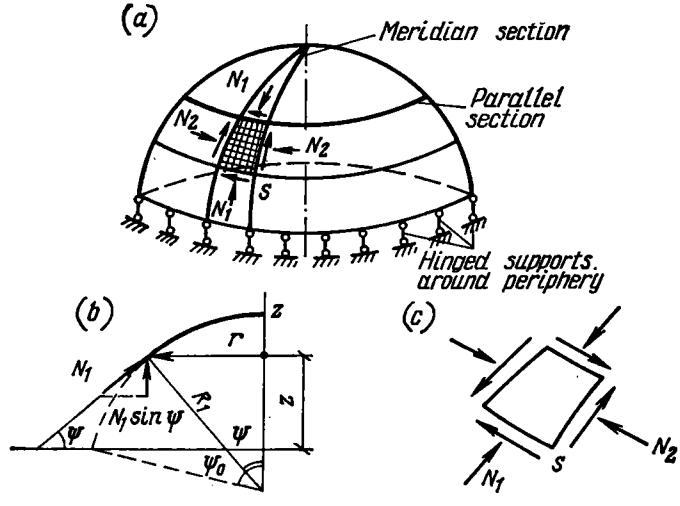


Fig. V.27. To the design of a dome
(*a*) dome resting on a continuous hinge movable support; (*b*) dome part bounded by a plane parallel to the base; (*c*) dome element and the internal forces

lorce, S (Fig. V.27*c*), referred to a unit length of the section. If the load imposed on a dome is axisymmetric, S is equal to zero.

Let us denote the current angular coordinate as ψ and the load imposed on the segment bounded by ψ as Q_ψ. From the equilibrium of forces in the dome element (Fig. V.27*b*), we find that

$$N_1 = Q_\psi/2\pi r \sin\psi \tag{V.48}$$

The horizontal projection, H, of the force, N_1, is termed the thrust:

$$H = N_1 \cos\psi = (Q_\psi \operatorname{ctg}\psi)/2\pi r \tag{V.49}$$

On setting

$$N_x = -N_1; \qquad N_y = -N_2; \qquad N_{xy} = S = 0$$

$$k_x = 1/R_1 \quad \text{and} \quad k_y = 1/R_2$$

we get from Eqs. (V.1) and (V.2)

$$N_1/R_1 + N_2/R_2 = q_\psi \tag{V.50}$$

where q_ψ is the load component normal to the surface of the dome.

Consider a spherical dome in which $R_1 = R_2 = R$. Let us designate the load due to the self-weight of the spherical dome per unit surface as g; then we have (Fig. V.28*a*)

$$q_\psi = g \cos \psi \quad \text{and} \quad Q_\psi = 2\pi Rag \tag{V.51}$$

Using Eqs. (V.51) and the relations

$$a = R\,(1 - \cos \psi) \quad \text{and} \quad r = R \sin \psi$$

we find from Eqs. (V.48) and (V.50) that

$$N_1 = Rg/(1 + \cos \psi) \quad \text{and} \quad N_2 = gR \cos \psi - Rg/(1 + \cos \psi) \tag{V.52}$$

The diagrams for N_1 and N_2 for a semi-spherical dome are shown in Fig. 5.28*b* and *c*. It should be noted that when $\psi = 0$, $N_1 = Rg/2$

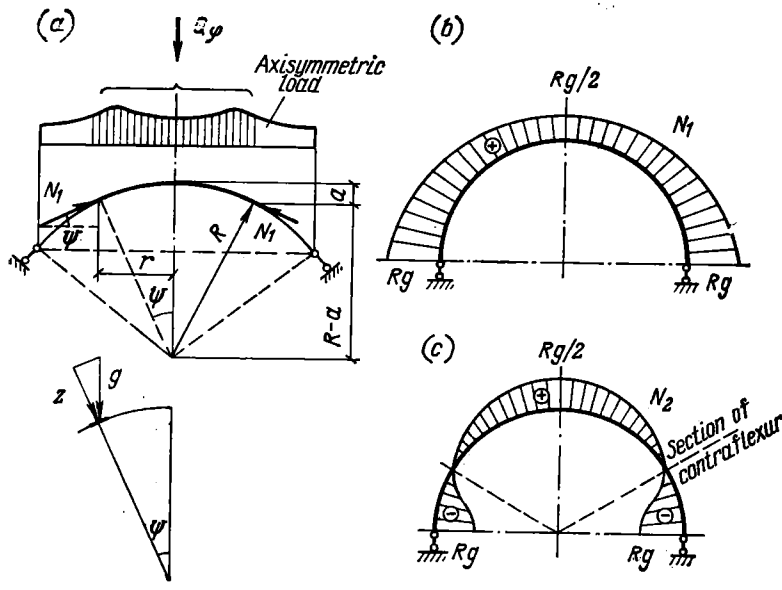


Fig. V.28. Statically determinate spherical dome
(*a*) loading system; (*b*) and (*c*) diagrams for N_1 and N_2 induced in a hemispherical dome by the roof self-weight

(compression) and $N_2 = Rg/2$ (compression); and when $\psi = \pi/2$, $N_1 = Rg$ (compression) and $N_2 = -Rg$ (tension).

The circular section where $N_2 = 0$, known as the section of contraflexure, is determined by the opening angle $\psi = 51°49'$.

For a spherical dome subjected to snow load, p, which is assumed to be uniformly distributed over the horizontal projection of the

dome and to vary in proportion to $\cos\psi$ over the dome surface, the solution may be obtained in a similar way

$$N_1 = pR/2 \quad \text{(constant along a meridian)}$$
$$N_2 = (pR\cos 2\psi)/2 \qquad \text{(V.53)}$$

The main loads governing the size of a dome are the self-weight of the shell together with the heat insulation and roofing materials, and snow load. Both loads are deemed to be axisymmetric. For shallow dome roofs, wind load is of minor importance and may be

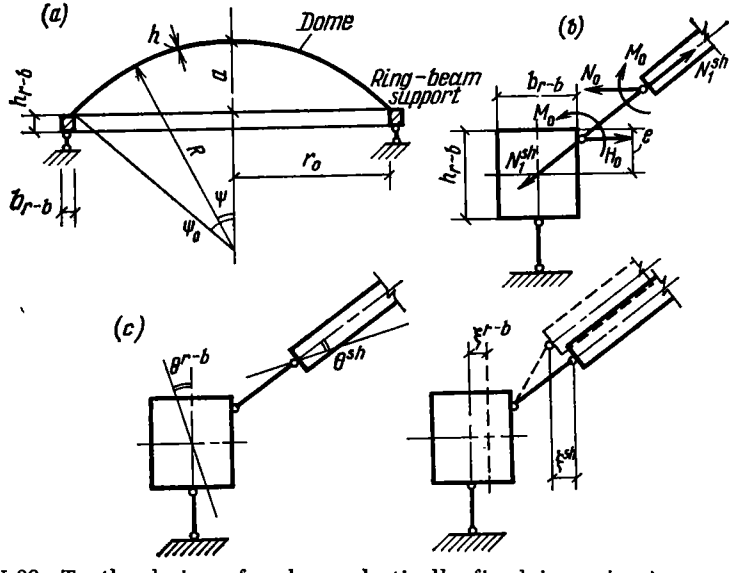


Fig. V.29. To the design of a dome elastically fixed in a ring-beam support (*a*) design system of the dome; (*b*) design system of a support joint; (*c*) positive directions of the angular and radial deflections of the shell and ring-beam support

neglected. In steep domes which are used less frequently, the forces due to wind load are determined by the methods discussed in a course on the theory of elasticity.

In real structures, a dome shell is elastically fixed in the ring-beam support, rather than simply supported (Fig. V.29*a*). Because of this, additional statically indeterminate forces—the meridian bending moment, M_0, and the radial thrust, H_0—arise at the shell edge (Fig. V.29*b*). Their values are found on the assumption that the shell and the ring-beam support deform together. The fixation has its effect on the shell only near the ring beam, which is superimposed on the moment-free state of stress of the shell.

The problem involving the edge forces in elastically fixed domes was first solved by P.L. Pasternak (USSR) in 1925-1927.

TABLE V.1. Formulas for Determining Deflections of Ring-Beam Support and Spherical Shell Edge

Forces and loads			
edge moment M_0	edge horizontal thrust H_0	roof self-weight g	snow load p
		Shell edge deflections	
$C_1 = M_0$	$C_1 = 0$	—	—
$C_2 = -M_0$	$C_2 = sH_0 \sin \psi_0$	—	—
$\theta_M^{sh} = -sM_0/D$	$\theta_H^{sh} = -s^2 H_0 \sin \psi_0/2D$	$\theta_g^{sh} = 2gR \sin \psi_0/Eh$	$\theta_p^{sh} = 3pR \sin 2\psi_0/2Eh$
$\xi_M^{sh} = -s^2 M_0 \sin \psi_0/2D$	$\xi_H^{sh} = -s^3 H_0 \sin^2 \psi_0/2D$	$\xi_g^{sh} = gR^2 \sin \psi_0 [-\cos\psi_0 + 1/(1+\cos \psi_0)]/Eh$	$\xi_p^{sh} = -pR^2 \sin \psi_0 \times \cos 2\psi_0/2Eh$
		Ring-beam deflections (at the joint)	
$\theta_M^{r\text{-}b} = 12r_0^2 M_0/Eb_k h_k$	$\theta_H^{r\text{-}b} = 12r_0^2 H_0 e/Eb_k h_k^3$	—	—
$\xi_M^{r\text{-}b} = 12r_0^2 M_0/Eb_k h_k^3$	$\xi_H^{r\text{-}b} = H_0 r_0^2/EF_k$	$\xi_g^{r\text{-}b} = R^3 g \cos \psi_0 \sin^2 \psi_0/EF_k(1 + \cos \psi_0)$	$\xi_p^{r\text{-}b} = R^3 p \cos \psi_0 \sin^2 \psi_0/2EF_k$

The state of the system in the local bending area is described by Eq. (V.23). The solution of this equation for the moment is given by Eq. (V.24), and for the deflections, w, normal to the shell surface, by the following equation

$$Dw = 0.5\, s^2\, (C_1 e^{-\varphi} \sin \varphi + C_2 e^{-\varphi} \cos \varphi) \qquad \text{(V.54)}$$

where

$s = 0.76\sqrt{Rh}$; $\varphi = x/s$ and $D = (\text{approx.})Eh^3/12$

Figure V.29*c* illustrates the positive directions of the angular deflections, θ^{sh} and θ^{r-b}, and the radial deflections, ξ^{sh} and ξ^{r-b}, of the shell edges and the ring-beam support at the joint, respectively.

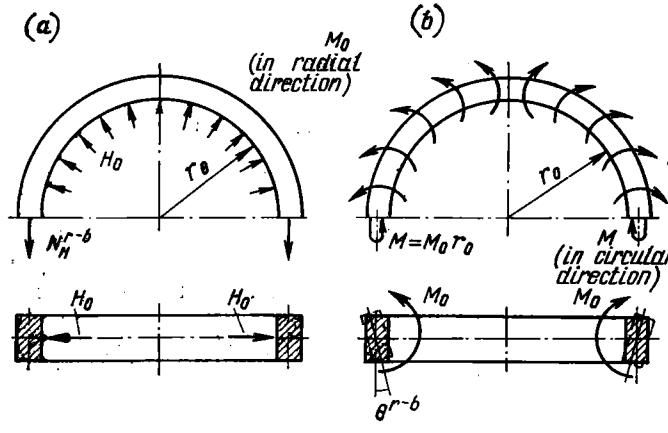


Fig. V.30. Design systems for a ring-beam support subjected to (*a*) thrust; (*b*) moments

Equation (V.54) makes it possible to find the deflections, θ^{sh} and ξ^{sh}, of the shell edge under load, moment, M_0, and thrust, H_0. Table V.1 gives the formulas for determining the deflections of the edge of a spherical shell. The shell parameters can be found in Fig. V.29*a*.

The joint between the shell and the ring-beam support is usually designed so that, with a moment-free support, the shell meridian forces, N_1^{sh}, induced by p and g would pass through the centroid of the ring-beam cross section (Fig. V.29*b*), thereby giving rise to only axial tension without bending. The forces, N_1^{sh}, bring about radial deflections of the ring beam, ξ_p and ξ_g (no angular deflections result in this case).

The thrust H_0 applied to the ring-beam support (Fig. V.30*a*) induces the tensile force, N_H^{r-b}, which gives rise to the radial deflection, ξ_H^{r-b}, of the ring-beam axis. The thrust, H_0, applied with eccen-

tricity, e (see Fig. V.29*b*), brings about the moment, H_0e, which causes the ring beam to rotate through $\theta_H^{r\text{-}b}$.

The moment, M_0, uniformly distributed over the ring beam causes its cross sections to rotate through the same angle, $\theta_M^{r\text{-}b}$ (Fig. V.30*b*). In this case, the ring-beam layer at the centroid is not deformed, the part of the section above the centroid is in tension, and that below the centroid in compression. As a whole, the ring beam is subjected to bending in a vertical axial direction.

The formulas for computing the ring-beam deflections are given in Table V.1.

The angular deflections of the shell edge and the ring-beam support at the joint, brought about by the load and the unknown M_0 and H_0, should be equal to

$$\theta_p^{sh} + \theta_g^{sh} + \theta_M^{sh} + \theta_H^{sh} = \theta_M^{r\text{-}b} + \theta_H^{r\text{-}b} \tag{V.55}$$

The same applies to the radial deflections

$$\xi_p^{sh} + \xi_g^{sh} + \xi_M^{sh} + \xi_H^{sh} = \xi_p^{r\text{-}b} + \xi_g^{r\text{-}b} + \xi_M^{r\text{-}b} + \xi_H^{r\text{-}b} \tag{V.56}$$

Substituting the deflections calculated by the formulas given in Table V.1 (taken with appropriate deflection signs) yields two equations in the unknowns, M_0 and H_0. Solving these equations for M_0 and H_0 gives the sought values.

The bending moments in the local bending area are given by Eq. (V.24) which, after rearrangement, takes the following form

$$M_x = M_0 (\cos\varphi + \sin\varphi)\, e^{-\varphi} - sH_0 \sin\varphi_0 \sin\varphi e^{-\varphi} \tag{V.57}$$

The circular force is the sum of the forces caused by the load, M_0 and H_0

$$N_2 = N_2^{(p)} + N_2^{(g)} + N_2^{(M_0)} + N_2^{(H_0)} \tag{V.58}$$

that is,

$$\begin{aligned} N_2 = {} & gR\,[\cos\psi - 1/(1 + \cos\psi)] + pR\cos 2\psi/2 \\ & + 2RM_0e^{-\varphi}\,(\sin\varphi - \cos\varphi)/s^2 \\ & + [2RH_0 \sin\psi_0 e^{-\varphi}\cos\varphi]/s \end{aligned} \tag{V.59}$$

In Eq. (V.59), the first and second terms are calculated for the argument, ψ, beginning at the apex of the shell, and the third and fourth terms, for the argument, $\varphi = x/s$, beginning at the shell edge. Outside the area of local bending, the third and fourth terms are zero very nearly.

Figure V.31 shows the meridian moment and circular force diagrams typical of in-situ r.c. domes elastically fixed around the edge.

The ring beam is subjected to the axial force, N, and the bending moment, M, given by

$$N = (N_{\mathrm{I}}^{(p)} + N_{\mathrm{I}}^{(g)}) \cos \psi_0 r_0 - H_0 r_0 \tag{V.60}$$

$$M = M_0 r_0 \tag{V.61}$$

The ring-beam support is in eccentrical tension. The bending moment, however, is small, so the support may be analysed as a member in axial tension.

In domes made of prefabricated members, M_0 should be taken as zero if the joint between the shell and the ring-beam support is designed as a moment-free unit.

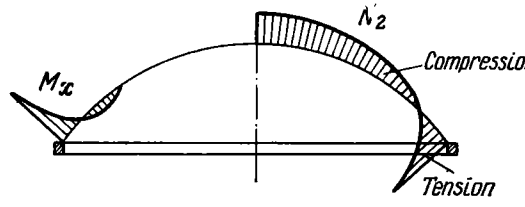


Fig. V.31. Diagrams for bending moments and circular forces in a dome elastically fixed in a ring-beam support

The stability of the dome shell is deemed guaranteed if the magnitude of the total design load does not exceed

$$q = E_c (h/R)^2/20 \tag{V.62}$$

and the total design load imposed on the shell is not more than

$$P = 0.314\ E_c h^2 \tag{V.63}$$

where h is the thickness of a plain shell. For ribbed shells, use should be made of the fictitious values h_f and $E_{c,f}$ given by Eqs. (V.31).

In-situ r.c. domes are usually cast plain. The shells of shallow domes, except the areas near the supports, are in compression. They are reinforced to suit structural requirements with a single layer of fabric consisting of bars 5 to 6 mm in diameter spaced 15 to 20 cm apart. The areas near the supports are additionally reinforced with bars usually 6 to 8 mm in diameter, to resist the moment at the support, M_x, along the meridians, and with reinforcing steel to take the tensile circular forces, N_2, along the parallels (Fig. V.32*a*). The load-bearing reinforcement of the ring beam is composed of circular bars 20 to 30 mm in diameter, welded together along their length.

In present-day practice, the ring-beam supports of domes are prestressed by tensioning their circular load-bearing reinforcement (Fig. V.32*b*). The prestress considerably reduces the cross-sectional

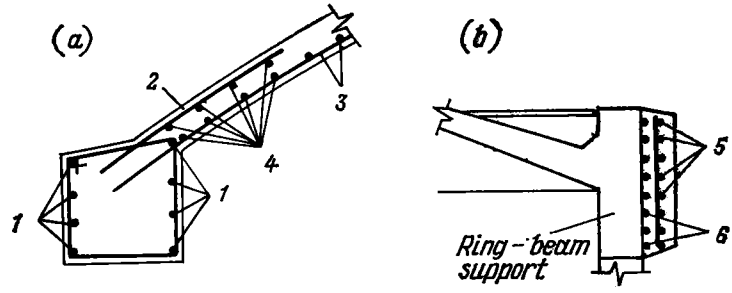


Fig. V.32. Reinforcement of an in-situ r.c. dome

(a) ordinary reinforcement; (b) prestressed circular reinforcement; *1*—load-bearing reinforcement in the ring beam; *2*—additional reinforcement to resist M_x; *3*—fabric placed for structural reasons over the entire shell; *4*—circular reinforcement to resist N_2; *5*—prestressed reinforcementt *6*—shotcrete plaster

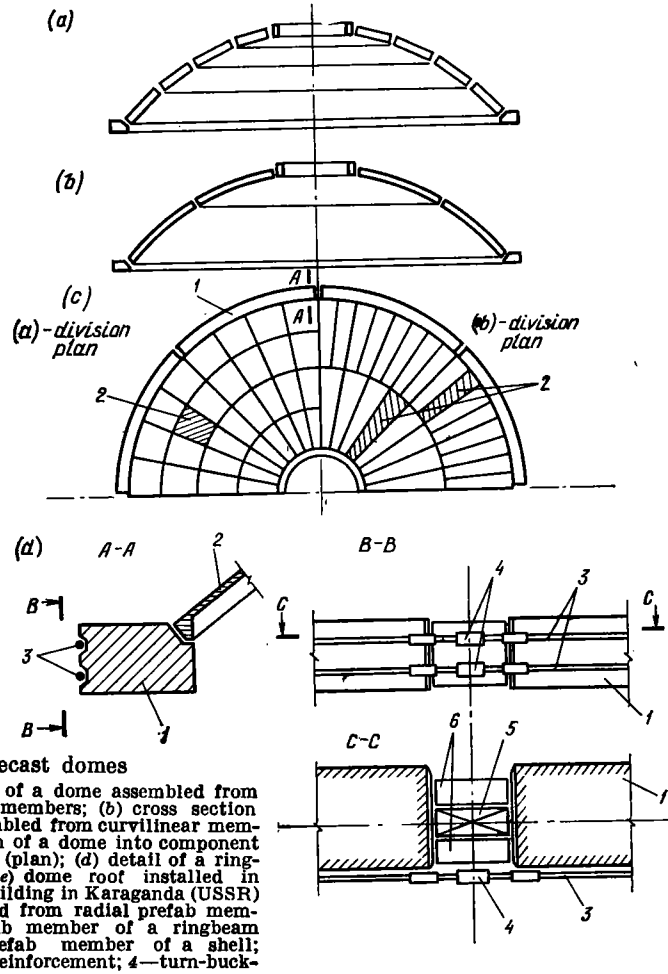
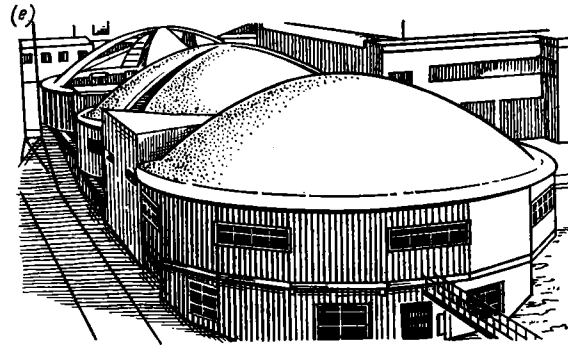


Fig. V.33. Precast domes

(a) cross section of a dome assembled from flat trapezoidal members; (b) cross section of a dome assembled from curvilinear members; (c) division of a dome into component prefab members (plan); (d) detail of a ring-beam support; (e) dome roof installed in an industria lbuilding in Karaganda (USSR) (shells assembled from radial prefab members); *1*—prefab member of a ringbeam support; *2*—prefab member of a shell; *3*—prestressed, reinforcement; *4*—turn-buck-



les for prestressed reinforcement; *5*—prestressing jack; *6*—concrete inserts to be installed after tensioning

area of ring-beam supports due to increased crack resistance, and cuts down steel consumption owing to the use of high-strength wire.

Domes may be assembled from flat and curvilinear prefabricated members as shown in Fig. V.33. The prefabricated members consist of slabs of minimum thickness (3 to 4 cm), strengthened with ribs. They are joined by welding together the reinforcement stick-outs or embedded steel parts. Ring-beam supports are likewise made of prefabricated members. Their load-bearing reinforcement (bars or cables) is placed either in the outer slots (Fig. V.33*d*) or in ducts left in ring beams.

V.7. CORRUGATED ROOFS

Corrugated roofs include corrugated and hipped-plate vaults in which the corrugation width is small in comparison with the longitudinal span of the roof. The vaults are supported on walls or columns, or directly on footings (Fig. V.34). In cross section, an individual corrugation may have the form of a sinusoid, curvilinear tray, or triangular or V-shaped folded plate. As a consequence, the component prefab members of a corrugated roof may be straight or curvilinear.

Corrugated vaults are used to roof over industrial and civil buildings having a span of 12 to 100 m or even more. The rise, f, may range between 1/2 and 1/10 of the span. The vaults are shaped so that their configuration would match as closely as possible the bending moment diagram due to main (usually dead) load.

In large-span buildings, the roofs are made of several identical members, for spans of up to 24 m, they are made of two halves. The end-face edges of the prefabricated members are strengthened

with ribs to resist rough handling in transit and erection, and also to facilitate concreting the joints that are to transfer considerable compressive forces.

In large-span and wide-corrugation thin-walled vaults, the stability of the cross section in the direction of the corrugation is ensured by transverse diaphragms, tie rods or spreaders.

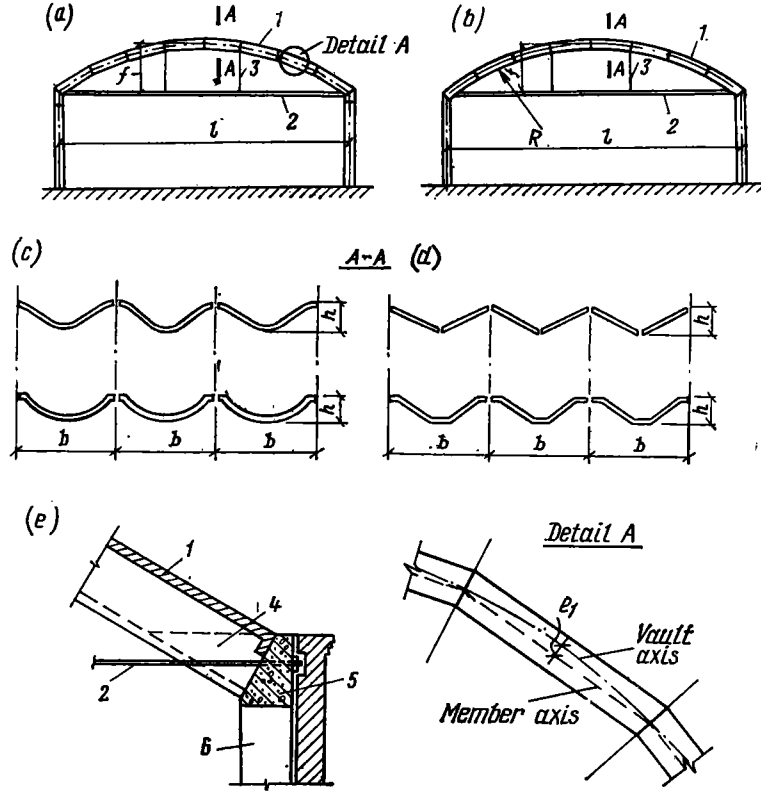


Fig. V.34. Corrugated vault roofs

(*a*) assembled from straight prefab members; (*b*) assembled from curvilinear prefab members; (*c*) curvilinear cross sections of vaults; (*d*) folded-plate cross sections of vaults; (*e*) support joint of a corrugated vault; *1*—prefab vault members; *2*—vault tie rod; *3*—vault suspension; *4*—spandrel backfilling; *5*—support beam; *6*—column

To take up the lateral thrust, roofs with supports placed at a high level are equipped with tie rods (Fig. V.34); those with supports located at a low level are strengthened with buttresses, or annexes if buttressing does not provide the necessary stability.

At a support joint (Fig. V.34*e*), the corrugated plate adjoins the supporting beam; if the centre-to-centre distance between the sup-

ports is the same as the corrugation width, it is good practice to strengthen the edge of the marginal prefabricated member, rather than to provide a supporting beam.

In analysis, each corrugation is regarded as an independent thin-walled arch with a span equal to the corrugation width. All the recommendations given in Ch. IV for arches hold in this case, too.

The corrugations of a vault are checked for strength as eccentrically compressed members at the sections where bending moments reach their maximum. In vaults made of straight-line members, account should be taken of an additional bending moment, $M_1 = Ne_1$ (Fig. V.34*a*).

The slabs of corrugated roofs are reinforced with welded-wire fabric, and the end-face ribs of the component prefab members with welded-bar mats. In the longitudinal direction, the amount of reinforcement placed on the topside and underside of the members is found by design, or this may be done for structural reasons (the latter case applies when the eccentricity of the longitudinal force about the centroid of the section does not exceed 0.35 of the corrugation rise).

The longitudinal joints between the prefabricated members are concreted, and the concrete is compacted. The ends of the members are strengthened with local reinforcing steel. At the joints, the members are connected to each other by welding together the stick-outs or embedded parts.

The thickness of the longitudinal edges of the prefabricated members should be two times or half as large again as that in the span.

V.8. SUSPENDED ROOFS

Suspended roofs are used to span over especially large areas (for example, stadiums, indoor playgrounds, exhibition pavilions, markets, motion-picture theatres, and large industrial buildings). A suspended roof consists essentially of a system of guys (flexible cables) supported on a rigid edge structure (ring, frame or arch) and prefabricated roof slabs (made of lightweight reinforced concrete, fine-mesh wire-fabric reinforced concrete, or other materials).

Suspended roofs may use a single system of guys supporting positive- or negative-curvature surfaces (Figs. V.35 and V.36), or a double system of guys (Fig. V.37). Suspended roofs may be installed in buildings of any configuration in plan (rectangular, round, oval, etc.).

Figures V.35 through V.37 show only the main types of suspended roofs; actually, there are a great many more varieties.

Suspended roofs are made rather shallow, the sag, f, at the centre of a roof usually ranges between 1/10 and 1/25 of the main dimension in plan.

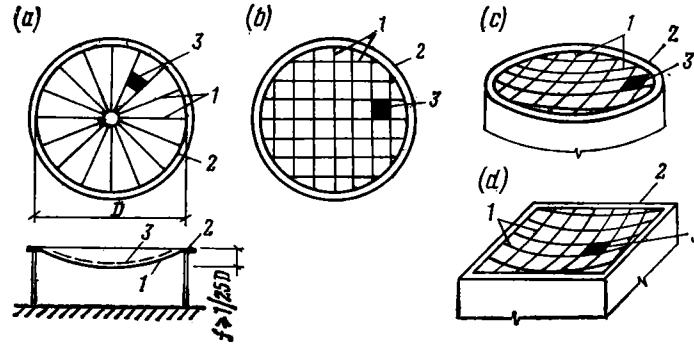


Fig. V.35. Positive-curvature suspended roofs using a single system of guys
(*a*) round-plan roof with radial guys; (*b*) same with orthogonal guys; (*c*) oval-plan roof (*d*) rectangular-plan roof; *1*—guys; *2*—rigid ring support (closed frame); *3*—roof slabs

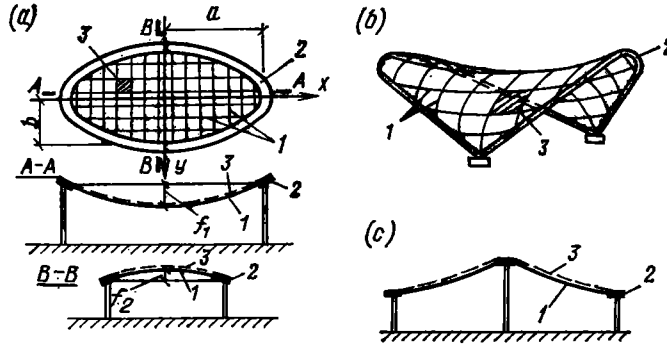


Fig. V.36. Negative-curvature suspended roofs using a single system of guys
(*a*) supported around the periphery; (*b*) supported at two points; (*c*) supported around the periphery and by an intermediate support; *1*—guys; *2*—ring support (arches); *3*—roof slabs

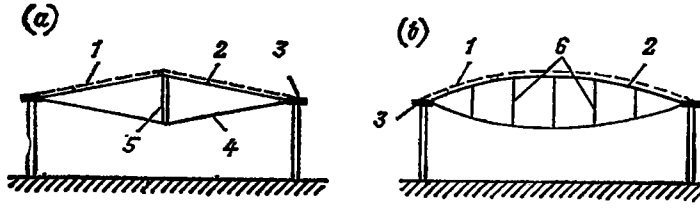


Fig. V.37. Suspended roofs using a double system of guys
(*a*) with one spreader; (*b*) with several spreaders; *1*—roof slabs; *2*—prestressing guys; *3*—rigid ring support (closed frame); *4*—load-bearing guys; *5*—central drum; *6*—struts

The guys may be radial (Figs. V.35*a* and V.37*a*) mostly for roofs round in plan, or orthogonal (V.35*b*).

Suspended roofs require no falsework or scaffolding during erection. This is a major advantage of suspended roofs over other roof types.

The roof membrane freely suspended on the rigid edge has a negligible flexural rigidity and is, therefore, rather deformable. Variations in external load affect the geometrical shape of the membrane, the effect being most pronounced with snow concentrated on concave surfaces or with wind suction on the leeside of suspended roofs. To ensure the stability of the geometrical shape, reinforced concrete suspended roofs should be prestressed.

The roofs shown in Fig. V.35 may be prestressed by one of the two following methods.

Method One. The guys are tensioned by jacks after the joints between the roof slabs have been concreted. The guys are placed in the ducts which are then grouted.

Method Two. The guys are tensioned by means of erection load applied to the roof slabs or suspended from the guys prior to concreting the joints between the roof slabs. After the concreted joints have attained the design strength, the load is removed, and the roof thus becomes prestressed.

In either method, the roof slabs play an important part.

In the roofs of Fig. V.36*a* and *b*, the geometrical shape is stabilized by tensioning transverse guys placed on longitudinal guys freely suspended on the edge structure.

In suspended roofs with a double system of guys (Fig. V.37), the bottom load-bearing system is prestressed when tensioning the top guy system.

With such methods of prestressing, more steel is required for guys, because roof slabs are less active in this case.

Suspended roofs are rather economical, being close in performance to the best types of other thin space roofs.

Suspended roofs are designed on the assumption that all of the roof load is taken up by the guys. The roof slabs are assumed to be only in compression, and the guys only in tension. In addition, the guys are supposed to be absolutely flexible (that is, to have no flexural rigidity) and nonextensible.

In the general case, the design of suspended roofs for any load is complex. There exist, however, simple approaches to individual symmetrical structures subjected to some loads.

Design of Roofs with Radial Guys. Consider a round-plan roof with the guys spaced a distance b apart (around the circumference of the roof), subjected to a load, q, uniformly distributed over the roof projection (Fig. V.38*a*). With such a load, each guy is designed individually (Fig. V.38*b*).

Since the guy supports are placed at the same level and the support reaction is directed along the tangent to the guy axis at the point of fixation, we may, owing to the symmetry of the loading system, write for the vertical components of the support reactions

$$A = B = qba/2$$

From the equation of moments about the point O on the left-hand side of the guy (Fig. V.38*b*), we find

$$H = qba^2/6f \qquad (V.64)$$

The guys are designed to resist the force

$$N_g = \sqrt{A^2 + H^2}$$

The compressive force, $N_{r\text{-}b}$, in the ring beam (Fig. V.38*c*) subjected to the distributed radial pressure, $H_1 = H/b$, is given by

$$N_{r\text{-}b} = H_1 a = qa^3/6f \qquad (V.65)$$

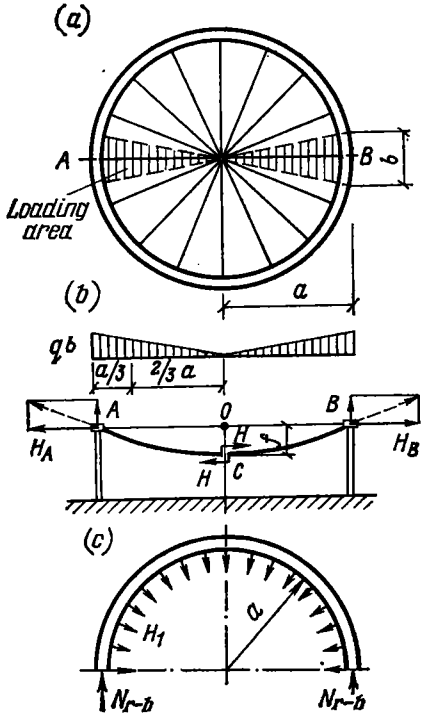


Fig. V.38. To the design of a round-plan suspended roof with radial guys
(*a*) plan; (*b*) loading system for a guy; (*c*) loading system for a ring-beam support

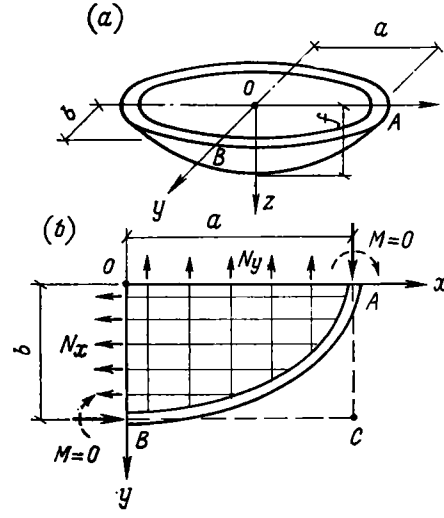


Fig. V.39. To the design of an elliptic-plan suspended roof with orthogonal guys
(*a*) geometric scheme; (*b*) loading system for the roof in plan

Design of Roofs with Orthogonal Guys. Consider an elliptical shallow roof subjected to a load, q, uniformly distributed over the roof projection (Fig. V.39).

As previously noted, suspended roofs can only be in the moment-free state of stress. This is described by Eq. (V.1) in which $N_{xy} = 0$ (because nothing is provided to take up shearing forces). Recalling Eqs. (V.2) and (V.3) we get

$$N_x \, \partial^2 z/\partial x^2 + N_y \, \partial^2 z/\partial y^2 = -q \qquad \text{(V.66)}$$

The above equation leads to two alternative solutions.

Alternative I. Given: load q and tensile force $N_x = N_y = N$ (the same in both directions). *To find*: The surface equation.

The surface of an elliptical suspended roof subjected to a uniformly distributed load is close to that of the elliptical paraboloid (Fig. V.39*a*):

$$z = f\,(1 - x^2/a^2 - y^2/b^2) \qquad \text{(V.67)}$$

where a and b are the ellipse parameters in plan, known in advance, and f is the sag of the surface.

The curvatures of this surface are described as

$$\partial^2 z/\partial x^2 = -2f/a^2 \quad \text{and} \quad \partial^2 z/\partial y^2 = -2f/b^2 \qquad \text{(V.68)}$$

The sag may be determined using the specified tensile force and the curvatures of Eq. (V.68) in Eq. (V.66)

$$f = qa^2b^2/2N\,(a^2 + b^2) \qquad \text{(V.69)}$$

Alternative II. Given: magnitude of load and surface equation *To find*: Forces N_x and N_y in the roof (Fig. V.39*b*).

Assume that the load, q, imposed on the roof is uniformly distributed over its surface which is described by Eq. (V.67).

Consider a quarter of the roof (Fig. V.39*b*) on the assumption that the bending moment in the ring-beam support is zero at points A and B. From the equation of moments about point C acting on this part of the roof surface, we obtain

$$N_x b^2 = N_y a^2 \qquad \text{(V.70)}$$

Using this equation in Eq. (V.66) and recalling the curvatures given by Eq. (V.68), we find

$$N_x = qa^2/4f \quad \text{and} \quad N_y = qb^2/4f \qquad \text{(V.71)}$$

Despite the difference between N_x and N_y, the roof load, q, is equally distributed among the guys in both directions. So, recalling Eqs. (V.68) and (V.71), we may write

$$q_x = -N_x \, \partial^2 z/\partial x^2 = (qa^2/4f)\; 2\; (f/a^2) = q/2$$

In a similar manner, we find that $q_y = q/2$.

It is an easy matter to prove that under these conditions the ring-beam support is in axial compression not only at points A and B, but also along the whole length.

For a hyperbolic paraboloid (see Fig. V.36*a*), the equation of, and the tensile force in, the surface are

$$z = f_1 x^2/a^2 - f_2 y^2/b^2 \tag{V.72}$$

$$N_x = (q + p_{ten})\, a^2/2f_1; \qquad N_y = p_{ten} b^2/2f_2 \tag{V.73}$$

Here, the longitudinal guys are subjected to the load, q, and the additional load, p_{ten}, due to the tensioning of the transverse guys.

The forces in the guys of a hyperbolic paraboloid are greater than those in the guys of an elliptical paraboloid. The ring-beam support is subjected to eccentrical compression.

It should be noted that, in the above problems, the thrust, H, does not vary along the length of the guys, and the force in the guys is

$$N = \sqrt{Q_c^2 + H^2} \tag{V.74}$$

Here, Q_c is the shearing force calculated for the guys, using a similar beam subjected to the share of load imposed on the guys of a given direction.

CHAPTER VI

DESIGN OF MULTI-STOREY BUILDINGS OF SKELETON AND BEARING-WALL CONSTRUCTION

VI.1. DESIGN OF MULTI-STOREY INDUSTRIAL BUILDINGS

1. Construction Types

Multi-storey buildings are widely used to accommodate plant and processes in the mechanical engineering, instrument-making, chemical, electrical engineering, radio engineering, textile, food, and some other industries, and also warehouses, cold-storage houses, garages, and the like. As a rule, such buildings are of the skeleton construction with filler wall panels.

According to the industrial process to be accommodated, industrial buildings are generally from 3 to 7 storeys high (with a total height of up to 40 m). In some cases, buildings intended for lightweight equipment may be 12 to 14 storeys high. The width of industrial buildings may be from 18 to 36 m or even more. The height per storey and the column grid size are assigned so that the component members and the overall dimensions of the assembly units can be standardized. The height per storey is taken as a multiple of 1.2 m, being 3.6, 4.8 and 6 m; the ground storey may sometimes be as high as 7.2 m. The most widely used column grid sizes are 6×6 m, 9×6 m and, though seldom, 12×6 m. The small column grid sizes are chosen in anticipation of heavy live loads imposed on the floors, which may reach 15 kN/m^2 or, in some cases, 25 kN/m^2 and more.

For industrial purposes, the most convenient buildings are of the multi-storey skeleton-type without any vertical diaphragms that might take up some of the area necessary for process equipment and services. The main load-bearing elements of a multi-storey skeleton-type building are reinforced concrete frames and floors combining the frames into a single structure. In the transverse direction, a building derives its resistance to lateral loading from the rigidity of the connections between the columns and girders of multi-storey frames, so that the building behaves as a frame-type structure. In the longitudinal direction, the building owes its space

rigidity to vertical steel braces or reinforced concrete shear walls installed in the column rows and in the plane of the exterior walls; in this case, the building acts as a shear wall structure (Fig. VI.1).

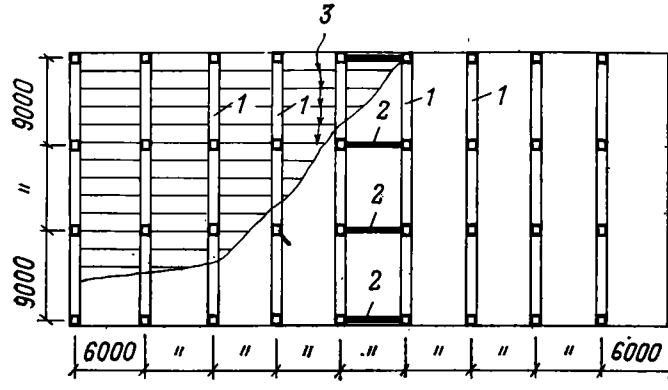


Fig. VI.1. Layout of a skeleton-type multi-storey industrial building
1—transverse frames; *2*—longitudinal shear walls; *3*—floor slabs

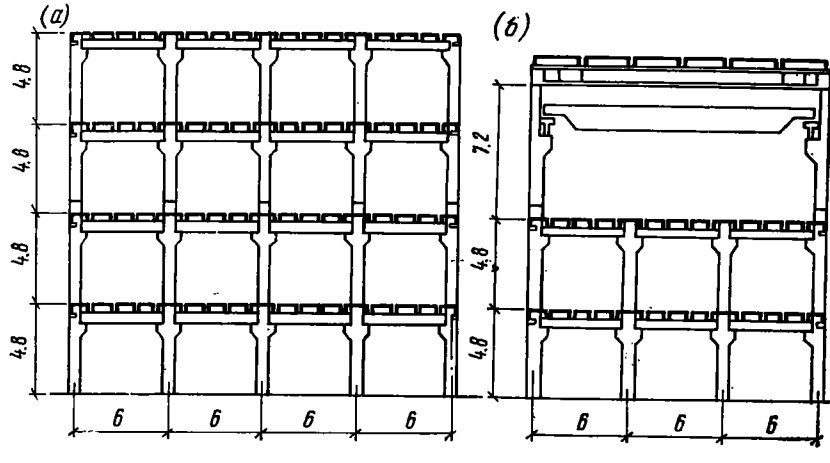


Fig. VI.2. Multi-storey industrial buildings
(*a*) regular; (*b*) with overhead travelling cranes in the upper storey

If, for some reasons, braces or shear walls cannot be installed in a building, they must be replaced by longitudinal girders. In this case, the building will behave as a frame-type structure in both transverse and longitudinal directions.

A likely design of a multi-storey industrial building with beam and girder floors is given in Fig. VI.2. If overhead travelling cranes

will be installed (say, in chemical industry buildings), the upper storey is assembled from columns, girders and crane beams similar to those used in one-storey industrial buildings.

The girders are supported on the column (strut) cantilevers; the reinforcement stick-outs are welded in a pool of molten metal within a reusable mould, and the joint is concreted during erection (Fig. VI.3). The floors are made of ribbed slabs 1 500 mm wide. The floor slabs installed along the column rows serve as girders

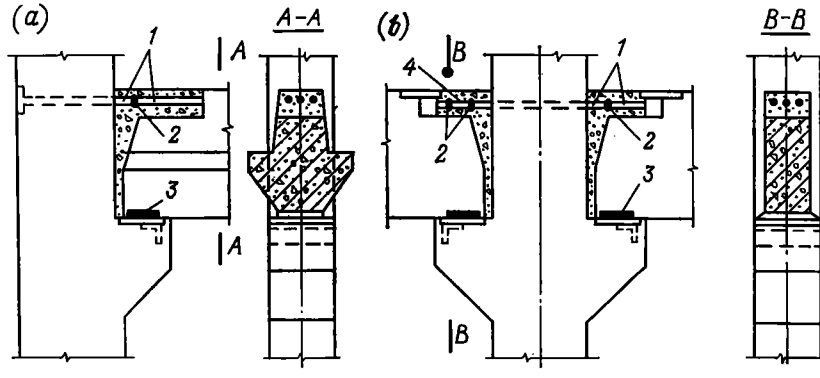


Fig. VI.3. Connection between the girders and columns
a) at an outer joint; (*b*) at an intermediate joint; *1*—reinforcement stick-outs; *2*—field-welded reinforcement stick-outs; *3*—field-welded embedded parts; *4*—reinforcement inserts

(spreaders) which contribute to the rigidity of the building in the longitudinal direction during erection. In such buildings, floor slabs may be supported either on the flanges of T-section girders (when the floors are intended to carry machine tools, in which case the load may roughly be regarded as uniformly distributed), or on the top of rectangular girders (this mostly applies to chemical industry buildings where the process equipment extends from floor to floor, and a large concentrated load is transferred to an individual column). In both cases, type-design girders for spans of 6 or 9 m are made with the same depth (800 mm) and rib width (300 mm).

Type-design multi-storey industrial buildings with beam and girder floors have been developed for column grid sizes of 6 × 6 and 9 × 6 m, and for live loads ranging between 5 and 25 kN/m^2.

Figure VI.4 shows a likely construction of a multi-storey industrial building with flat-slab floors. The girders of the transverse and longitudinal multi-storey frames are flat slabs rigidly connected to the columns by the capitals. In both directions, the skeleton-type building behaves as a frame structure. To standardize the dimensions of the slabs and capitals in the outer and middle spans of the flat-slab floor, the exterior self-supporting walls are displaced

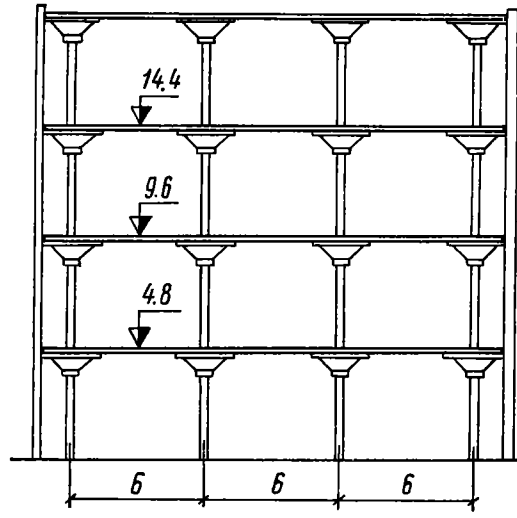


Fig. VI.4. Multi-storey industrial building with flat-slab floors

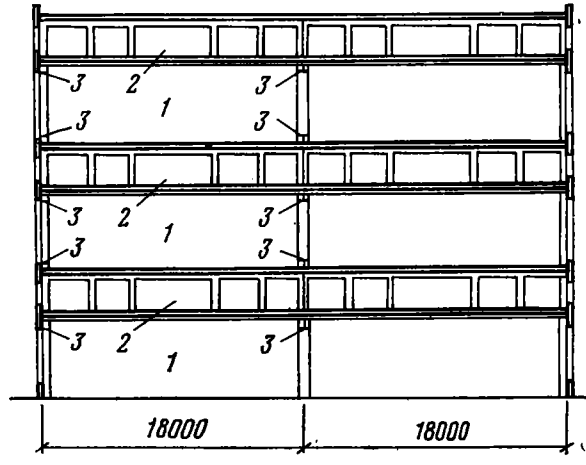


Fig. VI.5. Multi-storey industrial building with intertruss storeys
1—main storeys; *2*—intertruss storeys; *3*—connections between columns and trusses without web diagonals

from the centre line of the outer column row for a distance equal to one half of the drop panel width.

Multi-storey industrial buildings with closely spaced supports and column grid sizes of 6 × 6 and 9 × 6 m are not always convenient from the view-point of plant layout, and future expansion or re-tooling might require expensive reconstruction. Therefore,

they should be confined to heavy live floor loads running over 10 kN/m².

A distinction of universal industrial buildings having additional floors in the intertruss space is a larger column grid size which may be 18 × 6, 18 × 12 and 24 × 6 m. The larger spans are bridged by trusses without diagonal web members. The space between the top and bottom chords of such trusses is used to accommodate engineering facilities, utilities or welfare services; it may also be used as storage space or for other auxiliary purposes. The height of intertruss storeys may be 2.4, 3 or 3.6 m.

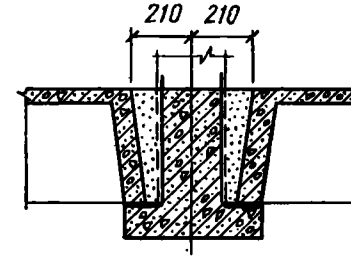


Fig. VI.6. Floor slabs supported on the bottom chord of a truss without web diagonals

A likely construction of a universal industrial building is shown in Fig. VI.5. The building has three main and three intertruss storeys. The trusses without web diagonals are rigidly connected to the columns. They are members of the multi-storey skeleton, and act as frame girders. The outer struts of the trusses have projections on the top and bottom for connection to the columns of the neighbouring storeys. The main floors are made of ribbed slabs laid on the top chords of the trusses. The additional floors use hollow-core or ribbed slabs supported on the flanges of the bottom truss chords (Fig. VI.6).

2. Construction Types of Multi-Storey Frames

Multi-storey frames are assembled from off-site prefabricated members adapted to streamlined manufacture and erection. Frame

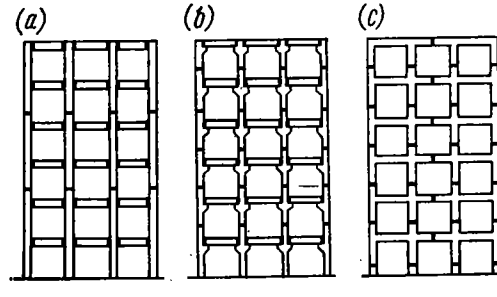


Fig. VI.7. Multi-storey frames assembled from several prefab members

girders are generally designed to be built up from rectilinear members joined together in the plane of the column side by concealed or cantilever joints (Fig. VI.7*a* and *b*). Columns are likewise designed

from several rectilinear members to be joined at every other or every third storey above floor level. Sometimes, in order to enhance the continuity of structure and reduce the number of prefabricated members, multi-storey frames are designed as assemblies of single-span one-storey frames (Fig. VI.7*c*). The joints in multi-storey frames using prefabricated members are generally made rigid. Hinged joints would impair the overall rigidity of the building and its resistance to lateral loading. This is especially pronounced with the increasing number of storeys in a skeleton-type building designed to act as a frame structure.

Standard 6-m girders are reinforced with nonprestressed steel, and 9-m girders with prestressed steel placed in the span (Fig. VI.8). Two-storey columns are congested with longitudinal and transverse reinforcement as eccentrically compressed members (Fig. VI.9).

There are several types of rigid joints in multi-storey frames, intended to take up the longitudinal force, N, and the bending moment, M, transferred through embedded parts, reinforcing steel and concrete.

Figure VI.10 shows a concreted joint between the end faces of two columns, produced by welding together embedded angles and plates. For ease of column alignment, the casing formed by the angles and plates has a welded centering pad 30 to 40 mm thick. The longitudinal load-bearing bars of the columns are factory-welded to the

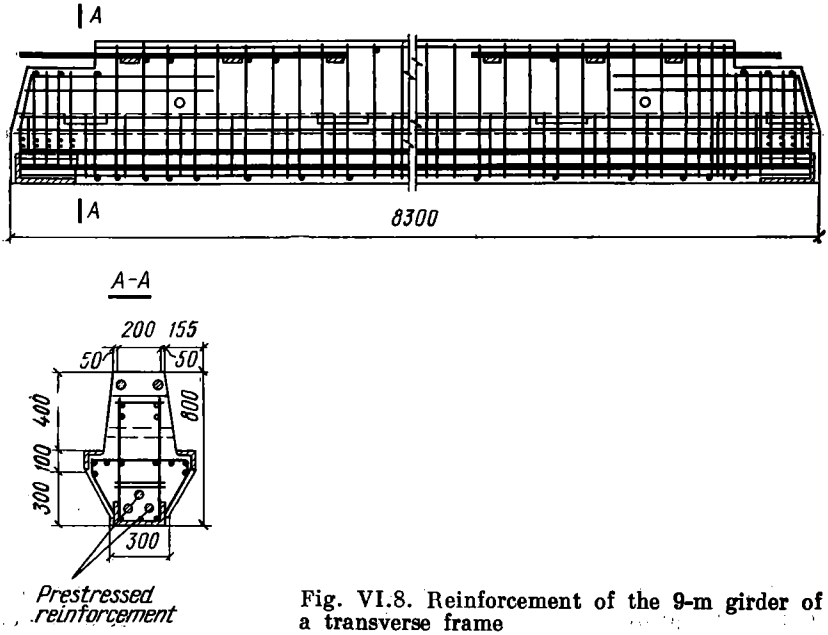


Fig. VI.8. Reinforcement of the 9-m girder of a transverse frame

casings from the outside. The joint bars placed between the load-bearing bars are welded to the casings during erection. The gap between the column ends is caulked with stiff mortar, and concrete is placed over welded-wire fabric. This type of joint has the same strength as the columns, but requires a relatively large amount of steel and is too labour consuming.

Figure VI.11 shows a welded joint using end-face steel sheets and a centering pad. To the steel sheets 10 to 20 mm thick are welded four deformed anchor bars and a centering pad 3 to 4 mm thick. The steel sheets of the columns being joined are welded together around their periphery during erection. For this purpose, the edges of the steel sheets are bevelled to obtain the desired weld throat. At the ends where the longitudinal reinforcement is cut off, the columns are additionally strengthened with transverse fabric which is designed using the equations given in Ch. I; here, the crushing area is taken as $F_{cr} = F_{col}$. This type of joint is rather deformable and may be used when the eccentricity of the longitudinal force is $e_0 \leqslant 0.2h$.

The welds in the above joint are designed approximately on the assumption that the stress in the concrete outside the centering pad and the circumferential welds is low owing to the flexibility of the steel sheets which are deformed during the transfer of the pressure. As a point of departure, we shall assume that the stress in the concrete is the same over the entire contact area, and is zero outside (Fig. VI.12). In the end-face

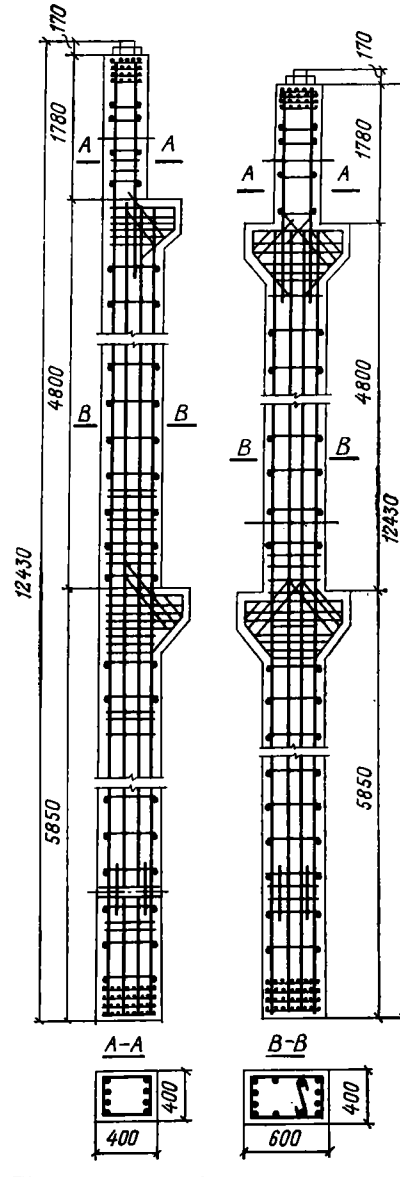


Fig. VI.9. Reinforcement of transverse frame columns

steel sheets of thickness δ, the pressure is assumed to be propagating at an angle whose tangent is 1.5. Under the circumstances, the

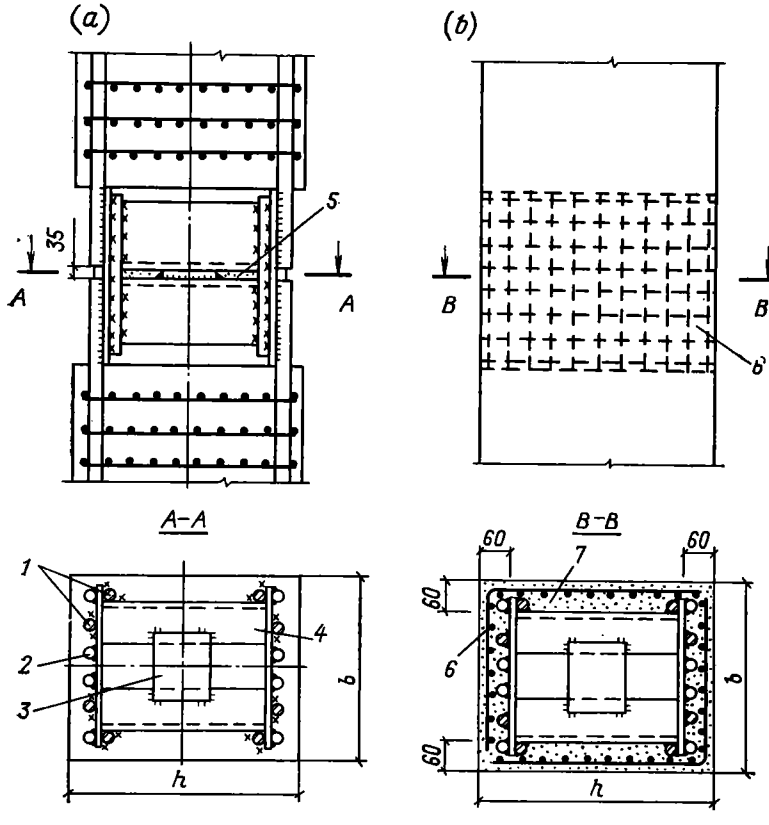


Fig. VI.10. Concreted joint between the end faces of two columns produced by welding together embedded casings
(*a*) joint before concreting; (*b*) concreted joint; *1*—fishplates; *2*—longitudinal steel; *3*—steel centering pad; *4*—steel casing formed by welded angles and plates; *5*—gap between column ends caulked with stiff mortar; *6*—welded-wire fabric; *7*—in-situ concrete

contact area bounded by the circumferential welds is

$$F_w = 5\delta\,(h_1 + b_1 - 5\delta) \qquad \text{(VI.1)}$$

and the contact area beneath the centering pad is given by

$$F_p = (d + 3\delta)\,(c + 3\delta) \qquad \text{(VI.2)}$$

Here, b_1 and h_1 are the dimensions of the end-face steel sheet; and c and d are the dimensions of the centering pad. Usually, when the centering pad is thin, $c = b_1/3$ and $d = h_1/3$. The total contact area

in the joint is

$$F_{cont} = F_w + F_p \tag{VI.3}$$

The force transferred to the welds is determined using the ratio of the areas

$$N_w = NF_w/F_{cont} \tag{VI.4}$$

The throat of the weld is

$$h_w = N_w/0.7R_w\Sigma l_w \tag{VI.5}$$

where Σl_w is the total length of the welds around the periphery of the steel sheet with allowance for poor penetration; and R_w is the design strength of the weld.

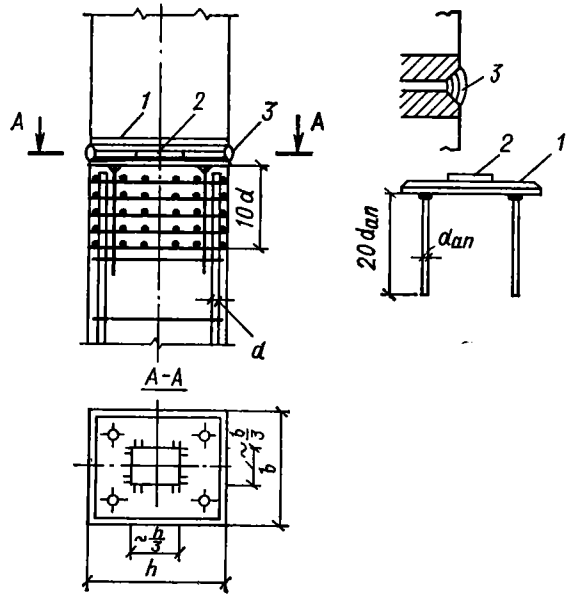


Fig. VI.11. Welded joint using end-face steel sheets and a centering pad
1—steel sheets; *2*—centering pad; *3*—welds

If the joint resists bending moments, the additional force is transferred to the transverse welds (normal to the plane of flexure). In this case,

$$N_{tr} = M/h_1 \tag{VI.6}$$

The total force transferred to the transverse weld is given by

$$N_{tr} = N_w b_1/2\,(b_1 + h_1) + M/h_1 \tag{VI.7}$$

The throat of the weld is

$$h_w = N_{tr}/0.7R_w l \qquad \text{(VI.8)}$$

where $l = b - 1$ is the length of the transverse weld in cm with allowance for poor penetration.

In most cases, the bending moment at the joints between the columns in multi-storey skeleton-type buildings may be reduced by

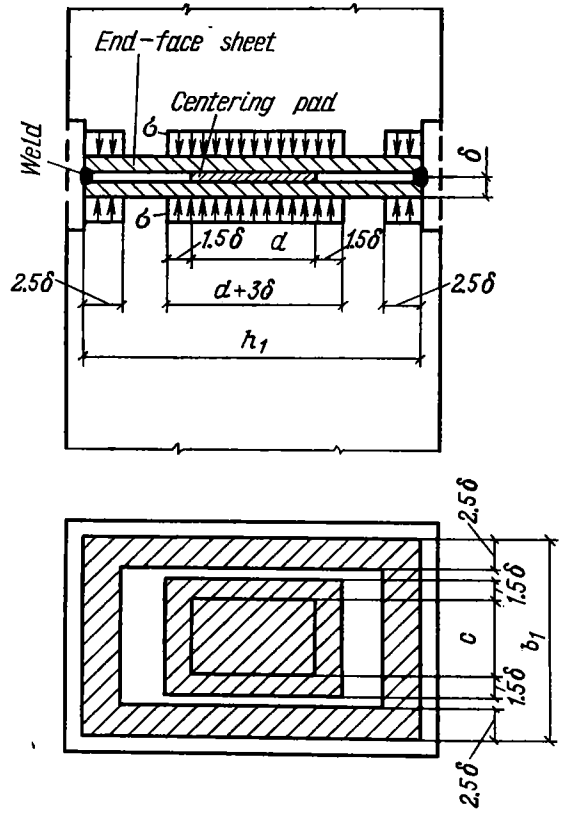


Fig. VI.12. To the design of a joint using end-face steel sheets

making the joints closer to the middle of the storey height where the bending moments due to vertical and lateral loads tend to zero, and where it is easier to join the columns.

In-Situ and Precast/In-Situ Multi-Storey Frames. The girder of an in-situ multi-storey frame is reinforced similarly to the girder of an in-situ beam and girder floor, except the marginal support where the girder is rigidly connected to a column (Fig. VI.13*a*).

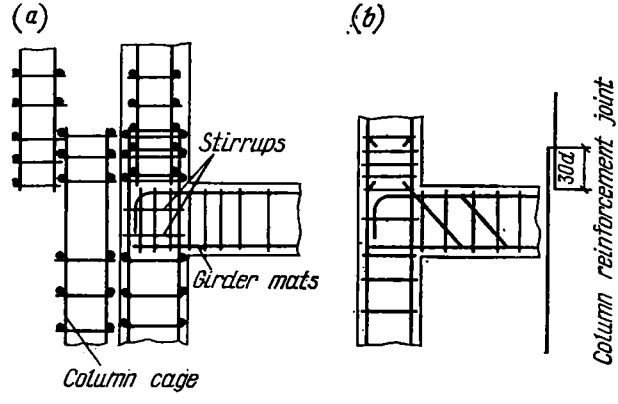


Fig. VI.13. Reinforcement of the joints of an in-situ multi-storey frame

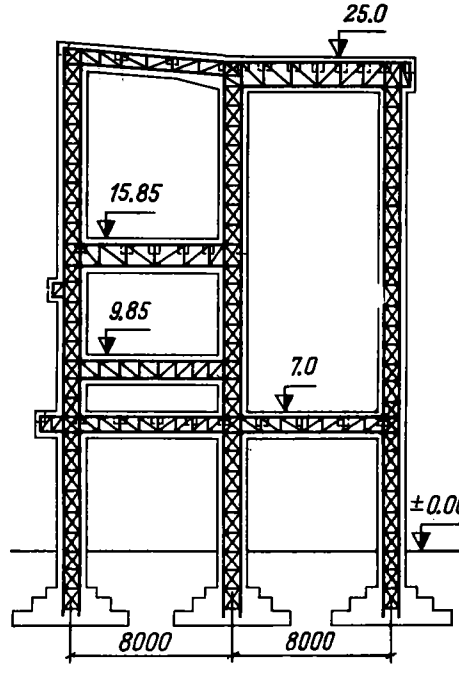


Fig. VI.14. Load-bearing reinforcing cage of an in-situ multi-storey frame

When designing a frame, "bonding" joints must be provided. The purpose of suhc joints is to bond fresh concrete to that already placed, should an interruption occur in pouring the concrete. In columns, such joints should be located on the topside of the floor. Here, the reinforcement of the lower-storey columns must be extended as appropriate and connected to the reinforcement of the columns of the storey above (Fig. VI.13*b*).

It is good practice to reinforce large-span and high-storey in-situ buildings with load-bearing cages. Figure VI.14 illustrates the load-bearing reinforcing cage of a multi-storey frame for a power station building.

For each span of the girder, the welded mats are made in the form of plane trusses with diagonals, and are assembled into a single

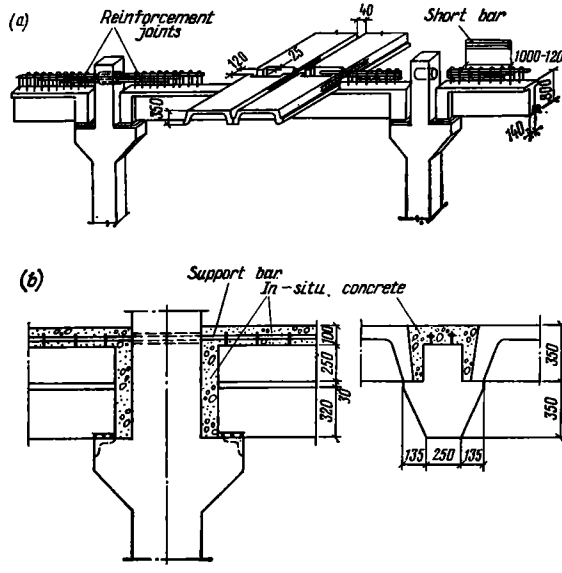


Fig. VI.15. Joints in a precast/in-situ multi-storey frame
(*a*) before concreting; (*b*) after concreting

reinforcing cage with lateral braces at the top and on the underside. For a column the reinforcement is in the form of a cage made of longitudinal bars, stirrups and lateral braces located at the sides.

Precast/in-situ frames are likewise made with rigid connections. The T-section girder has protruding stirrups and bare supporting reinforcement on top of its flange (Fig. VI.15*a*). The girder gives support to ribbed slabs placed with a gap of 12 cm between their end faces. The joint between the girder and the columns is made rigid by connecting the top reinforcement of the girder to the column rein-

forcement. For this purpose, the supporting bars of the joint are passed through a hole in the column. The girders may have ledges to carry the slabs (Fig. VI.15*b*). After the prefabricated members have been installed, and the supporting reinforcement of the girder placed and welded, the cavities between the slabs and the gaps between the girder ends and the columns are concreted to provide monolithic connections. Owing to their interaction with the slabs, the girders are regarded as T-sections.

VI.2. DESIGN OF MULTI-STOREY CIVIL BUILDINGS

1. Construction Types

For mass construction, civil buildings of the skeleton and bearing-wall construction are designed in the USSR to have 12 to 16 storeys. In some cases, they may be high-rise buildings with 20 storeys or even more. The column grid size, the centre-to-centre distance between the bearing walls, and the storey height are chosen so that the component members can be adapted to industrialized manufacture and their dimensions standardized. Civil buildings assembled from prefabricated members have a number of advantages over other construction types in that they are uniform in height, use standardized component members, and are clear-cut in layout.

Skeleton-type construction is used for administrative, public and other buildings where large floor area unobstructed by partitions is required. Sometimes, it may be used in apartment houses over 25 storeys high. The main bearing elements in a skeleton-type multi-storey civil building are reinforced concrete frames, shear walls and floors.

Under lateral load, the various vertical members of a multi-storey building are made to act together owing to the high in-plane flexural rigidity of the floors which behave as lateral diaphragms. The flexural rigidity of the prefab floors is ensured by welded embedded parts and concreted joints between the slabs.

In this way, one ensures that multi-storey buildings have ample resistance to lateral loading. More specifically, the required space rigidity may be achieved in any one of several ways, according to the manner in which the structure resists lateral loads.

For example, in a skeleton-type building having transverse multi-storey frames and transverse shear walls, lateral loads are taken up by the interacting vertical members, and the building behaves in the transverse direction as a shear wall-frame structure. In the longitudinal direction, the building has only shear walls and, therefore, acts as a shear wall structure (Fig. VI.16*a*).

A building having transverse shear walls and longitudinal multi-storey frames behaves as a shear wall structure in the transverse direction, and as a frame structure in the longitudinal direction

(Fig. VI.16*b*). A skeleton-type building in which the girders and the columns are connected by hinges acts as a shear wall structure in both directions.

Shear wall-frame multi-storey buildings of skeleton construction are the best choice from the technical and economic view-point and are widely used in construction practice.

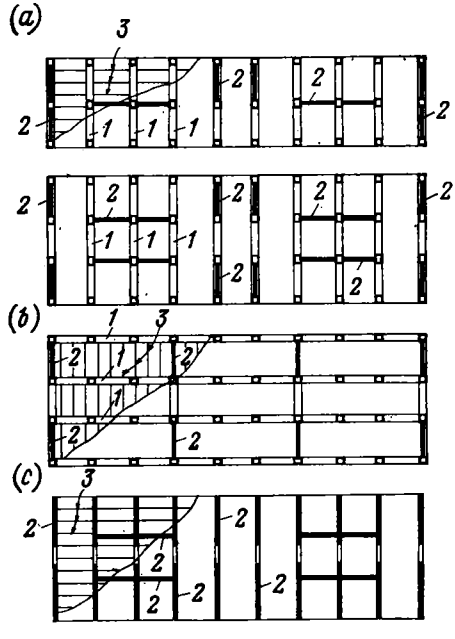


Fig. VI.16. Plan layouts of multi-storey civil buildings

(*a*) skeleton-type with transverse frames; (*b*) skeleton-type with longitudinal frames; (*c*) bearing-wall type; *1*—frame girders; *2*—shear walls; *3*—floor slabs

Bearing-wall (large panel) construction is used in apartment buildings, hotels, motels and similar buildings with closely spaced partitions and walls. In such buildings, the main bearing elements are shear walls formed by the panels of the interior bearing walls placed in the transverse direction (or, sometimes, in the longitudinal direction), and floors. The exterior wall panels are suspended on the end faces of the transverse bearing-wall panels. Multi-storey bearing-wall (large panel) buildings resist lateral loading as shear wall structures in both directions (Fig. VI.16*c*).

Figure VI.17 shows some other construction types of multi-storey buildings. These include, for example, skeleton-type single-core buildings, where the internal shear walls are made of combined elevator and ventilation shafts and staircases (Fig. VI.17*a*); skeleton-type double-core buildings in which the open-work cores are made of channels and I-sections (Fig. VI.17*b*); and skeleton-type double-core buildings of complex plan layout tailored to individual architectural designs (Fig. VI.17*c*). The above construction types* resist lateral loading as shear wall or shear wall-frame structures.

For freedom in layout, a large-size column grid size is used in single-core buildings. Also, in some cases, the internal columns may be omitted so that floor members are supported on the outer

* In the UK and US literature of the subject, they are also referred to as framed-tube-with-core structures.—*Translator's note.*

columns and the core. Floor girders 12 to 15 m in span are prestressed and connected to columns by means of hinges. Floor slabs are made hollow-cored or channeled. Such buildings resist lateral loading as shear wall structures.

The floors in double-core buildings with complex plan layout consist of in-situ flat slabs installed without capitals. Such buildings are constructed by the lift slab method of construction. With this method, slabs are cast at the ground level, one on top of another. Only side forms are necessary. Specially designed steel collars for column connections are embedded in the slab. After the slabs attain the desired strength, they are lifted hydraulically upon the columns and fastened to the columns after reaching their final position by welding the steel collars to the embedded steel parts in the columns. This construction type resists lateral loading as a shear wall structure. If, on the other hand, the floor slabs are rigidly connected to the columns so that the flat slabs behave as girders, the building acts as a shear wall-frame structure.

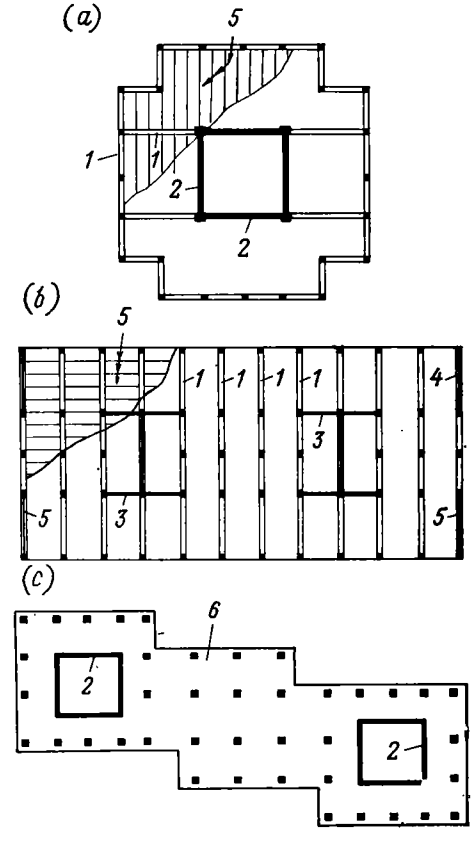


Fig. VI.17. Skeleton-type multi-storey buildings

(*a*) single-core type; (*b*) double-core type; (*c*) double-core type of complex plan layout, constructed by the lift-slab method; *1*—frame girders; *2*—closed stiffening core; *3*—I-section stiffening core; *4*—shear walls; *5*—floor slab; *6*—in-situ flat-slab floor

2. Main Vertical Structural Members

Multi-storey frames up to 16 storeys high have uniform-section columns throughout the building height (Fig. VI.18*a*). The load-bearing capacity of the lower-floor columns is improved by using a high-brand concrete, a larger amount of reinforcing steel, and stiff reinforcement. To reduce labour consumption during erection, the component members of precast columns are made to extend for two to four storeys.

Combined shear walls, comprising wall and frame parts have regular member and span dimensions throughout the building height (Fig. VI.18*b*). Shear walls with openings contain reinforced concrete lintels rigidly connected with the wall parts at the supports. They also have regular dimensions throughout the building height (Fig. VI.18*c*).

The hinged or rigid joints between the girders and the columns are made with or without concealed cantilevers. The rigid joints

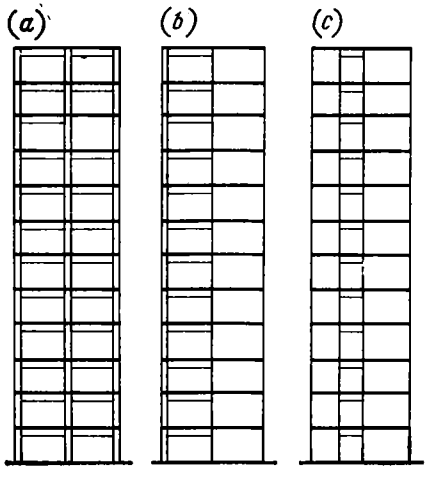


Fig. VI.18. Main vertical structural members of multi-storey buildings
(*a*) regular multi-storey frames; (*b*) combined shear walls; (*c*) open-work shear walls

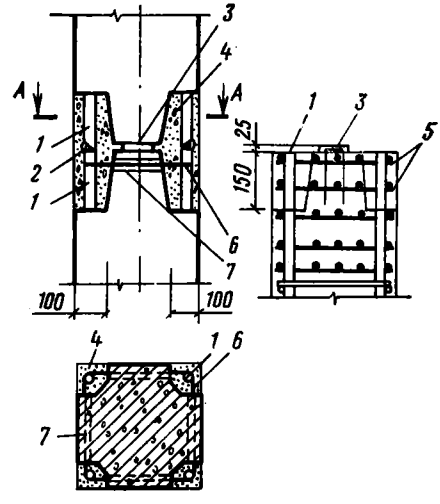


Fig. VI.19. Joint between columns made by welding together the reinforcement stick-outs in a pool of molten metal contained within a reusable mould
1—reinforcement stick-outs; *2*—to be welded during erection; *3*—concrete centering projection; *4*—in-situ concrete; *5*—transverse fabric; *6*—additional stirrup; *7*—slots in column

considerably increase the overall rigidity of the building and cut down the amount of steel in the girders (because such girders have an improved strength and crack resistance and an increased limiting sag).

The joints between the columns are made by welding together the reinforcement stick-outs up to 40 mm in diameter in a pool of molten metal contained within a reusable mould (Fig. VI.19). At the ends, the column members are additionally strengthened with transverse fabric and cast to form centering projections 25 mm high for ease of alignment during erection. The sides of these pro-

jections are made 150 mm long so that the reinforcement stick-outs can conveniently be welded together. After the column parts have been installed and aligned, the reinforcement stick-outs are successively welded along the diagonal, and an additional erection stirrup 10 to 12 mm in diameter is placed in the joint area. The joint cavities are concreted in-situ under pressure in a reusable formwork to constitute shear keys. Experience has shown that such joints

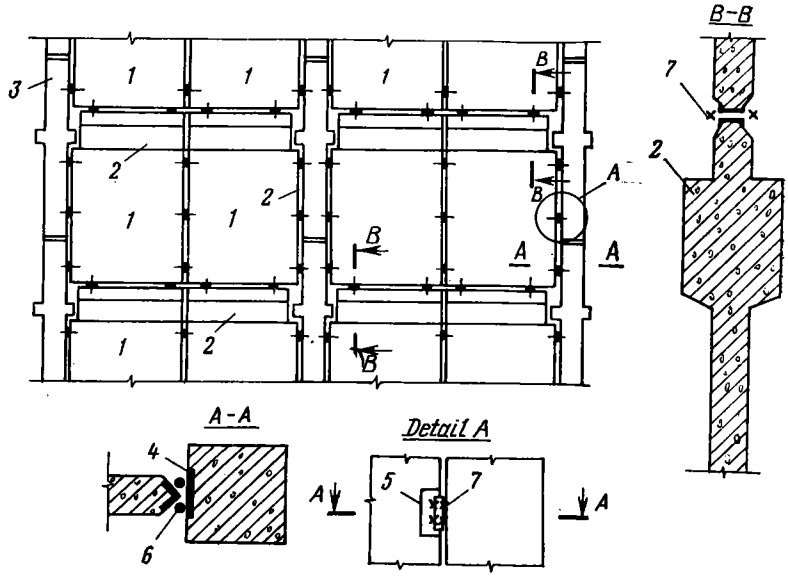


Fig. VI.20. Connection of shear wall members
1—shear wall panels; *2*—flanges supporting floor slabs; *3*—skeleton columns; *4*—column embedded parts; *5*—embedded parts of shear wall panels; *6*—joint bars; *7*—to be welded during erection

are strong and reliable. As compared with other types of joints, they take less steel and less labour to make them. At one time, the column members were made with spherical, rather than pointed ends, and the reinforcement stick-outs were welded together in a pool of molten metal contained within a reusable mould to form a spherical joint. This practice required more complex formwork, so it is no longer used in multi-storey building construction.

Precast shear walls consist essentially of skeleton columns and panels with flanges intended to carry floor slabs (Fig. VI.20). The component members are joined by welding together embedded steel parts and concreting. Use is also made of panels cast in-situ after reinforcing fabric has been welded to the embedded parts of the columns.

Stiffening-core shear walls are most frequently cast in-situ in sliding formwork. In-situ cast structures are preferable because the component members of the cores cannot be sufficiently standardized to warrant prefabrication. Also, precast products would require a considerable number of welded joints to be made during erection to resist appreciable shearing forces arising at the corners of the shear walls.

To meet sound-insulation requirements, the panels of the interior bearing walls are made of heavy concrete 14 to 16 cm thick. This

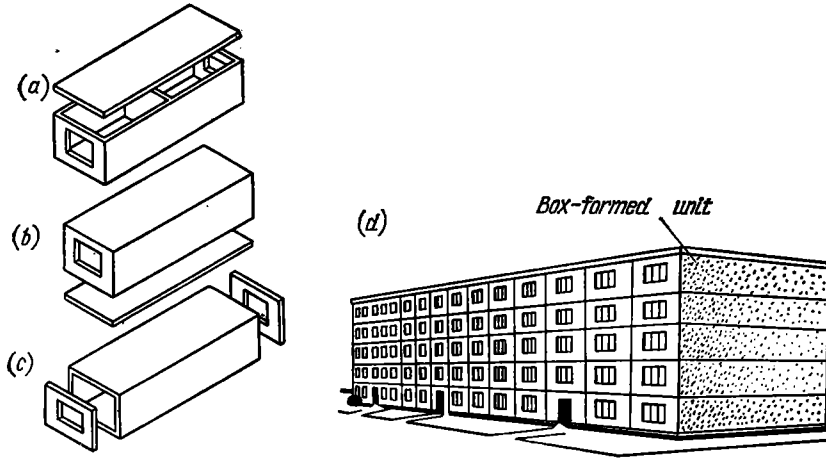


Fig. VI.21. Multi-storey apartment building built of box-formed units
(*a*) unit with a separate ceiling panel; (*b*) unit with a separate floor panel; (*c*) tube-like unit with separate end-wall panels; (*d*) multi-storey building

thickness ensures an adequate load-bearing capacity of the panels in buildings up to 16 storeys high. In higher buildings, the load-bearing capacity in the lower floors is enhanced by using higher brands of concrete and reinforced concrete or thicker panels.

The concrete panels of bearing walls are reinforced with vertical steel distributed for structural reasons at each face. The steel area is 0.3 cm^2 per metre run of the horizontal cross section of a panel. The amount of horizontal distribution reinforcement at each face should be at least 0.3 cm^2 per metre run of the vertical cross section of a panel. The reinforced concrete panels of bearing walls are reinforced with double vertical steel so that the minimum percentage of reinforcement in horizontal cross sections is 0.1 for M-200 concrete, and 0.15 for M-300 or M-400 concrete. To increase the strength of the support sections of reinforced concrete panels (that is, to compensate for the cut-off longitudinal reinforcement), they are indirectly reinforced with fabric.

An advanced type of bearing-wall buildings is one built of box-formed complete room- or apartment-size dwelling units. Such units are fully finished at an off-site factory and require minimum labour for erection. According to the manufacturing process involved, three types of box-formed units are available, namely a unit with a separate ceiling panel, a unit with a separate floor panel, and a tube-like unit with separate end-wall panels (Fig. VI.21). At an off-site factory, these dwelling units can be cast as monolithic structures or assembled from prefab panels. The behaviour of this type of building under service loads is governed by the manner in which the dwelling units rest one on top of another. If the units rest on in-situ concrete joint all the way around their perimeter, the building behaves as a bearing-wall type with the walls in compression. If the units are supported at the corners or rest on internal pilasters, the building behaves as a bearing-wall type with the walls subjected to bending in their plane.

VI.3. DESIGN OF MULTI-STOREY FRAMES

1. Choice of Trial Sections

Plane frames spaced a certain distance apart and connected by floors constitute a space frame block whose dimensions in plan are equal to the distance between expansion joints or exterior walls. Vertical dead and live loads and also lateral wind loads are simultaneously applied to all frames of a block, so the frames do not interact as in a space frame, and each plane frame can be analysed and designed individually.

A multi-storey reinforced concrete frame is statically indeterminate. Because of this, we should first choose the cross-sectional area of the girders and struts, determine their rigidity, and find the rigidity ratio. For this purpose, we may use as an example a similar previously designed structure, or find an approximate trial area for the component members. The depth of the girders is given by

$$h_0 = 1.8\sqrt{M/R_{pr}b}$$

where $M = 0.6M_0$ to $0.7M_0$; here, M_0 is the bending moment in the girder, calculated as for a simply supported single-span beam.

The cross-sectional area of the columns is approximately given by

$$F_c = 1.2N/R_{pr} \quad \text{to} \quad 1.5N/R_{pr}$$

The trial cross-sectional areas of the girders and columns thus found are coordinated and rounded off to the nearest standard size. The moments of inertia for the girder and column cross sections are determined as for a solid concrete section. If in-situ floors are used,

the moment of inertia for the girders is calculated as for a T-section whose flange width is equal to the centre-to-centre distance between the frames.

2. Forces Induced by Vertical Loads

The multi-storey multi-bay frames of skeleton-type buildings are analysed as repetitive regular structures or symmetric structures with a middle short bay (Fig. VI.22*a*). The load in each storey (tier)

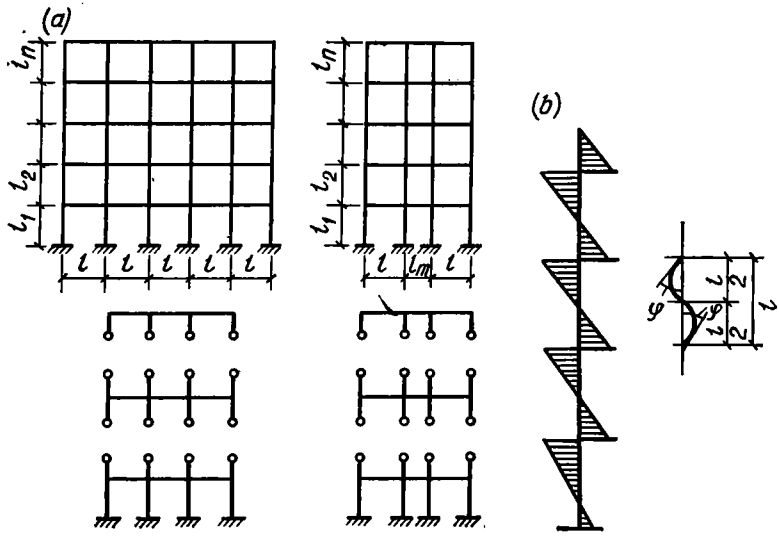


Fig. VI.22. (*a*) Design models for multi-storey frames, and (*b*) bending moment diagram for a multi-storey column

is assumed to be the same. The column joints in such frames, aligned along the same vertical, undergo approximately equal angular deflections and, as a consequence, are subjected to the same joint moments with the points of contraflexure lying in the middle of the storey height (Fig. VI.22*b*). Owing to this, we may design a multi-storey frame as a series of several one-storey frames in which all the struts or columns (except those in the ground floor) have hinges at their ends, and the height of the struts is equal to half the storey height.

For vertical load, three such one-storey frames must be analysed, namely the top-floor frame, the middle-floor frame and the ground-floor frame. If there are more than three bays in a frame, the frame is practically replaced by a three-bay frame, and the bending mo-

ments in the middle bays of the multi-bay frame are assumed to be the same as in the middle bay of the three-bay frame.

In the displacement method of design, the number of unknown angular deflections is equal to the number of joints in one storey of the frame. In the case of vertical loads, lateral deflections are usually ignored. In the force method of design, the unknowns are the support moments in the girder of one frame storey, and the problem reduces to the three-term equations for a beam on elastically rotating supports. Design calculations may also be carried out using the tables given in Appendix XIII.

The support moment in the girder of a frame having the columns of the same cross-sectional area within one storey is given by

$$M = (\alpha g + \beta p)\, l^2$$

where α and β are the tabular coefficients depending on the disposition of dead and live loads and the ratio of the sum of the rigidities per metre run of the struts to the rigidity per metre run of the girder; g and p are the dead and live loads per metre run of the girder; and l is the span of the girder (the centre-to-centre distance between the columns).

In each loading system of the frame, the bending moments in the struts are determined according to the difference in support moments between the girders in a joint and distributing it in proportion to the rigidities per metre run of the struts.

The bending moments in the span sections of the girders and also the shearing forces are found by usual methods as for a single-span beam exposed to an external load and bending moments applied at its ends.

In frame design, it is advisable to take into account the likely formation of plastic hinges and equalize bending moments to simplify the joints between the prefabricated members, standardize the formwork and the reinforcing steel, simplify the reinforcement of erection joints, improve concreting conditions, etc. For this purpose, the frame (as well as the girder in a beam and girder floor) is designed for dead load and various live load combinations as an elastic system. Then, an additional bending moment diagram is plotted for each of the above load combinations and the diagram for the elastic system so that the equalized moment in the design section is at least 70% of that in the elastic system.

For frame structures, it is good practice to deem that plastic hinges will form at the girder supports and to reduce the support moments. Assume that the frame has been designed as an elastic system, and a bending moment diagram has been plotted for a certain disposition of loads (Fig. VI.23*a*). If now we construct an additional bending moment diagram for the same loading system, the additional support bending moment, M_{add}, will be an independent variable,

and the frame in question and its set of canonical equations will have to be divided into two simpler systems with a smaller number of unknowns (Fig. VI.23*b*). An equalized bending moment diagram for the frame girders is shown in Fig. VI.23*c*.

In a simplified equalized-moment method, the girders of multi-storey and multi-bay frames are deemed to be subjected to live load applied at every second span and dead load applied at each span.

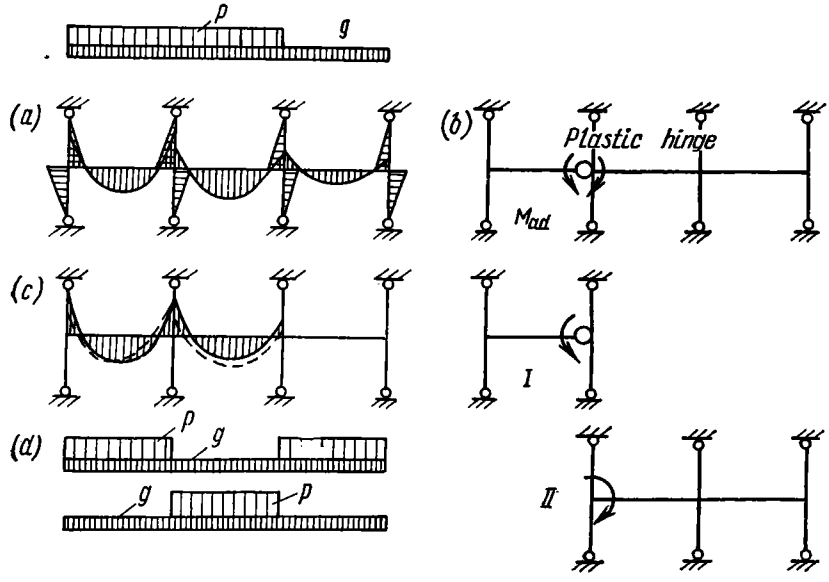


Fig. VI.23. To the design of multi-storey frames for vertical loads, using equalized moments

This disposition of loads yields a bending moment diagram with maximum moments in the spans and struts, which is taken as an equalized bending moment diagram (Fig. VI.23*d*). When the ratio of the intensities of live and dead loads is $p/g \leqslant 5$, the support girder moments in such an equalized-moment diagram are usually at least 70% of the maximum bending moments in the elastic system. In equalized-moment design, it is required that the moment due to the longitudinal force about the centroid of the compression zone taken up by the frame strut sections should be at least 70% of the corresponding moment in the elastic system. The frame struts behaving as in Case 2 should additionally carry all of the longitudinal force and at least half of the bending moment in the elastic system.

3. Forces Induced by Lateral Loads

Analysis for lateral (wind) loads is carried out approximately. The distributed lateral load is replaced by concentrated forces applied to the frame joints (Fig. VI.24). In the struts of all floors (except the ground floor), the points of contraflexure are assumed

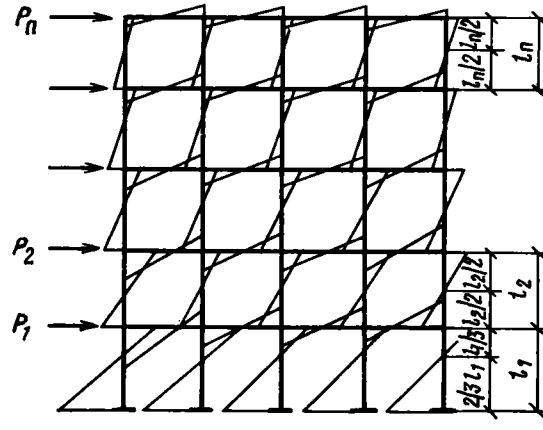


Fig. VI.24. To the design of multi-storey frames for lateral loads

to be in the middle of the storey height. In the ground floor where the strut is clamped in the footing, the points of contraflexure are taken to be at two thirds of the height as reckoned from the points of fixation.

The shearing forces in each storey of the frame are

$$Q_1 = P_1 + P_2 + \ldots + P_n$$

$$Q_2 = P_2 + P_3 + \ldots + P_n \quad \text{and so on}$$

They are distributed among the individual struts in proportion to their rigidities:

$$Q = Q_k B / \sum_1^m B_j \tag{VI.9}$$

where B is the rigidity of a strut, and m is the number of struts per storey.

The outer struts in the frame with a degree of fixation in a joint below that of the middle struts (because the outer joints have girders adjoining only from one side) take up a smaller share of the shearing force in each storey. In design calculations, this is taken care of by multiplying the rigidities of the outer struts by $\beta < 1$ looked up in Table VI.1.

TABLE VI.1. Coefficient β for Reducing Rigidities of Outer Struts in Multi-Storey Frames in Analysis for Lateral Loads

Coefficient	All floors, except ground floor, with i_g/i_b						Ground floor
	0.25	0.5	1	2	3	4	
β	0.54	0.56	0.62	0.7	0.75	0.79	0.9

Note. $i_g = B_g/l_g$ is the rigidity per metre run of the girder in the outer span, and i_b is the rigidity per metre run of the outer strut adjoining from beneath.

Once the shearing forces are found, it is necessary to determine the bending moments in the struts. For all floors, except the ground floor, the bending moments are given by

$$M = Q\,(l/2) \tag{VI.10}$$

For the ground floor, the strut bending moments in the top and bottom sections are determined as

$$M = Q\,(l/3) \qquad \text{and} \qquad M = Q\,(2l/3) \tag{VI.11}$$

When finding the support moments of the girders, the total moment induced in a frame joint by top and bottom struts is assumed to be distributed among the girders in proportion to their rigidities per metre run. In the outer joints, the moment in the girder is the sum of the strut moments.

4. Design Forces and Section Proportioning

On the basis of bending moment and shear diagrams for various dispositions of loads, we construct the resultant bending moment diagrams and calculate the corresponding longitudinal forces, N, for the main and additional load combinations.

The resultant diagrams are then used to find the values of M_{max} and M_{min} and the corresponding values of N, and also N_{max} and the corresponding M for the design sections. The design forces may alternatively be found from a table in which the values of forces for each disposition of loads are entered in appropriate columns. For the girders, the design sections are those at the supports and in the span, and for the columns, the design sections are those at the top, at the bottom and also one or two intermediate sections for long columns.

The sections of the girders and struts are proportioned as for bending and compression members. If the bending moments differ

in sign but are close in magnitude, the sections are reinforced symmetrically. The column section may be considered to be proportioned correctly if the percentage of reinforcement ranges between 1 and 2. The effective length of the struts is adopted according to their fixation in the joints. For the members of precast frames, the design should be extended to include the joints.

5. Lateral Deflections

For design purposes, a multi-storey multi-bay skeleton-type building behaving as a frame structure is treated as a multi-storey frame in which the rigidities of the girders and struts are equal

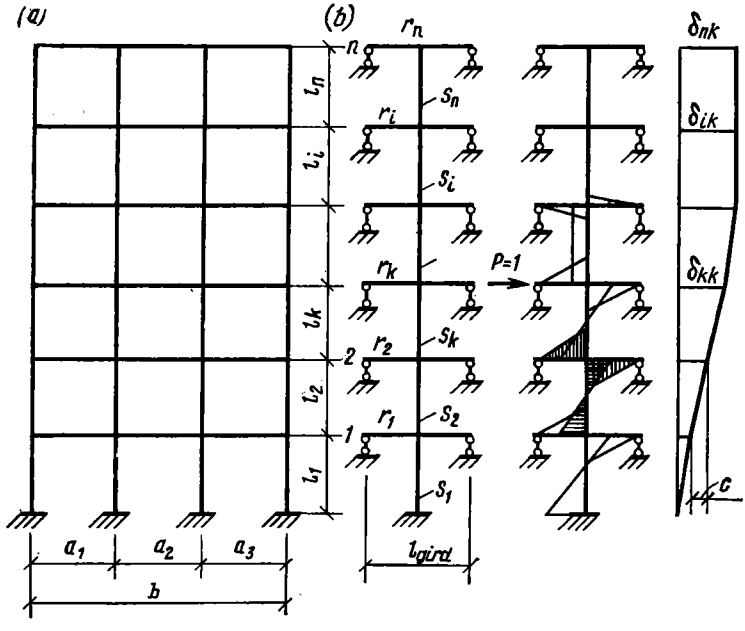


Fig. VI.25. Design models for and deflections of a multi-storey frame

to the total rigidities of all the frames of the building (Fig. VI.25*a*). Experiments have shown that in analysis for lateral deflections, the angular deflections of the joints in one storey of a multi-storey frame may be deemed equal and the design model for the frame may be assumed to be as shown in Fig. VI.25*b*. Here, s_i is the sum of the rigidities per metre run of the struts in one storey, r_i is the sum of the girder rigidities in this storey divided by the average girder span l_g (r_i may also be taken as the sum of the rigidities per metre

run of the girders in one storey), l_i is the storey height, and n is the number of storeys.

In the adopted design system, the lateral deflections induced by $P = 1$ are given by

$$\delta_{11} = (1/12)\,(S_1 + R_1) \tag{VI.12}$$

$$\delta_{kk} = (1/12)\,(S_k + R_k + l_k^2/4r_k), \quad k = 2, 3, \ldots, n \tag{VI.13}$$

$$\delta_{kl} = \delta_{ik} = \delta_{k,\,k+1} = \ldots = \delta_{kn} = \delta_{kk} + l_k l_{k+1}/48r_k \tag{VI.14}$$

where

$$S_k = \sum_1^k l_i^2/s_i \tag{VI.15}$$

$$R_1 = l_1^2/(4r_1 + 0.33s_1) \tag{VI.16}$$

$$R_2 = (l_1 + l_2)^2/(4r_1 + 0.33s_1) \tag{VI.17}$$

$$R_k = R_{k-1} + (l_{k-1} + l_k)^2/4r_{k-1}, \quad k = 3, 4, \ldots, n \tag{VI.18}$$

In Eq. (VI.13), the first term, S_k, shows the effect that the strut rigidity has on the deflections of the multi-storey frame, and the second term, R_k, gives the effect of the girder rigidity. When finding deflections on the assumption that the frame struts are fully fixed in the joints, R_k should be taken as zero. This, however, may introduce a significant error (the frame deflections thus found may be two, three or even more times the actual value).

The deflections of multi-storey frames due to lateral loads applied simultaneously to all storeys are given by

$$y = \delta_{k1}P_1 + \delta_{k2}P_2 + \ldots + \delta_{kn}P_n \tag{VI.19}$$

If the number of storeys is $n \geqslant 6$, and the shearing force per storey is

$$Q_k = \sum_{i=k}^{n} P_i \quad \text{where} \quad i \geqslant k$$

the frame deflections may be found as the sum of the relative displacements of the strut ends (sidesways) in each storey:

$$y = \sum_{i=1}^{k} Q_i c_i \quad \text{where} \quad i \leqslant k \tag{VI.20}$$

Here, c_i is the sidesway induced by a unit force applied at one of the top storeys.

For a multi-storey frame of regular structure where s and r remain constant with height, and the storey height l is uniform throughout the building, the sidesway induced by $P = 1$ may be found by multiplying the bending moment diagram by itself in the shaded

areas (Fig. VI.25*b*). Then,

$$c = \sum \int \overline{M}^2 \, dx/B = (l^2/12)\,(1/s + 1/r) \qquad \text{(VI.21)}$$

Designating the shear rigidity of the multi-storey frame as A and recalling that the unit angular deflection is $\psi = Ac/l = 1$, we may write

$$A = l/c \text{ or } A = 12/l\,(s^{-1} + r^{-1}) \qquad \text{(VI.22)}$$

If the number of storeys is $n \geqslant 6$, the discrete disposition of the girders may be replaced by a continuous disposition, the concentrated load, P_i, by a distributed load, $p\,(x)$, and the summation in Eq. (VI.20) by integration. Then, the deflection is

$$y = \int_0^x Q_0\,(c/l)\,dx = \int_0^x (Q_0/A)\,dx \qquad \text{(VI.20}a\text{)}$$

where Q_0 is the shearing force induced by the distributed load, and x is the coordinate of the horizontal section of the frame.

Differentiating Eq. (VI.20*a*) successively gives

$$Ay' = Q_0 \qquad \text{(VI.23)}$$

$$Ay'' = -p\,(x) \qquad \text{(VI.23}a\text{)}$$

As is seen, in a multi-storey frame subjected to bending, the relation between the lateral deflection and the shearing force is the first derivative, and the curvature, y'', is equal, accurate to the constant, A, to the external load taken with the "minus" sign.

The line of the total bending of the struts, *1* (the diagram of the frame storey deflections), concaves toward the initial strut position as in a system subjected to shear, whereas the line of local strut bending, *2*, oscillates about line *1*, deflecting on either side within each storey (Fig. VI.26). When the rigidity of the struts considerably exceeds the rigidity of the girders, the struts bend in a different manner which is shown in the figure by line *3*. In this case, the shearing force, $-By'''$, which is a function of the total flexural rigidity of the struts, $B = \Sigma B_j$, becomes rather significant. In addition, the longitudinal forces, N, bringing about elongation and shortening of the outer struts cause the frame to bend as a vertical cantilever structure in which the distance between the outer struts (chords) is b and the flexural rigidity is B_0. In the middle struts of a multi-storey frame with practically identical bays, the longitudinal forces are insignificant because they are equal to the difference in shearing force between the girders. Assuming that the frame experiences plane rotation, the additional angular deflection of the struts

induced by the moment of the vertical structure, $M = Nb$, is

$$\int_0^x (\overline{M}M/B_0)\,dx = (b/B_0)\int_0^x N dx$$

Then, according to Eq. (VI.23), the shearing force with allowance

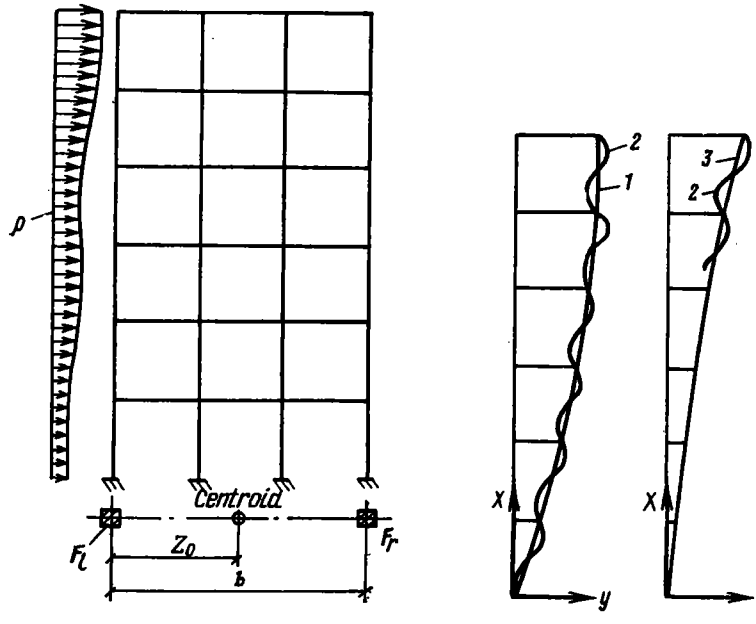


Fig. VI.26. Deflections of a multi-storey frame
1—line of total bending of the struts in a multi-storey frame deforming as a structure subjected to shear; *2*—line of the local bending of the struts; *3*—line of the total bending of the struts whose rigidity considerably exceeds the rigidity of the girders

for the deformations due to the longitudinal forces may be written as

$$A\left[y' + (b/B_0)\int_0^x N dx\right] = Q_0 \qquad \text{(VI.24)}$$

The general equation for the equilibrium of the shearing forces in a horizontal section of the frame is

$$-By''' + Ay' + (Ab/B_0)\int_0^x N dx = Q_0 \qquad \text{(VI.25)}$$

The value of N can be found from the moment equilibrium equation for the same horizontal section

$$N = (M_0 - M)/b = (M_0 - By'')/b$$

where M_0 is the moment due to the external load at x, and $M = -By''$ is the total bending moment in the frame struts.

On substituting for N in Eq. (VI.25) and differentiating with respect to x, the equation for bending takes the form

$$By^{IV} - Av^2y'' - AM_0/B_0 - p(x) = 0 \qquad \text{(VI.26)}$$

If we designate the deflection, y, multiplied by B as w, the above equation can be re-written as

$$s_2^2 w^{IV} - w'' - (v^2 - 1) M_0/v^2 - s_2^2 p(x) = 0 \qquad \text{(VI.27)}$$

where

$$s_2 = \sqrt{B/Av^2} \qquad \text{(VI.28)}$$

is the linear characteristic, and

$$v^2 = 1 + B/B_0 \qquad \text{(VI.29)}$$

Equation (VI.27) is the general equation for the design of frame, shear wall-frame and shear-wall structures. Since $M = -w''$, this equation may be reduced to a second-order differential equation.

The solution of Eq. (VI.27) for w is

$$w = C_1 + C_2 s_2 \varphi + C_3 \cosh \varphi + C_4 \sinh \varphi + C_0 \qquad \text{(VI.30)}$$

Here, C_i are the integration constants depending on the boundary conditions, C_0 is the partial solution depending on the load type, $\varphi = x/s_2$ is the dimensionless coordinate,

$$\lambda = H/s_2 \qquad \text{(VI.31)}$$

is the rigidity characteristic at $x = H$,

$$H = H_0 n/(n - 0.5) \qquad \text{(VI.32)}$$

is the design height of the building, and H_0 is the distance from the fixation to the centre line of the top-storey girder.

For ordinary frame structures, the first term in Eq. (VI.26) may be neglected, so

$$Ay'' + AM_0/B_0 + p(x) = 0 \qquad \text{(VI.33)}$$

Here, $v^2 = 1$ because $B = 0$.

On integrating Eq. (VI.33) twice and finding the integration constants from the boundary conditions: (1) $y(0) = 0$ and (2) $Ay'(0) = Q_0(0)$, we get an equation for the deflections of a multi-storey frame subjected to the uniformly distributed load, $p = p(x)$, and the moment due to the external load, $M_0 =$

$$= -0.5p\,(H - x)^2.$$

$$y = (pH^2/2A)\,(2\xi - \xi^2) + (pH^4/8B_0)\,(2\xi^2 - 4\xi^3/3 + \xi^4/3) \tag{VI.34}$$

where $\xi = x/H$ is the dimensionless coordinate.

For $\xi = 1$, the sag of the top storey of the frame is

$$f = pH^2/2A + pH^4/8B_0 = (pH^2/2A)\,(1 + \lambda_f^2/4) \tag{VI.35}$$

where λ_f is the rigidity characteristic of the frame with allowance for the longitudinal forces in the struts,

$$\lambda_f = H\sqrt{A/B_0} \tag{VI.36}$$

Experiments have shown that if $\lambda_f < 0.7$, the effect of the longitudinal forces in the struts of a multi-storey frame may be neglected, so, in calculations, $\nu^2 = 1$.

Let F_l and F_r designate the total cross-sectional areas of the left- and right-hand outer struts in a storey and z_0 the distance from the centre line of the left-hand struts to the centroid of the horizontal section (see Fig. VI.26). Then,

$$z_0 = F_r b/(F_r + F_l) = b/(1 + F_l/F_r) \tag{VI.37}$$

The moment of inertia of the horizontal section is

$$I_0 = F_l z_0^2 + F_r\,(b - z_0)^2 = F_l b^2/(1 + F_l/F_r) \tag{VI.38}$$

the flexural rigidity of the frame is

$$B_0 = E_c F_l b^2/(1 + F_l/F_r) \tag{VI.39}$$

and the flexural rigidity of a symmetrical frame at $F_l = F_r = F$ is

$$B_0 = E_c F b^2/2 \tag{VI.40}$$

Therefore, the rigidity, B_0, depends on the axial rigidity of the struts, $E_c F$.

6. Joint Compliance

The compliance or deformability of joints between prefabricated reinforced concrete members somewhat reduces the rigidity and increases horizontal sags in multi-storey skeleton-type buildings. Deformations at the joints between the girders and struts result from nonelastic strain in embedded parts, tie bars and anchors in the concrete. The bending moment, M, arising at a joint causes the initial angle between the faces of the members forming the joint to change by φ which is known as the angle of compliance. The average modulus of joint deformability or the rigidity coefficient of the joint is

determined experimentally as the slope of the secant in the M versus φ diagram (Fig. VI.27):

$$C = M/\varphi$$

The angle of joint compliance is given by

$$\varphi = (u_1 + u_2)/z$$

where u_1 and u_2 are the measured displacements of the tension and

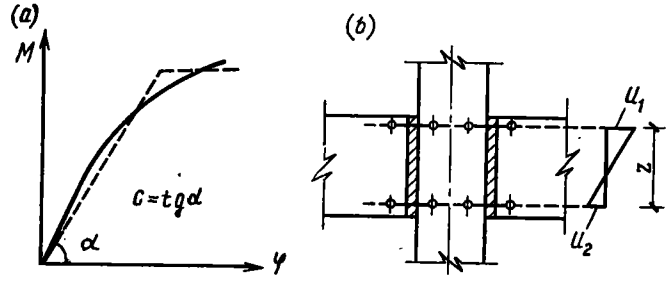


Fig. VI.27. Compliance of joints between girders and columns

compression zones of the joint minus the displacements produced in similar in-situ joints, and z is the distance between the instrument axes.

As a first trial, the coefficient of joint rigidity measured in kN/m may be determined as a function of the depth of the member being joined, using the following empirical formula

$$C = \eta 10^5 h^2 \quad \text{where} \quad \eta = \text{(approx.) } 7 \tag{VI.41}$$

When finding the shear rigidity, A, of a frame, the joint compliance is allowed for by reducing the rigidity per metre run of the members. If the joints between the columns are installed at each floor and the joints between the girders are made at each support, then

$$\left.\begin{aligned} s &= \sum i_{st}(1+\mu_{st})/(1+4\mu_{st});\ \mu_{st} = i_{st}/C_{st} \\ r &= \sum i_g/(1+6\mu_g);\ \mu_g = i_g/C_g \end{aligned}\right\} \tag{VI.42}$$

where C_{st} and C_g are the coefficients of joint rigidity in the struts and girders, respectively, and i_{st} and i_g are the rigidities per metre run of the struts and girders, respectively.

The joints between columns also experience axial compliance under the action of the longitudinal force, N. The longitudinal force is responsible for an additional longitudinal displacement, u, in the zone of the column joint. The coefficient of joint rigidity, $C_0 = N/u$, is found experimentally (Fig. VI.28). According to

experimental data, the coefficient of joint rigidity for type-design columns 40 cm square in cross section is $C_0 = 7 \times 10^6$ kN/m.

The longitudinal strain in the struts with allowance for the joint compliance (with the joints installed at each floor) is given by

$$\varepsilon = N/E_cF + N/lC_0 = N(1 + \mu_0)/E_cF \qquad \text{(VI.43)}$$

where

$$\mu_0 = E_cF/lC_0 \qquad \text{(VI.44)}$$

We may assume that the joint compliance causes the modulus of elasticity, E_c, to decrease to $E_c(1 + \mu_0)$ of its original value.

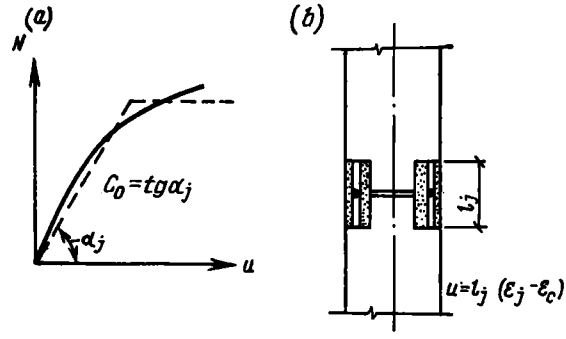


Fig. VI.28. Compliance of joints between columns

According to Eq. (VI.40), the flexural rigidity of a symmetric frame with allowance for the compliance of the strut joints can be written as

$$B_0 = E_cFb^2/2(1 + \mu_0) \qquad \text{(VI.40}a\text{)}$$

The value of B_0 for an asymmetric frame is determined in a similar manner.

Present evidence shows that the joint compliance may increase the lateral deflections in a multi-storey skeleton-type building by 20 to 40%.

7. Dynamic Characteristics

For a multi-storey skeleton-type building (Fig. VI.29*a*), the number of frequencies and the corresponding modes of free lateral vibrations is equal to the number of storeys (the number of degrees of freedom). The weight of the floors and columns is assumed to be applied at the joints (Fig. VI.29*b*). The storey mass of a multi-storey

building is

$$m = Q/g \tag{VI.45}$$

where Q is the load per storey due to the weight of the floor, columns and walls and the live load, and g is the acceleration due to gravity.

The frequencies and modes of free vibrations are found from frequency equations in which the unit deflections, δ_{ik}, are determined by Eqs. (VI.12) and (VI.13).

The fundamental frequency of vibrations in a multi-storey frame may be found as

$$\varphi_1 = a_1/\sqrt{f} \tag{VI.46}$$

where a_1 is a coefficient depending on the number of storeys, for $n = 3$, $a_1 = 1.08$, for $n = 4$, $a_1 = 1.1$, and for $n = 5$, $a_1 = 1.12$;

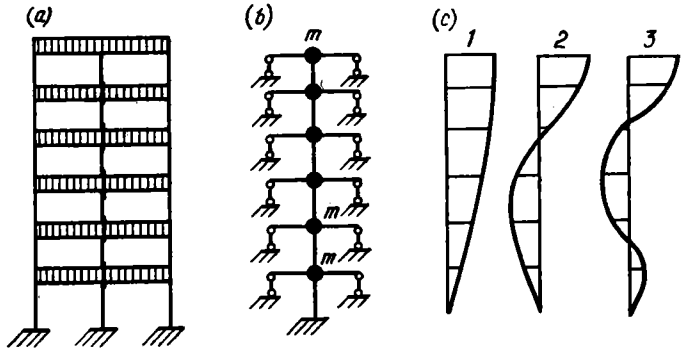


Fig. VI.29. To the dynamic design of a skeleton-type multi-storey building (*a*) transverse frame; (*b*) design model for determining the frequencies and modes of free lateral vibrations; (c) first three modes of free vibrations

and f is the sag of the top storey of the frame, given by Eq. (VI.19), induced by the lateral forces equal to the storey masses, $P_k = m_k$.

In the case of free horizontal vibrations in a multi-storey frame, the external load is the inertial forces of mass equal to $-m\partial^2 y/\partial t^2$. For $n \geqslant 6$, they may be regarded as a distributed load

$$p(x) = -(m/l)(\partial^2 y/\partial t^2) \tag{VI.47}$$

Recalling that $Ay'' = -p(x)$, we get a homogeneous differential equation for free vibrations

$$A\partial^2 y/\partial x^2 - (m/l)(\partial^2 y/\partial t^2) = 0 \tag{VI.48}$$

Substituting $y = XT$ and solving the above equation yields

$$X = C_1 \sin \alpha x + C_2 \cos \alpha x \tag{VI.49}$$

where X is the ordinate of the free vibration mode, T is a time function,

$$\alpha = \omega \sqrt{m/Al} \tag{VI.50}$$

and ω is the frequency of free horizontal vibrations.

The boundary conditions are: (1) $X(0) = 0$ and (2) $AX'(H) = 0$. As a consequence, we get two homogeneous equations: (1) $C_2 = 0$

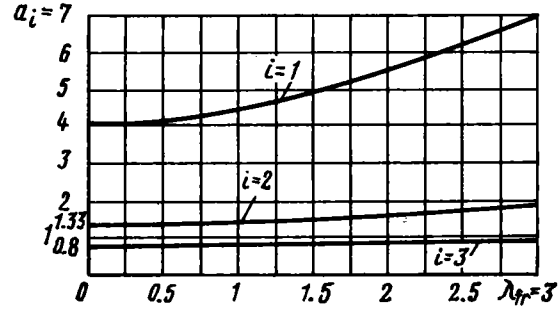


Fig. VI.30. To determining the period of free lateral vibrations in multi-storey frames with allowance for longitudinal forces in the struts

and (2) $C_1 \cos \alpha H = 0$. Solving these equations gives $\cos \alpha H = 0$ and $\alpha H = (2i - 1)\pi/2$, where $i = 1, 2, 3, \ldots$ is the harmonic number of free vibrations.

According to Eq. (VI.50), the period of free vibrations is

$$T_i = 2\pi/\omega = [4H/(2i-1)] \sqrt{m/Al} \tag{VI.51}$$

where H is as given by Eq. (VI.32).

According to Eq. (VI.49), the mode of free vibrations is

$$X = C_1 \sin \alpha x = C_1 \sin (2i - 1)\pi\xi/2 \tag{VI.51'}$$

When determining the mode of free vibrations, we need to know only the displacement ratios, so in Eq. (VI.51') $C_1 = 1$. The first three modes of free vibrations are shown in Fig. VI.29*c*.

If, according to Eq. (VI.36), the rigidity characteristic of the frame is $\lambda_f \geqslant 0.7$, the period of free vibrations is found with allowance for the longitudinal forces in the struts

$$T_i = a_i H \sqrt{m/Al} \tag{VI.51''}$$

where a_i is the coefficient determined using the plot of Fig. VI.30. Referring to the figure, the longitudinal forces mostly affect the fundamental frequency.

VI.4. DESIGN OF MULTI-STOREY SKELETON- AND BEARING-WALL-TYPE BUILDINGS FOR LATERAL LOADS

1. Design Models and Loads

Design models for multi-storey skeleton- and bearing-wall-type buildings are chosen according to their construction and the manner in which they resist lateral loads. As already mentioned, multi-storey buildings can be classified into frame, shear wall, and frame-shear wall structures. The floors are regarded as rigid horizontal

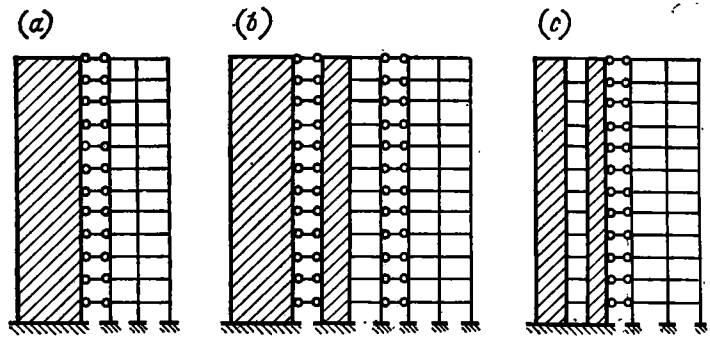


Fig. VI.31. Design models for shear wall-frame structures
(a) with a solid shear wall; (b) with a solid and a combined shear wall; (c) with an open-work shear wall

diaphragms nondeformable in their plane. The cases where the effect of in-plane bending should be taken into consideration are discussed later.

Frame-Shear Wall Structures. The design models for such structures are conceived to reflect the interaction of multi-storey frames and the various shear walls which may be solid, combined or open-work (Fig. VI.31). In the design model, the vertical structures which in actual buildings are aligned parallel to one another are shown standing in one plane and joined by link bars because their lateral deflections are the same at each storey level. Actually, the link bars shown between the multi-storey frame and shear wall are the floors. The link bars are deemed to be noncompressible and nonextensible. In the design model, the rigidity of the shear wall is taken equal to the total rigidity of the corresponding shear walls in the building block.

Shear Wall Structures. The design models for these structures are conceived to reveal the interaction between the various combinations of shear walls in multi-storey skeleton- or bearing-wall-type build-

ings. The shear walls may be with or without openings, or have one or several rows of openings (Fig. VI.32). Here, the shear walls which in actual buildings are aligned parallel to one another are shown standing in one plane and joined by link bars.

Since the longitudinal strains in the girders, lintels and link bars between the shear walls are small, they may be neglected. The shear strain in the frame struts and shear walls may likewise be ignored. The ratio of the shear wall section depth to the shear wall length is usually $h/H_0 \leqslant 1/4$.

The effect of the joint compliance in the struts and girders is taken care of by an appropriate reduction in their rigidity per metre run.

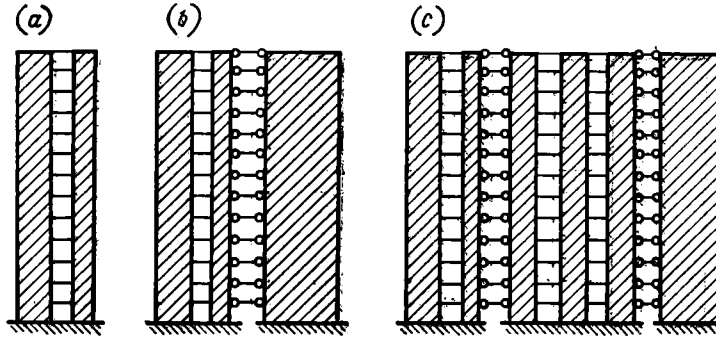


Fig. VI.32. Design models for shear wall structures
(a) with open-work shear walls; (b) with open-work and solid shear walls; (c) with mixed-type shear walls

As has been shown by experiments, the effect of the joint compliance in the shear walls may be taken into account in calculations by reducing their flexural rigidity by 30%.

In the design models for multi-storey buildings of regular structure where the rigidity of the members is uniform throughout the height, it is advisable to replace the discrete girders, lintels and link bars by a continuous arrangement. Then the design calculations may be based on the general differential equation (VI.27). The forces, deflections and dynamic characteristics of various multi-storey buildings can be found using the unified formulas and tables of solutions for the general equation. The design height of a building is given by Eq. (VI.32); when the number of storeys $n \geqslant 16$, $H = H_0$.

For the strength analysis of 12-storey buildings more than 40 m high, the design wind load is determined with allowance for the velocity pressure pulsations brought about by wind gusts. In addition, a building should be checked for the acceleration of vibrations, which should be $W \leqslant 15$ cm/s^2.

The sags in a multi-storey building are found for the basic wind load. The sag of the top storey, f, should not exceed $H/1\,000$.

For design purposes, the lateral wind load increasing upwards is replaced by an equivalent uniformly distributed load or an equivalent trapezoidal load. With a uniformly distributed load, the design formulas are more compact, and the deflections and forces in the design sections are practically exact. The equivalent uniformly distributed wind load depends on the moment in the footing

$$p = 2M_f/H^2$$

where M_f is the footing moment due to the actual wind load.

2. Frame-Shear Wall Structures

In frame-shear wall structures using solid shear walls (Fig. VI.33), the lateral deflections of all vertical members connected by the

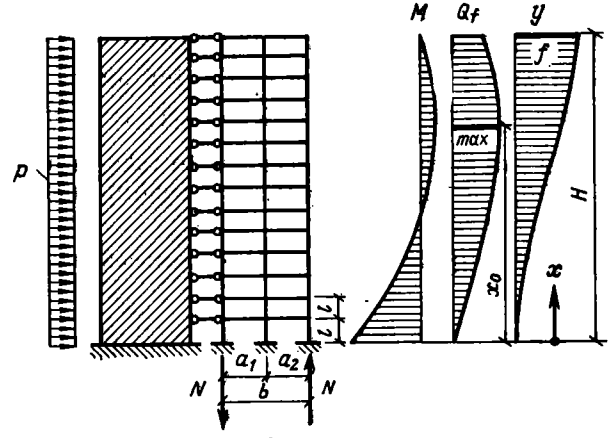


Fig. VI.33. To the design of a shear wall-frame structure

floors rigid in their plane are equal, so their total flexural rigidity is

$$B = \sum B_j + B_{sh\text{-}w}$$

where ΣB_j is the total rigidity of the frame struts, and $B_{sh\text{-}w}$ is the total rigidity of the shear walls.

The total rigidity of the struts is usually small as compared with the total rigidity of the shear walls and is, therefore, neglected. In design calculations, $B = B_{sh\text{-}w}$. This problem likewise involves Eq. (VI.27) and its solution given by Eq. (VI.30). The boundary conditions are as follows:

(1) $w(0) = 0$; (2) $w'(0) = 0$; (3) $-w'''(0) = Q_0(0)$ and

(4) $w''(\lambda) = 0$

With a uniformly distributed load $p(x) = p$, the moment and the shearing force are given by

$$M_0 = -0.5pH^2(1-\xi)^2; \qquad Q_0 = pH(1-\xi)$$

Then the value of C_0 in Eq. (VI.30) is

$$C_0 = -ps_2^4\varphi^2/2\nu^2 + [ps_2^4\lambda^4(\nu^2-1)/2\nu^2](\xi^2/2 - \xi^3/3 + \xi^4/12) \quad \text{(VI.52)}$$

Solving the set of linear equations gives

$$C_1 = -C_3 = -ps_2^4\varkappa/\nu^2 \quad \text{(VI.53)}$$

$$C_4 = -s_2C_2 = -ps_2^4\lambda/\nu^2 \quad \text{(VI.54)}$$

where

$$\varkappa = (1 + \lambda \sinh \lambda)/\cosh \lambda \quad \text{(VI.55)}$$

On substituting for C_i in Eq. (VI.30), the equation for deflections takes the following form

$$w = (ps_2^4/\nu^2)\{\lambda\varphi - \varphi^2/2 + \varkappa\cosh\varphi - \lambda\sinh\varphi - \varkappa + [\lambda^4(\nu^2-1)/2](\xi^2/2 - \xi^3/3 + \xi^4/12)\} \quad \text{(VI.56)}$$

When $\varphi = 1$ and $\xi = 1$, the sag of the top storey is

$$f = (pH^4/2\nu^2\lambda^2B)[1 - 2(\varkappa-1)/\lambda^2 + \lambda^2(\nu^2-1)/4] \quad \text{(VI.57)}$$

When finding the forces, it is necessary to take into account that $dx = s_2d\varphi = Hd\xi$ and that $\lambda s_2 = H$. The bending moments in the shear wall are given by

$$M = -w'' = -(pH^2/\nu^2)[0.5(1-\xi)^2(\nu^2-1) - (1 - \varkappa\cosh\varphi + \lambda\sinh\varphi)/\lambda^2] \quad \text{(VI.58)}$$

The shearing forces in the shear wall are

$$Q = M' = (pH/\nu^2)[(1-\xi)(\nu^2-1) + \cosh\varphi - (\varkappa/\lambda)\sinh\varphi] \quad \text{(VI.59)}$$

The shearing forces in the frame struts are given by

$$Q_f = Q_0 - Q = (pH/\nu^2)[1 - \xi + (\varkappa/\lambda)\sinh\varphi - \cosh\varphi] \quad \text{(VI.60)}$$

The longitudinal forces in the outer struts of the frames can be found from the equilibrium equation

$$N = (M_0 - M)/b = -(pH^2/b\nu^2)[0.5(1-\xi)^2 + (1 - \varkappa\cosh\varphi + \lambda\sinh\varphi)/\lambda^2] \quad \text{(VI.61)}$$

The bending moments, M, and the shearing forces, Q, are distributed among the individual shear walls of the structure according to their flexural rigidities.

The bending moment and deflection diagrams for the shear wall-frame structure are given in Fig. VI.33. In the shearing force diagram, Q_f is a maximum at the section with the coordinate, φ_0, where

$$Q_f' = -1 + \varkappa\cosh\varphi_0 - \lambda\sinh\varphi_0 = 0 \quad \text{(VI.62)}$$

It should be noted that at $\varphi = \lambda$ the shearing force, Q_f, is other than zero. The shearing force, Q_f, is distributed among the individual frame struts according to their rigidities. The bending moments in the struts and girders of the multi-storey frame are determined using the values of the shearing forces as described in Sec. VI.3.

The manner in which the line of bending due to the lateral load runs for a frame-shear wall structure depends on the rigidity characteristic, λ. In relatively rigid shear walls with $\lambda \leqslant 1$, the line

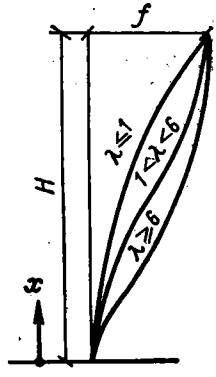


Fig. VI.34. Line of bending of a shear wall-frame structure as a function of rigidity characteristic λ

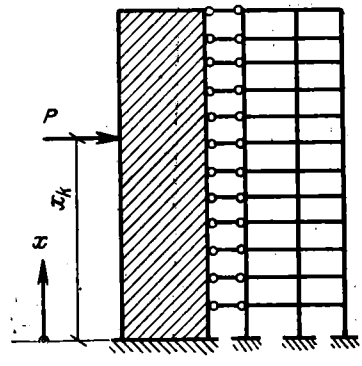


Fig. VI.35. To determining deflections of a frame-shear wall structure due to a lateral force

of bending convexes towards the initial position as in cantilever beams. As λ increases, the line of bending becomes convexo-concave, and, at $\lambda \geqslant 6$, concave (Fig. VI.34). The shape of the line of bending markedly affects the dynamic characteristics of a multi-storey building.

The lateral deflections of the frame-shear wall structure due to $P = 1$ applied at x_k (Fig. VI.35) are determined by solving Eq. (VI.25) at $p(x) = 0$, with the moment $M_0 = -(x_k - x)$ in the portion $x \leqslant x_k$, and $M_0 = 0$ in the portion $x \geqslant x_k$. The boundary conditions are supplemented by the constraints in terms of the linear and angular deflections, and the bending moment at x_k. Then, the deflection equation yields

$$\delta_{ik} = [\varkappa_1 + 0.5\varphi_k^2 (\nu^2 - 1)(\varphi_i - \varphi_k/3)] H^3/\nu^2\lambda^3 B; \quad i \geqslant k \qquad \text{(VI.63)}$$

where

$$\varkappa_1 = \varphi_k - \sinh \varphi_k + (\cosh \varphi_k - 1)(\sinh \varphi_i - \tanh \lambda \cosh \varphi_i + \tanh \lambda) \qquad \text{(VI.64)}$$

$$\varphi_k = x_k/s_2 \quad \text{and} \quad \varphi_i = x_i/s_2$$

3. Frame-Shear Wall Structures with Combined Shear Walls

In frame-shear wall structures with solid and combined shear walls (Fig. VI.36), the total flexural rigidity is $B = B_{sh\text{-}w} + B_{com}$, whereas in structures with combined shear walls only it is $B = B_{com}$.

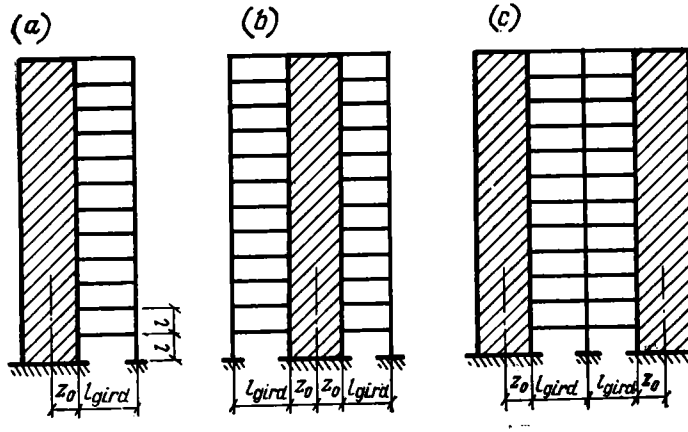


Fig. VI.36. To the design of frame-shear wall structures using combined shear walls
(a) frame part of the shear wall on one side; (b) frame part on either side; (c) frame part in the middle

The shear rigidity for the frame part of the structure is determined with allowance for the elastic rotation of the joints between the struts and girders (Fig. VI.37)

$$A_{com} = 3i_g(1+\eta_0)[i_g(1+\eta_0)+6i_{st}(1+2\eta_0)]/[l(i_g+3i_{st})] \quad \text{(VI.65)}$$

where i_g is the rigidity per metre run of the girder in the frame part, i_{st} is the rigidity per metre run of the strut in the frame part, and $\eta_0 = z_0/l_g$ (Fig. VI.36).

If the frames are arranged symmetrically on either side of the shear wall, the value of the shear rigidity in Eq. (VI.65) is doubled. If the structure is made up of two outer shear walls and a two-bay frame located in the middle (Fig. VI.36*c*), the shear rigidity in Eq. (VI.65) is doubled, too, but i_{st} is multiplied by 0.5.

The shear rigidity of a shear wall-frame structure with a combined shear wall is the sum of the shear rigidities of the frame proper and the frame part of the combined shear wall

$$A = 12/l\,(s^{-1} + r^{-1}) + A_{com} \quad \text{(VI.66)}$$

When $\lambda_f < 0.7$, the longitudinal forces in the struts of a multi-storey frame have negligible effect on the behaviour of the structure. If the longitudinal forces in the struts are neglected in design calculations and v^2 is assumed to be unity, the forces and deflections in the frame-shear wall structure using combined shear walls are found by the formulas derived above for the frame-shear wall structures.

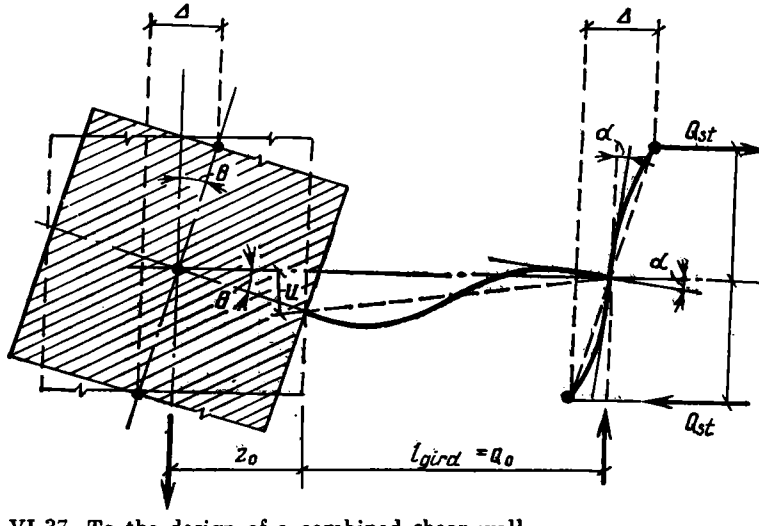


Fig. VI.37. To the design of a combined shear wall

The design of such structures with allowance for the longitudinal forces in the struts will be discussed in Subsec. 7 of the present chapter.

The total shearing force taken up by the struts of the frame proper, $Q_{f,\,st}$, and by the struts of the frame part of the shear wall, Q_{st}, is shared in proportion to the shear rigidities:

$$Q_{f,\,st} = Q_f (A - A_{com})/A \quad \text{and} \quad Q_{st} = Q_f A_{com}/A \tag{VI.67}$$

The support moments in the girders of the frame part at the face of the shear wall are related to Q_f as

$$M = [3i_g (1 + \eta_0) (6 + i_g/i_{st})/(3 + i_g/i_{st})] (Q_f/A) \tag{VI.68}$$

and those at the strut axis as

$$M = [18 (1 + \eta_0) i_g/(3 + i_g/i_{st})] (Q_f/A) \tag{VI.69}$$

The bending moment in the strut of the frame part of the combined shear wall is equal to half the support moment in the girder.

4. Shear Wall Structures Using Identical Open-Work Shear Walls

Consider shear wall structures using identical shear walls but having a different number of opening rows, say, with one asym-

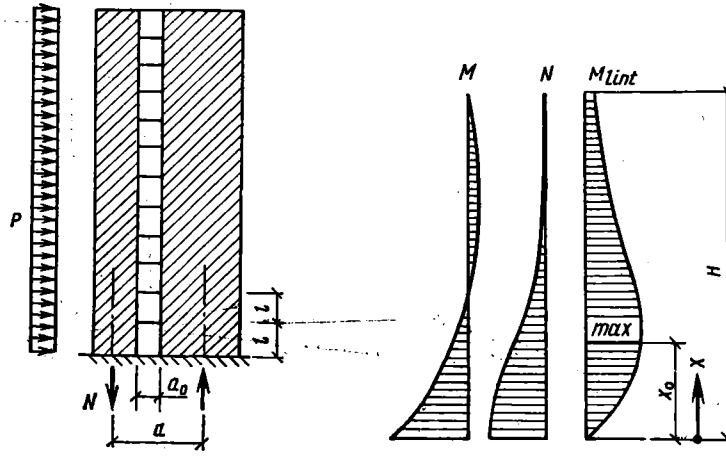


Fig. VI.38. To the design of an open-work shear wall with one asymmetrical row of openings

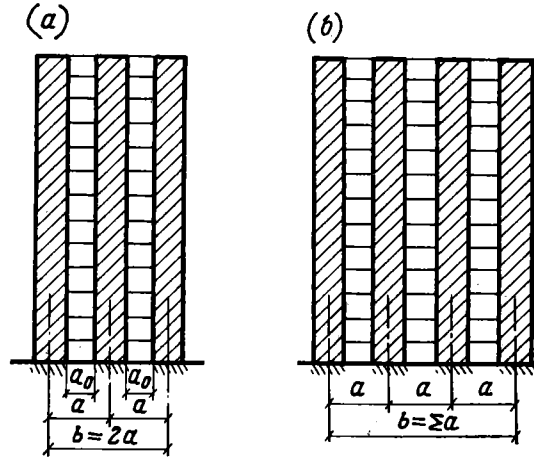


Fig. VI.39. To the design of an open-work shear wall with (*a*) two rows of openings; (*b*) several rows of openings

metrical row (Fig. VI.38) or with several rows slightly differing in opening width (Fig. VI.39).

We shall regard a shear wall with openings as a multi-storey (or multi-tiered) frame in which the vertical members or coupled walls act as struts and the lintels as girders. Since in such a frame the rigidity of the struts is many times that of the girders, the local bending of the struts between the joints within one storey may be neglected, and, when finding the shear rigidity, A, it may be deemed that $1/s$ is negligible in comparison with $1/r$. Then, according to Eq. (VI.22), the shear rigidity of the open-work shear wall is

$$A = 12r/l \tag{VI.70}$$

where $r = \Sigma i_{lint}$ is the total rigidity per metre run of the lintels in one storey (tier) of the shear wall having several rows of openings.

In addition, it should be kept in mind that the rigidity of the lintels, B_{lint}, is finite within the openings only, and that in each of the coupled walls or vertical members the lintels are perfectly rigid. Therefore, the lintel rigidity averaged over the entire span is $B_{lint}\gamma^3$, where $\gamma = a/a_0$, a is the centre-to-centre distance and a_0 is the clear space between the vertical members. Accordingly, the rigidity per metre run of the lintel is

$$i_{lint} = B_{lint}\gamma^3/ak \tag{VI.71}$$

The factor, k, takes care of the shear strain in the lintel. It is equal to

$$k = 1 + 2.4\,(h/a_0)^2 \tag{VI.72}$$

where h is the depth of the lintel cross section.

The total flexural rigidity of the vertical members of a shear wall is $B = \Sigma B_j$, where B_j is the flexural rigidity of one member. If the structure consists of solid and open-work shear walls (see Fig. VI.32*b*), the total flexural rigidity is $B = B_{sh\text{-}w} + \Sigma B_j$.

The flexural rigidity, B_0, of a shear wall (taken at the section through the openings minus the rigidity of the vertical members about their axes) is determined using Eq. (VI.39). In this equation, the centre-to-centre distance between the outer walls is $b = \Sigma a$; for one row of openings, $b = a$.

In the general equation (VI.27) and its solution given by Eq. (VI.30), the boundary conditions for open-work shear walls remain the same as for frame-shear wall structures. So, in this case, the design should be based on the deflection and sag equations (VI.56) and (VI.57) and the equation for the bending moments in the shear walls, (VI. 58).

The longitudinal forces in the outer vertical members of the shear wall are

$$N = -(1/l)\int_x^H Q_{lint}\,dx \tag{VI.73}$$

Hence, the shearing forces in the lintels are

$$N'l = -\frac{d}{dx}\int_{x}^{H} Q_{lint}\,dx = Q_{lint} \tag{VI.74}$$

Differentiating Eq. (VI.61) gives the shearing forces in the lintels

$$Q_{lint} = (pHl/bv^2)\,[1 - \xi + (\varkappa/\lambda)\sinh\varphi - \cosh\varphi] \tag{VI.75}$$

In a symmetric shear wall having two rows of openings, the shearing forces are the same in all the lintels of a tier (storey). For shear

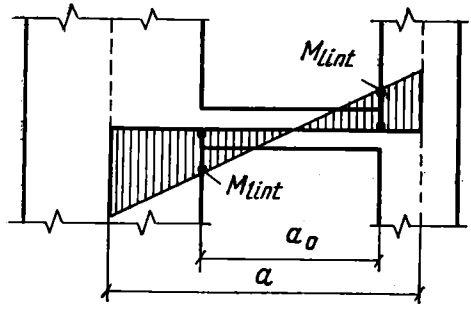


Fig. VI.40. Bending moment diagram for the lintel of an open-work shear wall

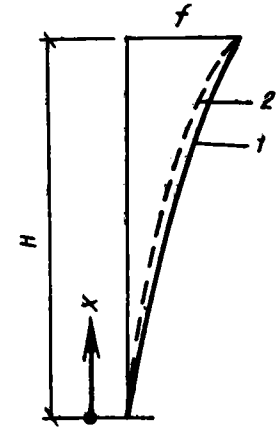


Fig. VI.41. Line of bending of a shear wall
1—with openings at $\lambda = 1$ to 9 and $v^2 = 1.1$; *2*—solid

walls having several rows of openings, this equality is taken as an assumption.

On the assumption that the point of contraflexure lies in the middle of the clear distance, the bending moment (Fig. VI.40) in the lintels at the side of the opening is

$$M_{lint} = Q_{lint}a_0/2 \tag{VI.76}$$

The force distribution diagrams for a shear wall with openings are shown in Fig. VI.38. In the M_{lint} diagram, the coordinate for a maximum moment is found by Eq. (VI.62) as for frame-shear wall structures. The bending moments in the individual vertical members are found by sharing the total moment M among them in proportion to their rigidities.

According to the equilibrium equation for the generalized shearing forces, the shearing force due to external load is balanced by the

derivative of the bending moment in the vertical members and the distributed moment in the lintels, $\overline{M}$, that is,

$$M' + \overline{M} = Q_0 \tag{VI.77}$$

where

$$\overline{M} = (\sum Q_{lint} a)/l = Q_{lint} b/l \tag{VI.78}$$

The shearing force in a vertical member of a shear wall is

$$Q_j = M' B_j/B + Q_{lint}\,(a_l + a_r)/l \tag{VI.79}$$

where a_l and a_r are the distances from the centre lines of the vertical members to the point of contraflexure in the lintel from the left and right.

The line of bending for an open-work shear wall is close in shape to the line of bending for a cantilever beam. Figure VI.41 illustrates the bending line for various open-work shear walls at $\nu^2 = 1.1$ and λ ranging between 1 and 9.

According to Eq. (VI.57), the sag of the top storey (tier) of an open-work shear wall may be taken as $f = f_1 + f_0$, that is, as the sum of two sags, namely

the sag due to the compliance of the lintels

$$f_1 = (pH^4/2\nu^2\lambda^2 B)\,[1 - 2\,(\varkappa - 1)/\lambda^2] \tag{VI.80}$$

and the sag due to the total bending of the shear wall

$$f_0 = pH^4\,(\nu^2 - 1)/8\nu^2 B = pH^4/8B_{sh\text{-}w} \tag{VI.81}$$

Here, the flexural rigidity of the shear wall taken at the section through the openings is

$$B_{sh\text{-}w} = \nu^2 B/(\nu^2 - 1) = B_0 + B$$

The deflections, δ_{ik}, of an open-work shear wall, induced by $P = 1$ applied at the level, x_k, are determined as for frame-shear wall structures, using Eq. (VI.63).

It should be noted that when $\lambda \geqslant 3$, we may assume for design purposes that

$$\cosh \lambda = \sinh \lambda \text{ and } \varkappa = \lambda$$

Let us relate the horizontal deflections of a shear wall to the shearing forces in the lintels. For this purpose, we write an equilibrium equation for the bending moments in the lintels and vertical members at the joints. Since the points of contraflexure for the struts (vertical members) lie in the middle of the storey (tier) height and those for the girders (lintels) in the midspan, we may write

$$2Q_0\,(l/2) = \sum 2Q_{lint}\,(a/2)$$

whence

$$Q_0 = Q_{lint}\,(\Sigma a/l) = Q_{lint} b/l \tag{VI.82}$$

Substituting this for Q_0 in Eq. (VI.24) gives

$$Q_{lint} = (l/b)\,Ay' + (lA/B_0)\int_0^x N\,dx \tag{VI.83}$$

Hence,

$$y' = bQ_{lint}/lA - (b/B_0)\int_0^x N\,dx \tag{VI.84}$$

On differentiating Eq. (VI.84) twice and recalling that $Q_{lint} = N'l$, we get

$$y''' = (b/l)\,(Q''_{lint}/A - Q_{lint}/B_0) \tag{VI.85}$$

5. Parameters λ and ν^2 from Design Experience

From experience with multi-storey skeleton-type building design, it is known that the rigidity characteristic, λ, for frame-shear wall structures usually ranges between 0.5 and 2. Also, analysis of various design models for skeleton-construction buildings reveals that when a building is 16 to 18 storeys high, the frame rigidity characteristic, λ_f, is below 0.7, and, as a consequence, the longitudinal strain in the struts has little effect on the forces and sags in a frame-shear wall structure. So when $\lambda_f < 0.7$, ν^2 is taken as unity in design formulas for forces and deflections.

The function of shear walls in a multi-storey skeleton-construction building is not only to unload the skeleton by taking up some of the load applied to the frames (usually from 10 to 25%), but mainly to change the shape of the shearing force diagram for the multi-storey frame struts. In these structures, the shearing forces, Q_f, reach their maximum at the top storey level and decrease downwards (see Fig. VI.33). In frame-type buildings, the bending moments in the struts and girders due to lateral loads increase downwards, and this calls for additional support reinforcement to be placed in the girders, and for increased cross-sectional area of the girders in the lower floors. In contrast, the bending moments in the skeleton members of a frame-shear wall structure decrease downwards which makes it possible to maintain the same cross-sectional area of the girders and amount of reinforcement at supports throughout the building height. Therefore, frame-shear wall structures are the best choice as regards the standardization of the structural members.

Experience shows that in structures using open-work shear walls the rigidity characteristic, λ, ranges between 3 and 9, depending

on the relative rigidity of the vertical members and lintels. When λ is from 12 to 15, the effect of the lintel compliance is negligibly small. When the rigidity characteristic is small (being from 1 to 2), the lintels are rather compliant and considerably reduce the lateral rigidity of the building. The longitudinal strain in the vertical members markedly affects the behaviour of open-work shear walls. Therefore, it should always be allowed for in design calculations. As often as not, the factor, ν^2, ranges between 1.1 and 1.3, although it may reach a greater value.

6. Tabular Design

The first thing to do in designing a multi-storey skeleton- and bearing-wall-type building is to determine the forces in the design sections. This is also necessary when choosing the most economical design among several alternatives. For this purpose, use may be made of the design formulas and tables compiled according to the basic equations (VI.57) through (VI.75).

The total bending moment in the solid shear walls of a frame-shear wall structure or the total bending moment at the fixation of the vertical members of an open-work shear wall structure is

$$M = -[(\nu^2-1)/2+\alpha_1](pH^2/\nu^2) \qquad \text{(VI.86)}$$

The total shearing force in the solid shear walls of a frame-shear wall structure or the total shearing force in the fixed vertical members of an open-work shear wall structure is $Q=pH$, or, in terms of the actual load,

$$Q = Q_{act} \qquad \text{(VI.87)}$$

The moment, M, and shearing force, Q, are shared by individual solid shear walls and the vertical members of an open-work shear wall in proportion to their flexural rigidities.

The maximum shearing force in frame struts is

$$Q_f = k_1 pH/\nu^2 \qquad \text{(VI.88)}$$

The maximum shearing force in the lintels of an open-work shear wall is

$$Q_{lint} = k_1 pHl/\nu^2 b \qquad \text{(VI.89)}$$

At the ground floor level, the longitudinal force in the outer struts of a multi-storey frame or that in the lintels of an open-work shear wall is

$$N = (M_0 - M)/b \quad \text{where} \quad M_0 = -0.5pH^2 \qquad \text{(VI.90)}$$

The sag at the top storey (tier) in a frame-shear wall structure or that in the vertical members of an open-work shear wall structure is

$$f=[(\nu^2-1)/8+\alpha_2](pH^4/B\nu^2) \qquad \text{(VI.91)}$$

The coefficients, α_1 and α_2, are given in Table VI.2, and the coefficient, k_1, in Table VI.3.

TABLE VI.2. Coefficients α_1 and α_2 for Determining Bending Moments and Sags in Shear Walls of Frame-Shear Wall Structures and Vertical Members of Open-Work Shear Wall Structures

λ	0	0.5	0.75	1	1.25	1.5	1.75	2	2.5	3	$\lambda \geqslant 4$
α_1	0.5	0.48	0.445	0.41	0.377	0.351	0.32	0.3	0.261	0.232	$\frac{\lambda-1}{\lambda^2}$
α_2	0.125	0.117	0.108	0.09	0.079	0.067	0.059	0.05	0.038	0.0298	$\frac{0.5-\alpha_1}{\lambda^2}$

TABLE VI.3. Coefficient k_1 and Coordinate ξ_0 for Determining Maximum Total Shearing Force in Frame Struts of Frame-Shear Wall Structures and Maximum Shearing Force in Lintels of Open-Work Shear Wall Structures

λ	0.5	0.6	0.75	1	1.25	1.5	1.75	2	2.5	3
k_1	0.037	0.052	0.075	0.115	0.153	0.187	0.218	0.247	0.297	0.34
ξ_0	0.93	0.9	0.85	0.77	0.7	0.63	0.58	0.54	0.47	0.38

λ	4	5	6	7	8	10	12	15	20	30
k_1	0.43	0.48	0.54	0.58	0.62	0.67	0.71	0.75	0.8	0.89
ξ_0	0.35	0.32	0.3	0.28	0.26	0.23	0.21	0.18	0.15	0.11

Example VI.1. *Given*: A 16-storey precast reinforced concrete building acting in the transverse direction as a frame-shear wall structure (Fig. VI.42); column

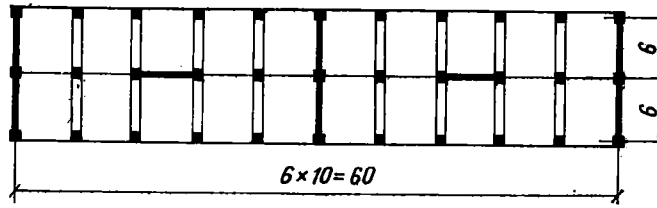


Fig. VI.42. To Example VI.1

grid size: 6 m square; storey height: $l = 3$ m; building height: $H_0 = 48$ m; transverse frame girders: 25 × 50 cm in cross section; columns: 45 cm square in cross section in all storeys; the building has three transverse and two longitudinal shear walls 14 cm thick; floors: large-size slabs; joints: welded embedded parts, in-situ concrete; concrete brand number: M-300.

To find: The sag and forces in the component members due to lateral loads.
Solution. The rigidity of the component prefab members is determined as for solid concrete sections, and given in Table VI.4.

TABLE VI.4. Design Data for Example VI.1

Rigidity	Struts	Girders
Flexural	$B_{st}=9.1\times10^4$ kN m^2	$B_g=7\times10^4$ kN m^2
Per metre run	$i_{st}=3.03\times10^4$ kN m	$i_g=1.17\times10^4$ kN m
Total per metre run	$s=73\times10^4$ kN m	$r=18.7\times10^4$ kN m
Total axial, outer struts	$E_cF=4.3\times10^7$ kN	—

The flexural rigidity of the three transverse shear walls is $B = 3 \times 87 \times 10^2 = 261 \times 10^7$ kN m^2.
According to Eq. (VI.22), the shear rigidity of the multi-storey frames is

$$A = 12/l\,(s^{-1}+r^{-1}) = (12\times10^4)/[3\,(73^{-1}+18.7^{-1})] = 60\times10^4 \text{ kN}$$

According to Eq. (VI.40), the flexural rigidity of the multi-storey frames is

$$B_0 = 0.5E_cFb^2 = 0.5\times4.3\times10^7\times12^2 = 310\times10^7 \text{ kN m}^2$$

According to Eq. (VI.32), the design height of the building is

$$H = H_0n/(n-0.5) = (48\times16)/(16-0.5) = \text{(approx.)}\ 48 \text{ m}$$

According to Eq. (VI.36), the rigidity characteristic of the multi-storey frames in the skeleton is

$$\lambda_f = H\sqrt{A/B_0} = 48\sqrt{(60\times10^4)/(310\times10^7)} = 0.67 < 0.7$$

So, the effect of the longitudinal strain in the struts may be neglected, and ν^2 may be taken as unity. According to Eq. (VI.31), the rigidity characteristic of the frame-shear wall structure is

$$\lambda = H\sqrt{\nu^2A/B} = 48\sqrt{(1\times60\times10^4)/(261\times10^7)} = 0.73$$

The sag of the building due to the basic load is $f \leqslant$ H/1 000. Equation (VI.91) and Table VI.2 give

$$f = \alpha_2pH^4/B = (0.109\times48^4p)/(261\times10^7) = 2.20\times10^{-4}\ p$$

where p is the load per metre of height, applied over the entire length of the building.
The forces in the building members are determined for the design load. According to Eq. (VI.86) and Table VI.2, the total bending moment in the shear walls at the point of fixation is

$$M = -\alpha_1pH^2 = 0.45\times48^2p = -1\,035\ p$$

the moment per shear wall being

$$M = -1\,035p/3 = -345\ p$$

The shearing force in the shear walls at the point of fixation is $Q = pH$ and that due to the actual load is $Q = Q_{act}$.

The total (maximum) shearing force in the frame struts is given by Eq. (VI.88) and Table VI.3

$$Q_f = k_1 pH = 0.072 \times 48\ p = 3.46\ p$$

the shearing force per frame being

$$Q_f = 3.46p/8 = 0.43\ p$$

According to Eq. (VI.90), the longitudinal force in the outer frame struts at the ground floor is

$$N = (M_0 - M)/b = -(0.5 - 0.45)\ 48^2 p/12 = -9.6\ p$$

the force per strut being

$$N = -9.6p/8 = -1.2\ p$$

According to Table VI.1, the reduction factor for the rigidity of the outer struts at $i_g/i_{st} = 1.17/3.03 = 0.39$ is $\beta = 0.55$.

The shearing force in an outer strut is

$$Q = Q_f \beta/(1 + 2\beta) = (0.55 \times 0.43p)/2.1 = 0.11\ p$$

and the shearing force in an intermediate strut is

$$Q = 0.21\ p$$

The bending moments in the frame struts are found using the shearing force Q, according to Eqs. (VI.10) and (VI.11).

Example VI.2. *Given*: A 16-storey precast reinforced concrete building using identical shear walls, acting in the transverse direction as a shear wall structure

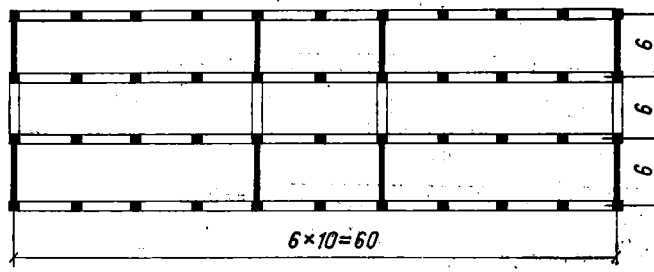


Fig. VI.43. To Example VI.2

(Fig. VI.43); column grid size: 6 m square; storey height: 4.2 m; building height: $H_0 = 67.2$ m; columns: 45 cm square in cross section in all storeys; the building has four transverse coupled shear walls and four longitudinal multi-bay frames; lintels: 30 × 120 cm in cross section; floors: large-size slabs; joints: welded embedded parts, in-situ concrete; conrete brand number: M-300.

To find: The sag and forces in the component members due to lateral loads.

Solution. The rigidity of the component r.c. members is determined as for solid concrete sections: $B_{lint} = 116 \times 10^4$ kN m². According to Eq. (VI.72),

$$k = 1 + 2.4\ (h/a_0)^2 = 1 + 2.4\ (1.3/5.555)^2 = 1.13$$

Taking into account the shear strain, the rigidity of the lintels is

$$B_{lint} = (116 \times 10^4)/1.13 = 103 \times 10^4 \text{ kN m}^2$$

The centre-to-centre distance between the vertical members of the coupled shear walls is $b = a = 12$ m. The clear span of the lintels is $a_0 = 5.55$ m, so $\gamma = a/a_0 = 2.16$. According to Eq. (VI.71), the rigidity per metre run of the lintel is

$$t_{lint} = B_{lint}\gamma^3/a = (103 \times 10^3 \times 2.16^3)/12 = 86 \times 10^4 \text{ kN m}$$

According to Eq. (VI.70), the shear rigidity of a shear wall having one row of openings is

$$A = 12r/l = (12 \times 86 \times 10^4)/4.2 = 247 \times 10^4 \text{ kN}$$

The flexural rigidity of the vertical members in the shear wall is

$$B_1 = B_2 = 15.8 \times 10^7 \text{ kN m}^2$$

The total flexural rigidity of the two vertical members is

$$B = 31.6 \times 10^7 \text{ kN m}^2$$

The axial rigidity of the vertical members is

$$E_cF = 3.5 \times 10^7 \text{ kN}$$

According to Eq. (VI.40),

$$B_0 = 0.5E_cFb^2 = 0.5 \times 3.5 \times 10^7 \times 12^2 = 252 \times 10^7 \text{ kN m}^2$$

The flexural rigidity of the shear wall at the section through the openings is

$$B_{sh\text{-}w} = B_0 + B = 283.6 \times 10^7 \text{ kN m}^2$$

According to Eq. (VI.29),

$$\nu^2 = 1 + B/B_0 = 1 + (31.6 \times 10^7)/(252 \times 10^7) = 1.125$$

At $n = 16$, the design height of the building is $H = H_0 = 67.2$ m. According to Eq. (VI.31), the rigidity characteristic of the shear wall is

$$\lambda = H\sqrt{\nu^2 A/B} = 67.2\sqrt{(1.125 \times 247 \times 10^4)/(31.6 \times 10^7)} = 6.3$$

The lateral load, p, per shear wall is equal to 1/4 of the total load applied to the longitudinal facade of the building, because the building has four identical shear walls. The sag of a shear wall is determined under the action of basic loads, and the forces in the component members are found by strength analysis for design loads.

According to Eq. (VI.91) and Table VI.2,

$$f = [(\nu^2 - 1)/8 + \alpha_2]\,(pH^4/B\nu^2) = [(1.125 - 1)/8 + 0.0092] \times [67.2^4 p/(31.6 \times 10^7 \times 1.125)] = 13.9 \times 10^{-4} p$$

The sag of a shear wall with nondeformable lintels is

$$f = pH^4/8B_{sh\text{-}w} = 67.2^4 p/(8 \times 283.6 \times 10^7) = 9 \times 10^{-4} p$$

Consequently, due to the compliance of the lintels the sag of the shear wall is 1.53 times the usual value.

According to Eq. (VI.86), the total bending moment in the vertical members at the point of fixation is

$$M = -[(\nu^2 - 1)/2 + \alpha_1]\,(pH^2/\nu^2) = -[(1.125 - 1)/2 + 0.134] + (67.2^2 p/1.125) = -790\ p$$

the bending moment per vertical member being $M = -395\ p$.

The shearing force in the vertical members at the point of fixation is $Q = pH$, or due to the actual load $Q = Q_{act}$.

The maximum shearing force in the lintels, given by Eq. (VI.89) and Table VI.3, is

$$Q_{lint} = k_1 pHl/bv^2 = (0.55 \times 67.2 \times 4.2p)/(12 \times 1.125) = 11.6\ p$$

The bending moment in the lintels is

$$M = Q_{lint} a_0/2 = [(11.6 \times 5.55)/2]\ p = 32.2p$$

According to Eq. (VI.90), the longitudinal force in the vertical members at the ground floor level is

$$N = (M_0 - M)/b = -(0.5 \times 67.2^2 p - 790p)/12 = -123\ p$$

7. Systems Using Mixed-Type Vertical Structural Elements

These systems include shear wall structures varying in types of shear walls used and in the number of opening rows provided, frame-shear wall structures with combined shear walls varying in the

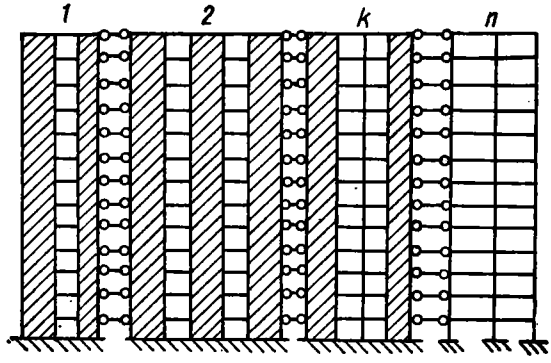


Fig. VI.44. To the design of shear wall structures using mixed-type vertical members

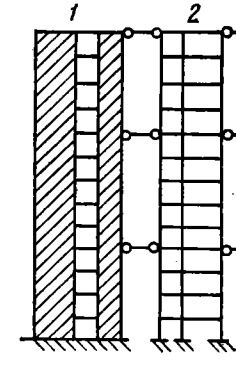


Fig. VI.45. To the design of structures with discrete constraints

longitudinal forces in the struts, frame-shear wall structures using open-work shear walls, and some others (Fig. VI.44). Such systems are treated on the basis of the general equation (VI.27). If a system contains m different types of vertical structural elements, we may write a set of m differential equations (VI.27) and two additional equations

$$\left.\begin{aligned} p_1(x) + p_2(x) + \ldots + p_m(x) &= p(x) \\ M_{01} + M_{02} + \ldots + M_{0m} &= M_0 \end{aligned}\right\} \qquad \text{(VI.92)}$$

Design experience shows that with a large number of different types of vertical structural elements, the best approach is to use

a design model involving a limited number of discrete restraints imposed on the vertical elements (Fig. VI.45) and to solve a set of algebraic equations. An adequate accuracy is achieved with three or four restraints distributed over the building height. In this case, the unit deflections of the vertical structures, δ_{ik}, are found by Eq. (VI.63).

If a system consists of only two different types of vertical structural elements, the forces and deflections may be determined using the standard design formulas. In this case, the set of two differential and two additional equations (VI.92) reduces to a sixth-order even-derivative differential equation in terms of deflections. Recalling that the total bending moment in the vertical members of an open-work shear wall is $M = -By''$, we may derive a fourth-order differential equation in terms of moments, which, for the load, $p(x) = p$ has the form

$$M^{IV} - 2a^2M'' + b^4M - cM_0 - ep = 0 \tag{VI.93}$$

where

$$2a^2 = (\nu_I^2 A_1 + \nu_{II}^2 A_2)/B \tag{VI.94}$$

$$b^4 = A_1A_2(B + B_{01} + B_{02})/BB_{01}B_{02} \tag{VI.95}$$

$$c = A_1A_2/B_{01}B_{02};\ e = A_1/B_{01} + A_2/B_{02} \tag{VI.96}$$

$$\nu_I^2 = 1 + B/B_{01};\ \nu_{II}^2 = 1 + B/B_{02} \tag{VI.97}$$

B is the total flexural rigidity of the vertical members of an open-work shear wall and solid shear walls; A_1, A_2, B_{01} and B_{02} are the rigidities of the first and second shear walls, subject to the following condition

$$A_1/B_{01} - A_2/B_{02} \neq 0 \tag{VI.98}$$

In multi-storey buildings, b^2 is usually less than a^2. Therefore, the solution for Eq. (VI.93) is

$$M = pH^2 [C_1 \cosh \alpha_1 x + C_2 \sinh \alpha_1 x + C_3 \cosh \alpha_2 x + C_4 \cosh \alpha_2 x - 0.5(1 - \xi)^2 t_1 + t_2] \tag{VI.99}$$

where

$$t_1 = c/b^4;\ t_2 = (e - 2a^2t_1)/H^2b^4 \tag{VI.100}$$

$$\alpha_{1,2} = \sqrt{a^2 \mp \sqrt{a^4 - b^4}} \tag{VI.101}$$

The boundary conditions are

(1) $M'(0) = pH$; (2) $M''(0) - [(A_1 + A_2)/B]\, M'(0) = 0$

(3) $M(H) = 0$ and (4) $M''(H) = -p$

The first boundary condition arises from the fact that the angular deflection of shear walls at the point of fixation is $y'(0) = 0$, whence, according to Eq. (VI.25) and over the zero integration interval for N, we have $By''(0) = Q_0(0)$ or $M'(0) = pH$.

The second boundary condition is derived by differentiating twice the equilibrium equation for the generalized shearing forces in the system

$$M''' + \overline{M}_1'' + \overline{M}_2'' = 0$$

Recalling Eq. (VI.83) and $Q_{lint1}(0) = Q_{lint2}(0) = 0$, we obtain

$$M'''(0) + (A_1 + A_2)\, y''(0) = 0$$

whence

$$M'''(0) - [(A_1 + A_2)/B]\, M'(0) = 0$$

The third and fourth boundary conditions are self-explanatory.

The integration constants found by solving the linear equations, accurate to pH^2, are

$$C_1 = -[\mu_2 \sinh \lambda_1 - \lambda_1 (t_1 + t_2\lambda_2^2 - 1)]/\mu\lambda_1 \cosh \lambda_1$$

$$C_2 = \mu_2/\mu\lambda_1 \qquad \text{(VI.102)}$$

$$C_3 = [\mu_1 \sinh \lambda_2 - \lambda_2 (t_1 + t_2 \lambda_1^2 - 1)]/\mu\lambda_2 \cosh \lambda_2$$

$$C_4 = -\mu_1/\mu\lambda_2 \qquad \text{(VI.103)}$$

where

$$\mu = \lambda_1^2 - \lambda_2^2; \quad \lambda_1 = \alpha_1 H; \quad \lambda_2 = \alpha_2 H \qquad \text{(VI.104)}$$

$$\mu_1 = \lambda_0^2 - \lambda_1^2 (1 - t_1); \quad \mu_2 = \lambda_0^2 - \lambda_2^2 (1 - t_1) \qquad \text{(VI.105)}$$

$$\lambda_0^2 = H^2 (A_1 + A_2)/B \qquad \text{(VI.106)}$$

Integrating twice the bending moment equation (VI.99) and finding the integration constants from the conditions $y(0) = y'(0) = 0$ gives an equation for the deflections of the system. Then,

$$y = (pH^4/B)\, [(1 - \cosh \alpha_1 x)\, (C_1/\lambda_1^2) - (\alpha_1 x - \sinh \alpha_1 x)$$
$$\times (C_2/\lambda_1^2) + (1 - \cosh \alpha_2 x)\, (C_3/\lambda_2^2) + (\alpha_2 x - \sinh \alpha_2 x)$$
$$\times (C_4/\lambda_2^2) + (t_1/2)\, (\xi^2/2 - \xi^3/3 + \xi^4/12) - t_2 \xi^2/2] \qquad \text{(VI.107)}$$

The shearing forces in the shear wall lintels can more conveniently be found by another differential equation. The equilibrium equation for the generalized shearing forces in the horizontal cross section is

$$M' + \overline{M}_1 + \overline{M}_2 = Q_0 \qquad \text{(VI.108)}$$

Here, the distributed moments in the lintels, $\overline{M}_1$ and $\overline{M}_2$, are given by Eq. (VI.78). In a frame-shear wall structure with open-work

shear walls, $\overline{M}_2 = Q_f$, and in a frame-shear wall structure using combined shear walls, $\overline{M}_1 = Q_{st}$ and $\overline{M}_2 = Q_g$.

Recalling that $M' = -By'''$ and that, according to Eq. (VI.85), the value of y''' holds for each open-work shear wall (because their deflections, y, are the same), we get the following set of two differential equations

$$\left.\begin{aligned}(B/A_1)\,\overline{M}_1'' - \nu_{\mathrm{I}}^2\overline{M}_1 + \overline{M}_2 + Q_0 = 0\\ (B/A_2)\,\overline{M}_2'' - \nu_{\mathrm{II}}^2\,\overline{M}_2 - \overline{M}_1 + Q_0 = 0\end{aligned}\right\} \qquad \text{(VI.109)}$$

For each shear wall, the set of equations (VI.109) gives a fourth-order differential equation. For the first shear wall

$$\overline{M}_1^{\mathrm{IV}} + 2a^2\overline{M}_1'' + b^4\overline{M}_1 - c_1Q_0 = 0 \qquad \text{(VI.110)}$$

Here, $2a^2$ and b^4 are the same as in Eqs. (VI.94) and (VI.95);

$$c_1 = (\nu_{\mathrm{II}}^2 - 1)\,A_1A_2/B \qquad \text{(VI.111)}$$

The solution of the above equation is

$$\overline{M}_1 = C_1\cosh\alpha_1x + C_2\sinh\alpha_1x + C_3\cosh\alpha_2x + C_4\sinh\alpha_2x + t_3Q_0 \qquad \text{(VI.112)}$$

where

$$t_3 = (\nu_{\mathrm{II}}^2 - 1)/(\nu_{\mathrm{I}}^2\nu_{\mathrm{II}}^2 - 1) \qquad \text{(VI.113)}$$

The boundary conditions are: (1) $\overline{M}_1(0) = 0$; (2) $\overline{M}_1''(0) + (1/B)\,A_1Q_0(0) = 0$; (3) $\overline{M}_1'(H) = 0$; and (4) $\overline{M}_1'''(H) - (1/B)A_1p = 0$.

The integration constants found accurate to pH are

$$C_1 = -(\lambda_{01}^2 - t_3\lambda_2^2)/(\lambda_1^2 - \lambda_2^2); \quad C_2 = (\lambda_{01}^2 - t_3\lambda_2^2) \times (1 + \lambda_1\sinh\lambda_1)/\lambda_1(\lambda_1^2 - \lambda_2^2)\cosh\lambda_1 \qquad \text{(VI.114)}$$

$$C_3 = (\lambda_{01}^2 - t_3^2\lambda_2^2)/(\lambda_1^2 - \lambda_2^2)$$

$$C_4 = -(\lambda_{01}^2 - t_3\lambda_1^2)(1 + \lambda_2\sinh\lambda_2)/\lambda_2(\lambda_1^2 - \lambda_2^2)\cosh\lambda_2 \qquad \text{(VI.115)}$$

where

$$\lambda_{01}^2 = H^2A_1/B \quad \text{and} \quad \lambda_{02}^2 = H^2A_2/B \qquad \text{(VI.116)}$$

When determining the integration constants for the second shear wall, λ_{01} in Eqs. (VI.114) and (VI.115) should be replaced by λ_{02}.

Once the values of $\overline{M}$ are known, they are used to find the bending moments in the lintels of each shear wall. According to Eqs. (VI.76) and (VI.78),

$$M_{lint} = 0.5Q_{lint}a_0 = \overline{M}la_0/2b \qquad \text{(VI.117)}$$

The longitudinal forces in the outer vertical members of a shear wall are found by integrating Eq. (VI.112). According to Eqs. (VI.73) and (VI.78), for the outer vertical members of each shear wall

$$N = -0.5 \int_x^H \overline{M}\, dx = -(pH^2/b)\,[(\sinh \lambda_1 - \sinh \alpha_1 x)\,(C_1/\lambda_1) + (\cosh \lambda_1 - \cosh \alpha_1 x)\,(C_2/\lambda_1) + (\sinh \lambda_2 - \sinh \alpha_2 x)\,(C_3/\lambda_2) + (\cosh \lambda_2 - \cosh \alpha_2 x)\,(C_4/\lambda_2) + (1-\xi)^2\,(t_3/2)] \qquad \text{(VI.118)}$$

Example VI.3. *Given*: A 16-storey building acting in the transverse direction as a shear wall structure with different types of shear walls and having multi-

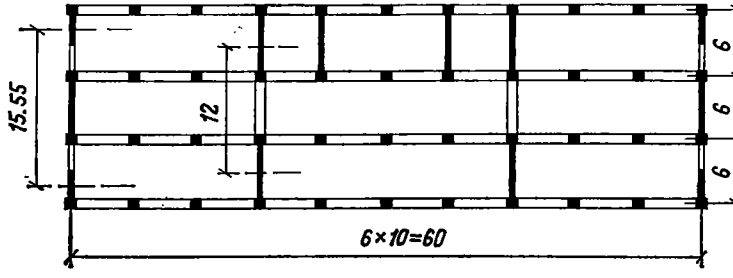


Fig. VI.46. To the design of a shear wall structure using mixed-type open-work shear walls

bay frames in the longitudinal direction (Fig. VI.46); column grid size: 6 m square; storey height: 4.2 m; building height: $H_0 = 67.2$ m; shear walls: solid and with one and two rows of openings; centre-to-centre distance between the outer vertical members: $b = 2a = 15.55$ m for the first shear wall, and $b = a = 12$ m, for the second.

To find: The sag and forces in the component members due to lateral loads.

Solution. The rigidities of the component members and the design parameters are given in Tables VI.5 and VI.6.

TABLE VI.5. Shear Wall Rigidities

Rigidity	Open-work shear wall		Solid shear wall
	first	second	
Vertical members, total flexural	$B_1 = 40.16 \times \times 10^7$ kN m^2	$B_2 = 63.68 \times \times 10^7$ kN m^2	—
Flexural	$B_{01} = 472 \times \times 10^7$ kN m^2	$B_{02} = 500 \times \times 10^7$ kN m^2	$B_{sh-w} = 31.84 \times \times 10^7$ kN m^2
Shear	$A_1 = 71.5 \times \times 10^4$ kN	$A_2 = 562 \times \times 10^4$ kN	—

TABLE VI.6. Design Parameters

Parameter	Equation
$\nu_I^2 = 1.29$; $\nu_{II}^2 = 1.29$	(VI.97)
$a^2 = 2.98 \times 10^{-3}$; $b^4 = 1.40 \times 10^{-6}$	(VI.94), (VI.95)
$t_1 = 122.30 \times 10^{-3}$; $t_2 = 85.5 \times 10^{-3}$	(VI.100)
$\alpha_1 = 15.5 \times 10^{-3}$; $\alpha_2 = 76 \times 10^{-3}$	(VI.101)
$\lambda_1 = \alpha_1 H = 1.041$; $\lambda_2 = \alpha_2 H = 5.110$; $\lambda_0^2 = 21.190$	(VI.104), (VI.106)
For design of sags and bending moments in vertical members:	
$C_1 = -86.153 \times 10^{-3}$; $C_2 = 67.004 \times 10^{-3}$	(VI.102)
$C_3 = -157.850 \times 10^{-3}$; $C_4 = 158.244 \times 10^{-3}$	(VI.103)
For design of lintel moments and longitudinal forces in vertical members of first open-work shear wall:	
$t_3 = 435 \times 10^{-3}$; $\lambda_{01}^2 = 2.39$	(VI.113), (VI.116)
$C_1 = -344.524 \times 10^{-3}$; $C_2 = 477.053 \times 10^{-3}$	(VI.114)
$C_3 = -77.138 \times 10^{-3}$; $C_4 = 77.014 \times 10^{-3}$	(VI.115)

The total flexural rigidity is $B = B_1 + B_2 + B_{sh-w} = 135.68 \times 10^7$ kN m^2. This satisfies the condition (VI.98) according to which $A_1/B_{01} \neq A_2/B_{02}$, that is, $1.15 \times 10^{-4} \neq 11.3 \times 10^{-4}$.

At $\xi = 1$, the sag of the top storey of the building is given by Eq. (VI.107)

$$f = 29.45 \times 10^{-3} \, (pH^4/B)$$

For the design of the moments in the lintels and the longitudinal forces in the vertical members of the second open-work shear wall, t_3, λ_{02}^2 and $C_{1,\,2,\,3,\,4}$ are calculated in a similar manner.

It should be noted that with hinged connections between the lintels and the vertical members of a shear wall, the sag is

$$f = 125 \times 10^{-3} \, (pH^4/B)$$

which is four times the original value.

The total bending moment in the vertical members of the open-work shear walls and in the solid shear walls is determined at the point of fixation by Eq. (VI.99): $M = -0.22 \, pH^2$. This moment is shared by individual members according to their flexural rigidities.

The distributed bending moments in the lintels are given by Eq. (VI.112): at $\xi = 0.4$, the maximum bending moment in the lintels of the first shear wall is $\overline{M} = 0.072 \, pH$, and, at $\xi_0 = 0.3$, the maximum bending moment in the lintels of the second shear wall is $\overline{M} = 0.300 \, pH$. The bending moments in the lintels at the side of the openings are given by Eqs. (VI.76) and (VI.78).

The longitudinal forces in the outer vertical members of a shear wall are found at $x = 0$ by Eq. (VI.118):

for the first open-work shear wall

$$N = -0.23 \, pH^2/b$$

for the second open-work shear wall

$$N = -0.05 \, pH^2/a$$

8. Effect of Soil Compliance

The nonuniform pressure exerted on the soil by a footing brings about a tilt of the footing and the shear wall it carries. The tilt entails additional deflections of the multi-storey building. In shear wall-frame structures, the compliant soil is responsible for an increased share of load taken by the frames, especially at the top

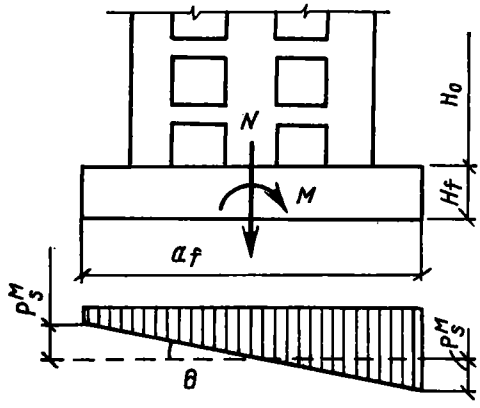


Fig. VI.47. Soil compliance

storey levels. Also, additional deflections are experienced by the frame skeleton, which fact, however, is of minor importance for frame-shear wall structures.

To raise the spatial rigidity of multi-storey buildings and eliminate additional deflections brought about by the compliant soil, it is necessary to compact the foundation material by piles or use mat foundations. The compliance of a pile footing, especially with end-bearing piles, is insignificant.

Under the action of M, the settlement at the edge of a shear wall footing (Fig. VI.47) is

$$u = \pm p_s/C_\varphi = \pm Ma_f/2C_\varphi I_f \qquad \text{(VI.119)}$$

Hence, the tilt of the footing (neglecting the footing strain)

$$\theta = 2u/a_f = M/B_f \qquad \text{(VI.120)}$$

where $B_f = C_\varphi I_f$ is the angular rigidity of the footing, C_φ is the modulus of subgrade reaction for a nonuniformly compressed foundation material, I_f is the moment of inertia of the footing bottom, and a_f is the footing side in the plane of flexure.

In shear wall structures with a single footing carrying all shear walls, the additional deflection brought about by the tilt of the

footing is

$$f_f = \theta H \tag{VI.121}$$

In solving Eq. (VI.30) for frame-shear wall structures, the boundary conditions should be adjusted so as to allow for soil compliance:

(1) $w(0) = 0$; (2) $w'(0) = -BM(0)/B_f = Bw''(0)/B_f$

the angular deflection of the shear wall at the footing is proportional to the bending moment*;

(3) $w'(0)/\nu^2 s_2^2 - w'''(0) = Q_0(0)$ and (4) $w''(\lambda) = 0$

The integration constants are determined by solving the set of equations:

$$C_1 = -C_3 = -ps_2^4 \varkappa \alpha_0/\nu^2; \quad C_2 = ps_2^3 \lambda/\nu^2 \tag{VI.122}$$

$$C_4 = -ps_2^4 [\alpha_0 1 + \lambda \sinh \lambda) - 1]/\nu^2 \sinh \lambda$$

where

$$\alpha_0 = \{\nu^2 + k_f \beta_1 [1 - (\nu^2 - 1)\lambda^2/2\nu^2]\}/(\nu^2 + k_f \beta_2) \tag{VI.123}$$

is the characteristic of soil compliance;

$$k_f = B/HB_f \tag{VI.124}$$

is the coefficient of soil compliance:

$$\beta_1 = \lambda \sinh \lambda/(1 + \lambda \sinh \lambda); \quad \beta_2 = \lambda \sinh \lambda/\cosh \lambda \tag{VI.125}$$

For a perfectly rigid foundation material, $\alpha_0 = 1$ and $k_f = 0$.

On substituting the integration constants into Eq. (VI.30), the equation for deflections of the frame-shear wall structure with allowance for soil compliance takes the form

$$w = (ps_2^4/\nu^2)\{\lambda\varphi - \varphi^2/2 + \alpha_0 \varkappa \cosh \varphi - [\alpha_0 (1 + \lambda \sinh \lambda) - 1] \sinh \varphi/\sinh \lambda - \alpha_0 \varkappa + 0.5 (\nu^2 - 1) \lambda^4 (\xi^2/2 - \xi^3/3 + \xi^4/12)\} \tag{VI.126}$$

Let us find the forces in the component members with allowance for soil compliance. The bending moment in the shear wall is

$$M = -w'' = -(pH^2/\nu^2)[0.5 (1 - \xi)^2 (\nu^2 - 1) - (1 - \alpha_0 \varkappa \cosh \varphi)/\lambda^2 - (\alpha_0 + \alpha_0 \lambda \sinh \lambda - 1) \sinh \varphi/\lambda^2 \sinh \lambda] \tag{VI.127}$$

The shearing force in the shear wall is

$$Q = M' = (pH/\nu^2)\{(1 - \xi)(\nu^2 - 1) + [\alpha_0 \times (1 + \lambda \sinh \lambda) - 1] \cosh \varphi/\lambda^2 \sinh \lambda - \alpha_0 \varkappa \sinh \varphi/\lambda\} \tag{VI.128}$$

* The term $H_f Q_0(0)$ in the expression for the bending moment is negligibly small and may be discarded.

The shearing force in the frame struts is

$$Q_f = Q_0 - Q = (pH/\nu^2)\{1 - \xi + (\alpha_0\varkappa/\lambda)\sinh\varphi - [\alpha_0(1 + \lambda\sinh\lambda) - 1]\cosh\varphi/\lambda\sinh\lambda \quad \text{(VI.129)}$$

Soil compliance reduces the characteristic, α_0, and the bending moments at the bottom of the shear wall (Fig. VI.48).

Example VI.4. *Given*: A 16-storey building acting in the transverse direction as a frame-shear wall structure; building particulars as in Example VI.1: transverse shear wall footings: 2×16 m; modulus of subgrade reaction for nonuniformly compressed soil: $C_\varphi = 0.5$ MPa cm^{-1} or 5×10^4 kN m^{-3}.

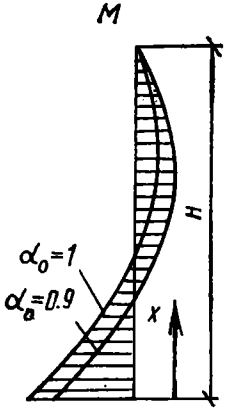


Fig. VI.48. Bending moment diagram for a shear wall in a frame-shear wall structure as a function of soil compliance characteristic

To find: The sag and forces in the component members due to lateral loads with allowance for soil compliance.

Solution. The angular rigidity of the footing carrying the three shear walls is

$$B_f = C_f I_f = 3 \times 5 \times 10^4 \times 2 \times 16^3/12 = 10.2 \times 10^7 \text{ kN m}$$

According to Eq. (VI.124), the modulus of subgrade reaction is

$$k_f = B/HB_f = (261 \times 10^7)/(48 \times 10.2 \times 10^7) = 0.53$$

At $\lambda = 0.73$, $\cosh\lambda = 1.28$ and $\sinh\lambda = 0.73$, Eq. (VI.125) gives $\beta_1 = 0.38$ and $\beta_2 = 0.48$, and Eq. (VI.55) gives $\varkappa = 1.24$. According to Eq. (VI.123), the characteristic of soil compliance at $\nu^2 = 1$ is

$$\alpha_0 = (1 + k_f\beta_1)/(1 + k_f\beta_2) = (1 + 0.53 \times 0.38)/(1 + 0.53 \times 0.48) = 0.95$$

According to Eq. (VI.126), the sag of the building at $\varphi = \lambda$ is

$$f = (pH^4/2\lambda^2 B)[1 - 2(\alpha_0\varkappa - 1)/\lambda^2] = 6.05 \times 10^{-4} p$$

So, due to soil compliance, the sag of the building is 2.75 times the sag, $f = 2.20 \times 10^{-4} p$.

According to Eq. (VI.127), at $\varphi = \xi = 0$ and $\nu^2 = 1$, the bending moment in the shear wall at the point of fixation is

$$M = -pH^2(\alpha_0\varkappa - 1)/\lambda^2 = -783\ p$$

For one shear wall, the bending moment is $M = -261\ p$, which is by 24% less than the bending moment for noncompliant foundation material $M = -345p$.

The shearing forces in the frame struts are determined in several sections disposed over the height, using Eq. (VI.128).

Example VI.5. *Given*: A 16-storey building acting in the transverse direction as a shear wall structure; building particulars as in Example VI.2; single footing carrying both vertical members of shear wall: 2.8×20 m; modulus of subgrade reaction for nonuniformly compressed soil: $C_\varphi = 0.5$ MPa cm^{-1} or 5×10^{-4} kN/m^3.

To find: The sag of the building due to lateral loads with allowance for soil compliance.

Solution. The angular rigidity of the shear wall footing is

$$B_f = C_\varphi I_f = 5 \times 10^4 \times 2.8 \times 20^3/12 = 10.5 \times 10^7 \text{ kN m}$$

The bending moment at the point of the shear wall fixation is

$$M = pH^2/2 = 67.2^2 p/2 = 2\,250\ p$$

(the "minus" sign is dropped).

According to Eq. (VI.120), the tilt of the footing is

$$\theta = M/B_f = 2\,250/(10.5 \times 10^7) = 2.15 \times 10^{-5}\ p$$

According to Eq. (VI.121), the additional sag of the shear wall due to the footing tilt is

$$f_f = \theta H = 67.2 \times 2.15 \times 10^{-5} p = 14.5 \times 10^{-4}\ p$$

The total sag is

$$f_{tot} = 13.9 \times 10^{-4} p + 14.5 \times 10^{-4} p = 28.4 \times 10^{-4}\ p$$

So, due to the soil compliance, the sag of the building is 2.05 times the original value.

9. Effect of Floor Sag under In-Plane Bending

In shear wall structures subjected to lateral loads, the floor sag under in-plane bending may be comparable with the sag of the shear walls if the latter are spaced relatively large distances apart (Fig. VI.49). In this case, the outer shear walls are unloaded and the intermediate shear walls have to carry an additional load, so the structure can no longer be analysed using a plane design model. It should be noted that the load distribution among open-work shear walls in proportion to M_{0j}, or among solid shear walls in proportion to B_j, takes place only when the floors are perfectly rigid.

Let the deflections of the outer shear walls be y, and those of the middle shear walls, $y + f$. Determine the sag of the floors, f, in a building of length L and width h due to a uniformly distributed lateral load $p(x)\, l/L$ and concentrated forces (the outer and middle shear wall reactions) as for a single-span beam. For three shear walls, the reactions of the outer solid shear walls are $0.5p_1(x)\, l$, and that of the middle open-work shear wall, $p_2(x)\, l$. Here, $p_1(x) + p_2(x) = p(x)$. The sag of the floors is

$$f = (pL^3/B_{fl})\,[5p(x)/384 - p_2(x)/48] = [p_1(x) - \alpha p(x)]/C \qquad \text{(VI.130)}$$

where

$$C = k_{fl} B_{fl}/lL^3 \qquad \text{(VI.131)}$$

is the floor thrust; B_{fl} is the floor rigidity under in-plane bending, which, with allowance for shear strain, is determined by dividing by $k = 1 + 2.4\,(h/L)^2$; $\alpha = 0.38$ and $k_{fl} = 48$ are numerical coefficients. For four shear walls, $\alpha = 0.27$ and $k_{fl} = 64.8$.

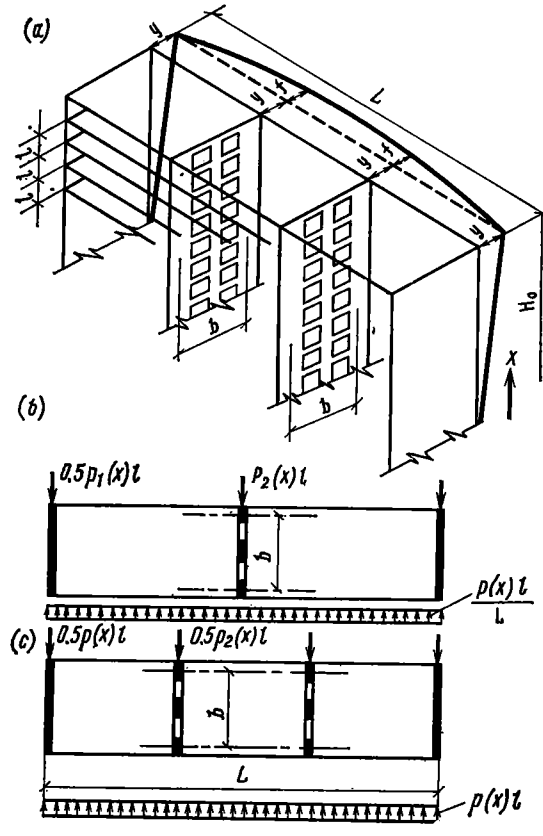


Fig. VI.49. To the space design of a multi-storey building with allowance for the floor sag under in-plane bending

In Eq. (VI.130), the reaction, $p_1(x)$, depends on the ratio of the elastic and geometrical characteristics of the floors and shear walls. Since $p_1(x) = B_1 y^{IV}$, Eq. (VI.130) gives

$$f = [B_1 y^{IV} - \alpha p(x)]/C \tag{VI.132}$$

where B_1 is the total flexural rigidity of the outer solid shear walls. Hence,

$$y^{IV} = [\alpha p(x) + Cf]/B_1 \tag{VI.133}$$

The equilibrium equation for the generalized shearing forces in the horizontal section of a building with solid outer and open-work middle shear walls has the form

$$-B_1 y''' - B_2 (y''' + f''') + \overline{M} = Q_0 \tag{VI.134}$$

where B_2 is the total flexural rigidity of the vertical members of the middle shear walls.

Re-arranging Eq. (VI.134) subject to Eq. (VI.83) and differentiating with respect to x gives

$$By^{IV} + B_2 f^{IV} - A\,(y'' + f'') - ANb/B_0 - p\,(x) = 0 \qquad \text{(VI.135)}$$

where $B = B_1 + B_2$.

The equation for the moments in the horizontal section yields

$$Nb = M_0 - M = M_0 + By'' + B_2 f'' \qquad \text{(VI.136)}$$

Substituting for Nb and f from Eqs. (VI.136) and (VI.132) for linearly varying distributed load $p\,(x)$ into Eq. (VI.135) gives a generalized equation for a space system in bending

$$(B_1 B_2/C)\,y^{VIII} - (B_1 A \nu_{st}^2/C)\,y^{VI} + By^{IV} - A\nu^2 y'' - AM_0/B_0 - p\,(x) = 0 \qquad \text{(VI.137)}$$

where

$$\nu_{st}^2 = 1 + B/B_0 \qquad \text{(VI.138)}$$

It should be mentioned that in a specific case where the floors are perfectly rigid and $C \to \infty$ the generalized equation (VI.137) can be re-arranged to yield Eq. (VI.26) for the plane design model.

Recalling that $By'' = -M$, Eq. (VI.137) may also be re-written as a sixth-order equation for moments.

For solid outer and middle shear walls, Eq. (VI.135) takes the form

$$By^{IV} + B_2 f^{IV} - p\,(x) = 0 \qquad \text{(VI.139)}$$

Substituting for y^{IV} from Eq. (VI.133) into Eq. (VI.139) yields an equation describing the floor sag, f, as a function of the coordinate, x,

$$B_2 f^{IV} + BCf/B_1 - (1 - \alpha B/B_1)\,p\,(x) = 0 \qquad \text{(VI.140)}$$

Equation (VI.140) is an equation for a beam on an elastic foundation with a modulus of subgrade reaction, C (see Ch. III); it can be reduced to the following form

$$(s_1^4/4)\,f^{IV} + f - (\mu/C)p\,(x) = 0 \qquad \text{(VI.141)}$$

where

$$s_1 = \sqrt[4]{4B_1 B_2/BC} \qquad \text{(VI.142)}$$

is a linear characteristic, and

$$\mu = B_1/B_2 - \alpha \qquad \text{(VI.143)}$$

If the beam on the elastic foundation is long ($\lambda = H/s_1 \geqslant 3$), the solution for Eq. (VI.141) is

$$f = C_1\eta_1 + C_2\eta_2 + (\mu/C)\, p\,(x) \qquad \text{(VI.144)}$$

where $\eta_1 = \exp(-\varphi)\cos\varphi$, $\eta_2 = \exp(-\varphi)\sin\varphi$ and $\varphi = x/s_1$. The boundary conditions are: (1) $f(0) = 0$ and (2) $f'(0) = 0$. The integration constants are

$$C_1 = -\mu p\,(0)/C; \quad C_2 = -\mu\,[p\,(0) + s_1 p'\,(0)]/C \qquad \text{(VI.145)}$$

Recalling Eqs. (VI.145), the solution (VI.144) can be re-written as

$$f = \mu\,[p\,(x) - \eta_3 p\,(0) - \eta_2 s_1 p'\,(0)]/C \qquad \text{(VI.146)}$$

where $\eta_3 = \eta_1 + \eta_2$.

Under a uniformly distributed load, the floor sag is

$$f = \mu p\,(1 - \eta_3)/C \qquad \text{(VI.147)}$$

Recalling Eqs. (VI.143) and (VI.147), the loads transferred to the outer and middle shear walls can be found from Eq. (VI.130):

$$p_1 = (B_1/B - \eta_3\mu)\, p \quad \text{and} \quad p_2 = (B_2/B + \eta_3\mu)\, p \qquad \text{(VI.148)}$$

If the nondimensional coordinate $\varphi > 1.5$, then $\eta_3 \to 0$ and the load is shared by the shear walls only in proportion to their rigidities, that is, as in the plane design model. Consequently, as the number of storeys and φ increase, the effect of the in-plane bending of the floors decreases, as do the deflections of a long beam on an elastic foundation. The in-plane sag of the floors becomes negligible in comparison with the sag of the vertical structural elements.

For $\lambda = 3$, the limiting number of storeys at which the behaviour of a building can be described by using a plane design model is

$$n_{lim} = H/l = \lambda s_1/l = (3/l)\sqrt{4B_1B_2/BC} \qquad \text{(VI.149)}$$

In regular buildings of a shear wall structure with three or four shear walls, n_{lim} often ranges between 12 and 14, depending on the relative rigidity of the shear walls and floors.

The effect of compliant joints between prefab floor members under in-plane bending may be evaluated on the basis of the curvature of the deflected axis

$$1/\rho = M\,(1/B + 1/aC_{fl}) \qquad \text{(VI.150)}$$

where $B = 0.85E_cI_c$, a is the distance between the joints of the floor slabs in the direction of L, and C_{fl} is the rigidity coefficient of the joints between prefab floor members (determined experimentally).

Hence, the rigidity of a precast floor under in-plane bending is

$$B_{fl} = B/(1 + B/aB_{fl}) \tag{VI.151}$$

Experiments have shown that for design purposes B_{fl} may be taken as $0.7\,B$.

10. Dynamic Characteristics

Frame-Shear Wall Structures. Under conditions of free lateral vibrations of a building, the load is the inertial forces of the mass. When the effect of the longitudinal strain in the struts is low and $\nu^2 = 1$ (that is, with $\lambda_f < 0.7$), Eq. (VI.26) may be re-arranged to yield an equation for the free vibrations of a frame-shear wall structure

$$B\partial^4 y/\partial x^4 - A\partial^2 y/\partial x^2 + (m/l)\,\partial^2 y/\partial t^2 = 0 \tag{VI.152}$$

On substituting $y = XT$, the above equation reduces to an ordinary differential equation of the form

$$X^{IV} - 2a^2 X'' - b^4 X = 0 \tag{VI.153}$$

where

$$2a^2 = A/B \quad \text{and} \quad b^4 = \omega^2 m/Bl \tag{VI.154}$$

The characteristic equation

$$r^4 - 2a^2 r^2 - b^4 = 0 \tag{VI.155}$$

has the following roots: $r_1 = -r_2 = \alpha_1$ and $r_3 = -r_4 = i\alpha_2$, where

$$\alpha_{1,2} = \sqrt{\sqrt{a^4 + b^4} \pm a^2}$$

The solution of Eq. (VI.153) is an equation for free vibration modes

$$X = C_1 \cosh \alpha_1 x + C_2 \sinh \alpha_1 x + C_3 \cos \alpha_2 x + C_4 \sin \alpha_2 x \tag{VI.156}$$

The boundary conditions are: (1) $X\,(0) = 0$; (2) $X'\,(0) = BX''lB_f$; (3) $BX''\,(H) = 0$ and (4) $AX'\,(H) - BX'''\,(H) = 0$.

This gives a set of four homogeneous equations:

(1) $C_1 + C_3 = 0$; (2) $C_2\lambda_1 + C_4\lambda_2 - C_1\lambda_1^2 k_f + C_3\lambda_2^2 k_f = 0$

(3) $C_1\lambda_1^2 \cosh \lambda_1 + C_2\lambda_1^2 \sinh \lambda_1 - C_3\lambda_2^2 \cos \lambda_2 - C_4\lambda_2^2 \sin \lambda_2 = 0$

(4) $C_1\lambda_2 \sinh \lambda_1 + C_2\lambda_2 \cosh \lambda_1 + C_3\lambda_1 \sin \lambda_2 - C_4\lambda_1 \cos \lambda_2 = 0$

where $\lambda_1 = \alpha_1 H$ and $\lambda_2 = \alpha_2 H$.

Expanding the determinant of the set of equations and equating it to zero gives a frequency equation in two unknowns, λ_1 and λ_2,

$$D(\omega) = k_f(\lambda_1^2 + \lambda_2^2)(\lambda_1^3 \sinh \lambda_1 \cos \lambda_2 - \lambda_2^3 \sin \lambda_2 \cosh \lambda_1) + 2\lambda_1^2\lambda_2^2 + (\lambda_1^4 + \lambda_2^4)\cosh \lambda_1 \cos \lambda_2 + \lambda_1\lambda_2(\lambda_1^2 - \lambda_2^2)\sinh \lambda_1 \sin \lambda_2 = 0 \quad \text{(VI.157)}$$

A second equation necessary to find λ_1 and λ_2 can be deduced by utilizing the property of the roots of the characteristic equation (VI.155) according to which

$$\alpha_1^2 + \alpha_2^2 = 2a^2 \quad \text{or} \quad \lambda_1^2 - \lambda_2^2 = \lambda^2 \quad \text{(VI.158)}$$

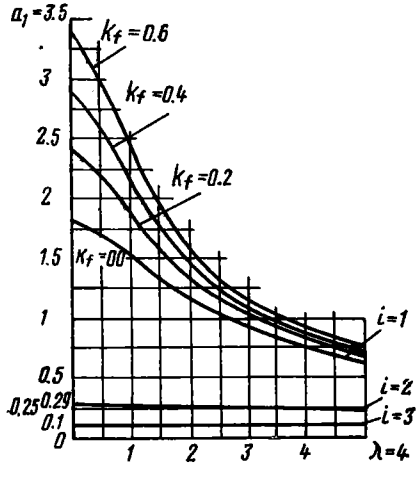


Fig. VI.50. To determining the periods of free lateral vibrations in frame-shear wall structures

According to another property of the roots of the characteristic equation

$$\alpha_1^2\alpha_2^2 = b^4 \quad \text{or} \quad \lambda_1^2\lambda_2^2 = b^4H^4$$

Hence,

$$b^4 = \lambda_1^2\lambda_2^2/H^4 \quad \text{(VI.159)}$$

For practical design purposes, Eqs. (VI.157) and (VI.158) are simultaneously solved for the product, $\lambda_1\lambda_2$, for various values of the rigidity characteristic, λ, and the soil compliance factor, k_f, which is then used to determine $a_i = 2\pi/2\lambda_1\lambda_2$. According to Eqs. (VI.154) and (VI.159), the period of free lateral vibrations of a frame-shear wall structure for the first three harmonics is

$$T_i = a_i H^2 \sqrt{m/Bl}; \quad i = 1, 2, 3 \quad \text{(VI.160)}$$

where a_i is the coefficient determined using the plot of Fig. VI.50, and B is the flexural rigidity of the solid shear wall.

Referring to the plot, the soil compliance affects only the first harmonic of free vibrations. Experience shows that in determining the period of free vibrations the effect of the rigidity of the filler walls and partitions in a skeleton-construction building may be taken care of by raising the shear rigidity, A, to 1.5 to 2 times its original value.

Shear Wall Structures. The equation for free lateral vibrations of open-work shear wall structures with allowance for the longitudinal forces in the vertical members may be derived from Eq. (VI.26)

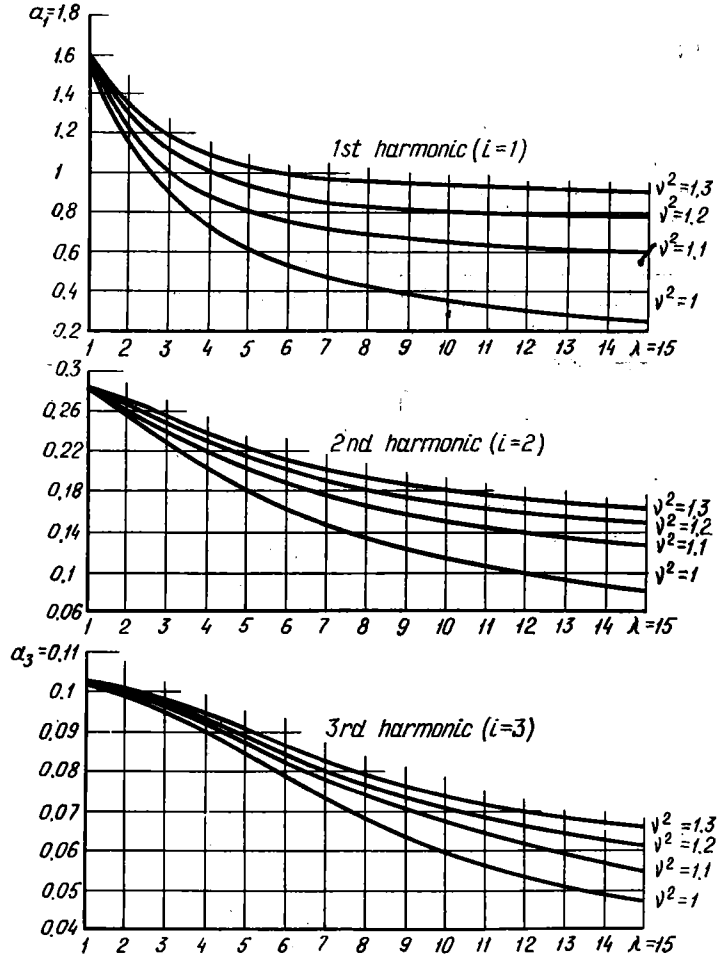


Fig. VI.51. To determining the periods of free lateral vibrations in open-work shear walls for the first three harmonics

by substituting into it the load, $p_x = -(m/l)\,(\partial^2 y/\partial t^2)$. For design puposes, the system with a distributed mass is replaced by a discrete system with concentrated masses and a limited number of degrees of freedom. The building is divided throughout its height into k equal portions in the middle of which is concentrated the distributed mass. The periods and modes of free vibrations can be found by solving the frequency equation drawn up for a system having ten degrees of freedom, using unit deflections, δ_{ik}. For the first three harmonics,

the period of free lateral vibrations of open-work shear walls is

$$T_i = a_i H^2 \sqrt{m/Bl}; \quad i = 1, 2, 3 \qquad \text{(VI.161)}$$

where a_i is the coefficient determined using the plot of Fig. VI.51, and B is the total flexural rigidity of the vertical members of the shear wall.

With allowance for soil compliance, the equation for the mode of free vibrations in solid shear wall structures is

$$X = C_1 \cosh \alpha x + C_2 \sinh \alpha x + C_3 \cos \alpha x + C_4 \sin \alpha x \qquad \text{(VI.162)}$$

where

$$\alpha = \sqrt[4]{\omega^2 m/Bl} \qquad \text{(VI.163)}$$

and B is the flexural rigidity of the solid shear wall.

The boundary conditions are (1) $X(0) = 0$; (2) $X'(0) = BX''(0)/B_f$; and (3): $BX''(H) = 0$, $BX'''(H) = 0$.

Hence, we have a set of four homogeneous equations:

$$\left.\begin{aligned} &\text{(1) } C_1 + C_3 = 0; \quad \text{(2) } C_2 + C_4 - (C_1 - C_3)\lambda_1 k_f = 0 \\ &\text{(2) } C_1 \cosh \lambda_1 + C_2 \sinh \lambda_1 - C_3 \cos \lambda_1 - C_4 \sin \lambda_1 = 0 \\ &\text{(4) } C_1 \sinh \lambda_1 + C_2 \cosh \lambda_1 + C_3 \sin \lambda_1 - C_4 \cos \lambda_1 = 0 \end{aligned}\right\} \qquad \text{(VI.164)}$$

where

$$\lambda_1 = \alpha H$$

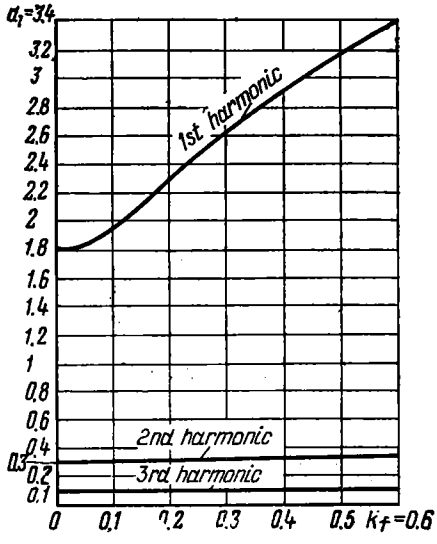


Fig. VI.52. To determining the periods of free lateral vibrations in shear walls with allowance for soil compliance

Expanding the determinant of the set of homogeneous equations gives the frequency equation

$$D(\omega) = 1 + \cosh \lambda_1 \cos \lambda_1 - k_f \lambda_1 (\cosh \lambda_1 \sin \lambda_1 - \sinh \lambda_1 \cos \lambda_1) = 0 \qquad \text{(VI.165)}$$

which is solved for λ_1 and α.

For the first three harmonics, the period of free lateral vibrations in the solid shear walls of shear wall structures is

$$T_i = a_i H^2 \sqrt{m/Bl}; \quad i = 1, 2, 3 \qquad \text{(VI.166)}$$

Here, the values of a_i are determined using the plot of Fig. VI.52.

Referring to the figure, soil compliance affects only the first harmonic of free vibrations in the shear wall.

If the vertical members of a shear wall are carried by a single footing, the period of free vibrations at the fundamental frequency in an open-work shear wall in a shear wall structure and also the period of vibrations in a frame-shear wall structure with allowance for the longitudinal forces in the struts may be determined using Eq. (VI.166). For this purpose, it is necessary to find the flexural rigidity of the open-work shear wall in the shear wall structure or the flexural rigidity of the solid shear wall in the frame-shear wall structure, equivalent to the rigidity of the solid shear wall in terms of the sag of the top storey due to a distributed load. Then, recalling that

$$f = pH^4/8B_{eq}$$

and that the sag of the above structures is given by the general equation (VI.57), we get

$$B_{eq} = B\nu^2\lambda^4/[4\lambda^2 - 8(\varkappa - 1) + (\nu^2 - 1)\lambda^4] \qquad \text{(VI.167)}$$

where B is the total flexural rigidity of the vertical members of the open-work shear wall or the flexural rigidity of the solid shear wall.

The value of B_{eq} thus determined should be substituted for B into Eq. (VI.166).

Example VI.6. *Given*: A 16-storey building acting in the transverse direction as a frame-shear wall structure; building particulars are as in Example VI1;. storey load (including the columns, wall and service load): $Q = 5\,500$ kN.

To find: The period of free lateral vibrations.

Solution: For the rigidity characteristic of the frame-shear wall building $\lambda = 0.73$ and the coefficients $\nu^2 = 1$ and $k_f = 0$, the plot of Fig. VI.50 gives $a_1 = 1.65$, $a_2 = 0.25$ and $a_3 = 0.1$.

According to Eq. (VI.160), the periods of the first three harmonics of free vibrations are

$$T_1 = a_1 H^2 \sqrt{m/Bl} = 1.65 \times 48^2 \sqrt{5\,500/(9.81 \times 261 \times 10^7 \times 3)} = 1.03 \text{ s}$$

$$T_2 = 0.156 \text{ s and } T_3 = 0.063 \text{ s}$$

For a compliant foundation material and $k_f = 0.53$ (see Example VI.4), the plot of Fig. VI.50 gives $a_1 = 2.7$ and Eq. (VI.160) yields $T_1 = 1.68$ s; in this case, the periods of free vibrations of higher harmonics do not change.

Example VI.7. *Given*: A 16-storey building acting in the transverse direction as a shear wall structure; building particulars are as in Example VI.2; storey load (including the columns, walls and service load): $Q = 12\,000$ kN.

To find: The period of free lateral vibrations.

Solution: For the rigidity characteristic of the open-work shear wall $\lambda = 6.3$ and the coefficient $\nu^2 = 1.125$, the plot of Fig. VI.51 gives $a_1 = 0.8$, $a_2 = 0.19$ and $a_3 = 0.082$.

For the total rigidity of the vertical members of the four shear walls $B = 4 \times 31.6 \times 10^7 = 126.4 \times 10^7$ kN m^2, Eq. (VI.161) gives the periods for the first three harmonics of free vibrations as

$$T_1 = a_1 H^2 \sqrt{m/Bl} = 0.8 \times 67.2^2 \sqrt{12\,000/(9.81 \times 126.4 \times 10^7 \times 4.2)} = 1.74 \text{ s}$$

$$T_2 = 0.41 \text{ s and } T_3 = 0.18 \text{ s}$$

Vibration Mode Coefficients. In the design of multi-storey buildings for dynamic loads, it is necessary to determine the vibration mode coefficient which, for constant storey masses, has the form

$$\eta_{ik} = X_{ik} \sum_{j=1}^{n} X_{ij} / \sum_{j=1}^{n} X_{ij}^2 \qquad \text{(VI.168)}$$

where i is the harmonic number, k is the number of the storey for which the mode coefficient is being determined, j is the number of any storey, and n is the number of storeys in the building.

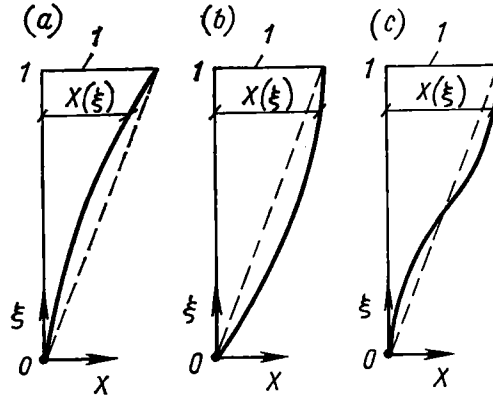


Fig. VI.53. Modes of free lateral vibrations
(*a*) in shear wall structures at $\lambda \leqslant 1$; (*b*) in frame structures; (*c*) in frame-shear wall structures at $\lambda > 1$

For regular structures, summation may be replaced by integration. Then, the vibration mode coefficient takes the form

$$\eta_{ik} = X_{ik} \int_0^1 X_i d\xi / \int_0^1 X_i^2 d\xi \qquad \text{(VI.169)}$$

According to the structural type of a multi-storey building, the first harmonic of free vibrations may be approximated by curves close to it in shape. For example, for shear wall and frame-shear wall structures having a rigidity characteristic of $\lambda \leqslant 1$ (Fig. VI.53*a*), we may assume

$$X = 1 - \cos(\xi\pi/2) \qquad \text{(VI.170)}$$

for frame structures (Fig. VI.53*b*),

$$X = \sin(\xi\pi/2) \qquad \text{(VI.171)}$$

and for frame-shear wall structures with $\lambda > 1$ (Fig. VI.53*c*),

$$X = 0.5\,(1 - \cos \xi\pi) \tag{VI.172}$$

According to Eq. (VI.169) and recalling Eqs. (VI.170) through (VI.172), the vibration mode coefficient is

$$\eta_{ik} = A_1 X \tag{VI.173}$$

where A_1 is a constant given by Table VI.7.

TABLE VI.7. Constant A_1 for Determining Vibration Mode Coefficient According to Structural Type of Multi-Storey Building

Coefficient	Structural type				
	shear wall	frame-shear wall with			frame
		$\lambda \leqslant 1$	$\lambda = 1$ to 6	$\lambda \geqslant 6$	
A_1	1.6	1.6	1.654.0.054 λ	1.33	1.27

11. Dynamic Effect of Wind Gusts

The wind load imposed on a building is a combination of a static and a dynamic component. The static component corresponds to the velocity pressure and should be taken into account in all design cases. The dynamic component is due to velocity pressure pulsations brought about by wind gusts and should be taken into account in buildings more than 40 m high depending on the period of free lateral vibrations.

The basic value of the static component is given by

$$q_{st}^{b} = q_0 kc \tag{VI.174}$$

where q_0 is the wind velocity pressure determined according to the locality of construction; k is the correction factor taking care of the increase in the velocity pressure related to the altitude and type oe locality: type A refers to open localities, and type B to cities, forest areas and similar localities with uniformly distributed obstructions more than 10 m high; and c is the aerodynamic coefficient for buildings, equal to 1.4.

For each ith mode of vibrations, the basic dynamic component is regarded as a system of inertial forces applied at the middle of the portions into which the building is divided for design purposes. The inertial force applied at the middle of the jth portion is given by

$$Q_{dyn,\,ik}^{b} = M_k \xi_{dyn,\,i} \eta_{ik} \nu \tag{VI.175}$$

where M_k is the mass of the kth portion, concentrated at the middle, $\xi_{dyn,\,i}$ is the dynamic coefficient, η_{ik} is the reduced acceleration at

the middle of the kth portion, and ν is the factor taking care of the spatial correlation of the wind velocity pulsation over the height and front of the building.

The dynamic coefficient, $\xi_{dyn,\ i}$, is determined from the standard diagram, according to the following parameter

$$\varepsilon_i = T_i v/1\ 200 \tag{VI.176}$$

where T_i is the period of free vibrations of a multi-storey building:

$$v = 4\sqrt{nq_0} \tag{VI.177}$$

is the design wind velocity; and n is the overload factor taken as 1.2.

The reduced acceleration is

$$\eta_{ik} = X_{ik} \sum_{j=1}^{n} X_{ij} Q^b_{st,j} m_j / \sum_{j=1}^{n} X^2_{ij} M_j \tag{VI.178}$$

Here, n is the number of portions into which the building is divided, $Q^b_{st,\ j}$ is the resultant of the basic wind load applied at the jth portion, and m_j is the coefficient of the velocity pressure pulsation for the middle of the jth portion, looked up in Table VI.8.

TABLE VI.8. Coefficient of Velocity Pressure Pulsation, m_j, for Middle of jth Portion

Locality type	Elevation above ground level, m						
	less than 10	20	40	60	100	200	350 and more
A	0.6	0.55	0.48	0.46	0.42	0.38	0.35
B	0.88	0.75	0.65	0.60	0.54	0.46	0.48

For multi-storey buildings with constant storey masses and sides in plan and constant rigidity throughout the height, the basic value of the dynamic wind load component may only be determined using the first mode of free lateral vibrations:

$$q^b_{dyn} = q^b_{st} \chi \nu \xi_{dyn1} m \tag{VI.179}$$

where χ is the coefficient given by Table VI.9, taking care of the mode of free lateral vibrations of a multi-storey building and variations in the pulsation coefficient over the building height; ξ_{dyn1} is the dynamic coefficient for the first mode of vibrations; m is the coefficient taking care of the type of building top; and ν is taken as 0.5 for buildings erected in type B locality.

TABLE VI.9. Coefficient χ Taking Care of Mode of Free Lateral Vibrations of Multi-Storey Building and Variations in Pulsation Coefficient over Building Height

$\xi = x/H$	0.1	0.2	0.3	0.4	0.5	0.6	0.8	0.9	1
χ	0.34	0.52	0.66	0.70	0.9	1	1.1	1.19	1.36

According to Eqs. (VI.174) and (VI.179), the basic value of the total wind load is

$$q^b = q_0 kc\,(1 + \chi\nu\xi_{dyn1}m) \tag{VI.180}$$

According to the relation between the force, mass and acceleration (with a pulsation coefficient of 0.2 and storey masses, m), the acceleration of vibrations of a multi-storey building may be determined using the following formula

$$W = 0.2q_{st,n}\eta_{1n}\xi_{dyn1}Ll/m \tag{VI.181}$$

where $q_{st,\,n}$ is the design static wind load component at the top storey level, L is the length of the building in a plane normal to the wind, and η_{1n} is the vibration mode coefficient given by Eq. (VI.173).

CHAPTER VII

DESIGN OF ENGINEERING STRUCTURES

VII.1. ENGINEERING STRUCTURES FOR INDUSTRIAL AND CIVIL PROJECTS

In addition to production, residential, office and other buildings, industrial and civil projects incorporate various engineering structures. These include elevated railway lines and motor roads in stockyards and warehouses, roofed and open-type materials handling galleries, pedestrian underpasses and overpasses, transformer substations, reservoirs, water towers, pumping stations, water-cooling towers, air compressor plants, heat supply systems, gas plants, settling or sedimentation tanks, filters, aeration tanks, retaining walls, and the like.

The more complex engineering structures are usually designed as part of transport, utility and water supply services on a city scale.

The less complex engineering structures are designed as part of a particular industrial or civil project. Most important of them are circular and rectangular water tanks or similar sewage disposal and water supply structures, water towers, silos (large storage containers for particulate materials), bunkers (small storage containers for particulate materials which are loaded and unloaded by gravity), retaining walls in warehouses and the like, and underground structures in production areas (conduits for services, deeply buried structures, etc.).

This text is devoted to engineering structures of the latter type; for the more complex engineering structures, the reader should turn to special courses.

VII.2. CIRCULAR TANKS

1. General

Water tanks may be cylindrical or prismatic (that is, circular or rectangular in plan), buried or installed on the ground surface, and covered or open-top. Tanks of more complex shape (spherical, toroid-

al, lenticular, etc.) are used less frequently. Most sewage disposal and waste water treatment structures are similar to water tanks in shape, construction and design.

In a general course on reinforced concrete structures, it is usual to deal only with circular and rectangular water tanks. Oil and oil product tanks installed at refineries, and thermal and electric power stations, and also tanks for process liquors used in the chemical, metal-making and food industries are covered in special courses.

The required tank capacity for water supply, sewage disposal or process systems is determined to meet appropriate process objectives. The shape and overall dimensions of tanks are chosen by comparing several alternative designs from technical and economic points of view. Experience has shown that for buried water tanks with a capacity of two to three thousand cubic metres circular structures are more economical than rectangular tanks, whereas for capacities of five to six thousand cubic metres, the latter are preferable.

The cost of construction and the amount of materials required considerably depend on the internal height and depth of a tank. As the height increases, the internal water pressure exerted on the walls rises, whereas the increasing depth brings about a rise in the lateral earth pressure.

In the Soviet Union, all water supply and sewage disposal structures have been standardized on the basis of large-scale technical and economic investigations. Water tanks are built in standard capacities and heights (Table VII.1).

TABLE VII.1. Capacity and Size of Standardized Circular Water Tanks

Capacity, m^3	100	150	250	500	1 000	2 000	3 000	6 000
Diameter, m	6.5	8	10	12	18	24	30	42
Height, m	3.6	3.6	3.6	4.8	4.8	4.8	4.8	4.8

Tanks may be of in-situ, precast or precast/in-situ construction. The amount of concrete and steel required by precast water tanks is 15 to 20% less than in the other types. The construction period is significantly shorter, and the cost of construction is by 5 to 7% lower.

Of late, water tanks have further been standardized in terms of construction types and component members.

The walls and roofs of tanks are made of heavy concrete. The concrete brand numbers in terms of compressive strength are M-200

to M-400, in terms of watertightness B4 to B8, and in terms of f ost resistance Mpa 100 to Mpa 150. The concrete brand is assigned according to the expected service and exposure conditions (design ambient temperature, soil moisture content, and ground water corrosiveness).

To meet the watertightness requirements, the concrete should contain a minimum amount of voids, which is achieved by thoroughly grading the aggregates, reducing the water-cement ratio, and careful compacting in the course of forming. The reinforcement is class A-I, A-II, A-III and B-I nonprestressed steel, and class A-IV (and higher) and Bp-II prestressed steel (for circular walls).

The joints between prefabricated members should be sealed with joint compounds, or use should be made of concretes based on expanding cement to provide tighter joints and to compensate for subsequent shrinkage strain in the prefab members.

To increase watertightness, tanks are given a coat of cement plaster on the inside, and the surface of the joints between wall slabs is shotcreted.

Tanks should be erected above the ground water level, because otherwise, it would be necessary to lower the ground-water level during construction, to install a heavier floor to resist water pressure from beneath, and to use pasted multi-layer insulation to protect the tank from the outside.

Buried tanks usually have flat roofs and floors. To maintain a desired temperature inside a tank, its roof is heat-insulated by a layer of soil 0.5 to 1 metre thick, or by effective heat-insulators such as cellular concrete, ceramsite concrete and the like.

Tank roofs have openings to provide access into tanks and pass ventilation shafts. Also, their floors have a pit up to 1 m deep for cleaning and total evacuation of tanks.

2. Construction Types

Let us consider some of the standardized construction types and component members used in Soviet practice.

Figure VII.1 shows an in-situ r.c. tank consisting essentially of a flat-slab roof supported on column having capitals at the top and reverse capitals at the bottom, a plain cylindrical wall, and a flat ribless floor. In low-capacity tanks, the required watertightness is secured without prestressing. If the capacity of a tank is 500 m^3 or more, wall prestressing is mandatory. The flat-slab roof is rather thin, owing to which the depth of the tank is minimal. Also, its bottom surface is smooth to improve ventilation above the level of the liquid being stored.

In the past, use was made of other types of circular tanks. They had beam and girder roofs supported on columns with a column grid size of 6 m square and more, dome roofs supported on walls, floors

sloping from the walls towards the centre, and so on. For some reasons, they have been superseded by type-design structures.

Precast circular tanks are illustrated in Figs. VII.2 through VII.4.

Figure VII.2 shows the roof of a circular tank, assembled from trapezoidal ribbed slabs laid on ring beams.

The wall of a tank consists of prefabricated panels of length equal to tank height. The panels are vertically installed in the slot between two circular floor ribs around the periphery of the tank (Fig.VII.3*a*).

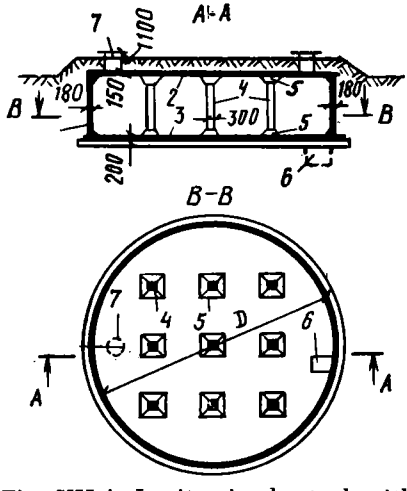


Fig. VII.1. In-situ circular tank with a flat-slab roof

1—wall; *2*—flat-slab roof; *3*—floor; *4*—columns; *5*—capitals; *6*—pit; *7*—manhole

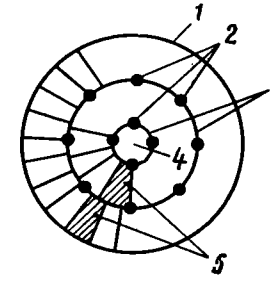


Fig. VII.2. Precast circular tank

1—cylindrical wall; *2*—columns; *3*—ring beams; *4*—round flat slab; *5*—trapezoidal slabs with ribs around the periphery

The vertical joints between the panels are concreted. After the concrete of the joints has attained at least 70% of its design strength, the wall is compressed by prestressed reinforcement wrapped around the tank. After the steel has been tensioned, it is protected by shotcreting.

The nominal width of wall panels is 3.14 or 1.57 m (Fig. VII.4*a*). With such a width, the periphery of a tank accommodates an integer number of panels, equal to D or $2D$, respectively (where D is the tank diameter). The structural width of the panels is 140 mm less than the nominal width. The 140-mm gap is concreted during erection. The brand number of the in-situ concrete should be not less than that for the panel concrete.

The thickness of the wall panels, δ, ranges between 120 and 200 mm (being a multiple of 20 mm). In tanks of radius $R \geqslant 12$ m, the outer

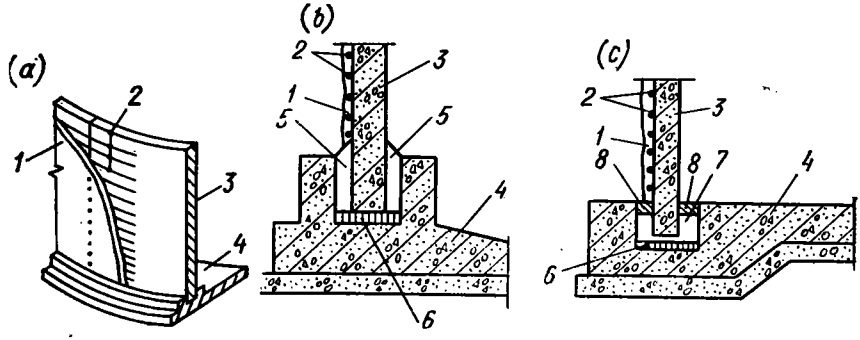


Fig. VII.3. Details of a precast circular tank

(*a*) wall construction; (*b*) rigid connection between wall and floor; (*c*) movable joint between wall and floor; *1*—shotcrete; *2*—prestressed hoop reinforcement; *3*—wall panel; *4*—floor; *5*—concrete using fine-grade broken stone as aggregate; *6*—levelling grout; *7*—asphalt putty; *8*—asbestos-cement grout

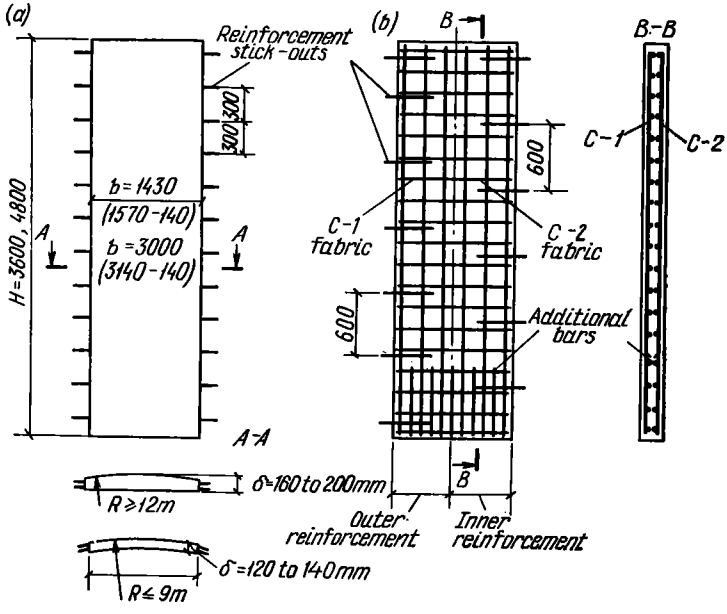


Fig. VII.4. Wall panels for circular tanks

(*a*) general view; (*b*) reinforcement

surface of the wall panels is made cylindrical, and the internal surface flat. In tanks of radius $R \leqslant 9$ m, both surfaces are cylindrical (Fig. VII.4*a*).

As already noted, the prestressed horizontal load-bearing reinforcement is placed on the outside of the wall panels (Fig. VII.3*a*).

The wall panels are reinforced with double fabric whose cross sectional area is taken to suit structural requirements (Fig. VII.4*b*). The reinforcement stick-outs of the neighbouring panels are welded to-

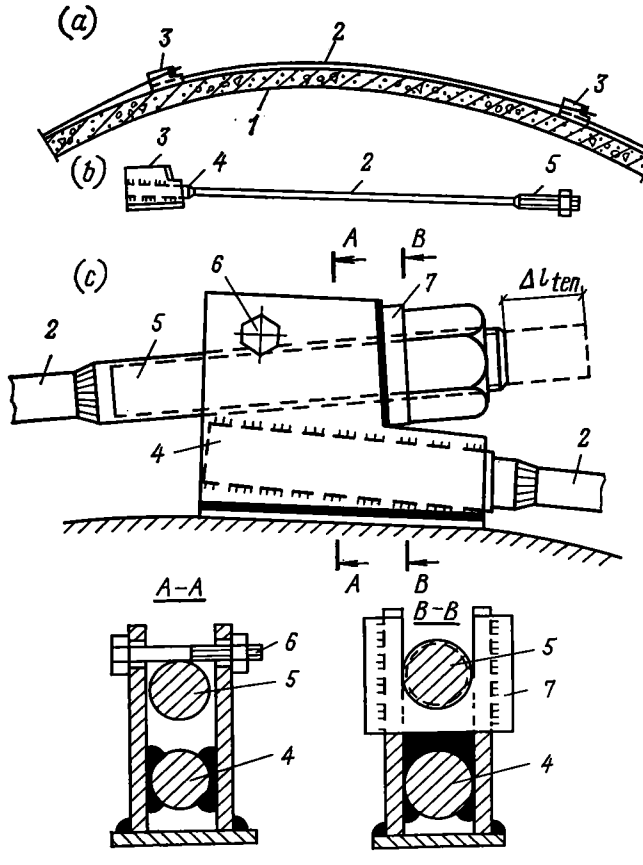


Fig. VII.5. Prestressed bars tensioned by electrical heat
(*a*) distribution of reinforcement over cylindrical wall; (*b*) reinforcing member; (*c*) anchor stop; *1*—wall; *2*—reinforcing bar; *3*—anchor stop; *4*—large-diameter short bar; *5*—screw-threaded large-diameter short bar; *6*—tie bolt; *7*—stop plate

gether so as to fix the panels in the design position and to prevent them from shrinkage and temperature cracking prior to the compression by the prestressed reinforcement.

The cross-sectional area of the vertical reinforcement in prefabricated wall panels is taken to provide the necessary strength and crack resistance in manufacture, transit and during erection. At the bottom, the panels have additional reinforcing bars (Fig.VII.4*b*) intended

to take up the bending moments (acting in a vertical direction) due to the interaction between the wall and the floor.

The precast wall panels may be connected to the floor rigidly so as to prevent the wall from radial deflection and angular deflection in the circular floor slot (Fig. VII.3*b*), or they may be hinged, in which case these deflections are likely to arise (Fig. VII.3*c*). In the former case, the gap between the panels and the floor is filled with high-strength concrete using fine broken stone as aggregate; in the latter case, the gap is filled with cold asphalt putty. The depth for which the wall panels should be embedded into the floor is determined by design, but it should be at least half as large again as the wall thickness.

The circular prestressed high-strength wire is tensioned on the walls by wrappers. The distance between the wire turns should be not less than 10 mm.

The bar reinforcement is tensioned by electrical heat. A circular (hoop) bar is cut into submultiple lengths (Fig. VII.5*a*); to the ends of each portion are welded short bars: one screw threaded, and the other plain-welded to an anchor stop (Fig. VII.5*b*). At the latter the reinforcement members are joined together (Fig. VII.5*c*). As electrical heat is applied to them, the bars elongate. After the bars have extended to the desired length, they are fastened by nuts to the stops. As the bars cool, the reinforcing circle shortens, the tank wall is compressed and the reinforcement remains in tension.

There also exist other methods of tensioning the reinforcement. The distance between the reinforcement bars ranges between 20 and 25 cm.

After the circular reinforcement has been prestressed, it is given several coats of shotcrete, so that the concrete cover is at least 25 mm.

The internal surface of the tank is plastered prior to tensioning the reinforcement so as to compress the plaster together with the wall panels.

3. Design Procedure

The liquid contained in a tank exerts a hydrostatic pressure on the tank walls, which increases linearly with rising depth. The design pressure at the depth $(l\text{-}x)$ below the liquid level (Fig. VII.6*a*) is

$$p_x = 1.1p_0\,(1 - x/l) \qquad \text{(VII.1)}$$

where $p_0 = \gamma l$, γ is the liquid density (for water, $\gamma = 1$), and 1.1 is the overload factor.

The hydrostatic pressure gives rise to circular tensile forces, N^0_{ctr}, in the wall. Their values are determined considering a half-ring of

unit chord height in the state of equilibrium (Fig. VII.6*b*)

$$N^0_{cir} = p_x R \qquad \text{(VII.2)}$$

where R is the radius of the ring.

For a wall separated from the floor, the diagram of forces in the ring is linear (Fig. VII.6*c*). Under the action of the circular forces,

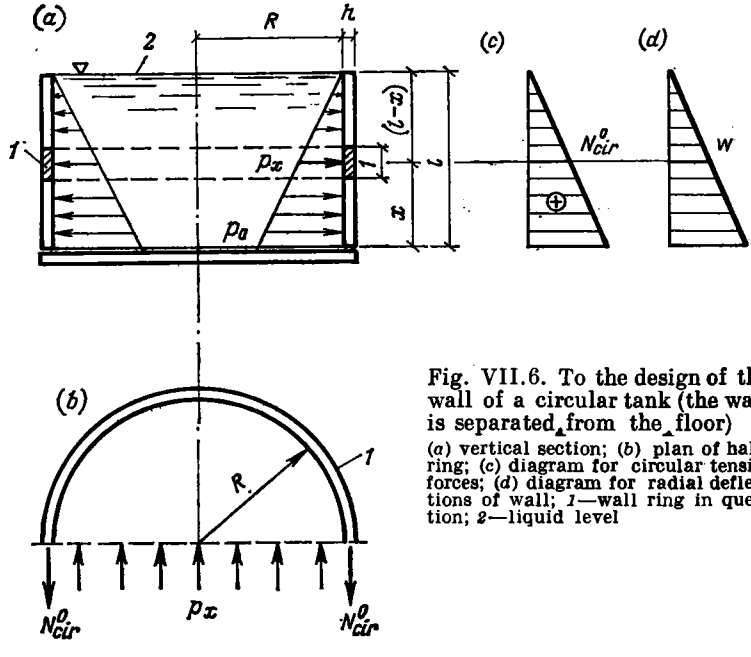


Fig. VII.6. To the design of the wall of a circular tank (the wall is separated from the floor)
(*a*) vertical section; (*b*) plan of half-ring; (*c*) diagram for circular tensile forces; (*d*) diagram for radial deflections of wall; *1*—wall ring in question; *2*—liquid level

the wall perimeter elongates, and the wall itself deflects in a radial direction. The diagram of the deflections, w, follows the diagram for N^0_{cir} (Fig. VII.6*d*).

When the wall is rigidly connected to the bottom (in in-situ tanks or tanks made of prefabricated members with the connection between the wall and the floor arranged as shown in Fig. VII.3*b*), the radial deflections at floor level are practically zero because of negligibly small in-plane deformability of the floor. Therefore, the vertical generatrix of the wall bends and gives rise to bending moments, M_x, acting along the generatrix, and to the corresponding shearing forces, Q_x.

The wall is an axisymmetric thin shell. So, like other thin shells, it is only subjected to local bending.

In areas of local bending, the wall can be described by Eq. (V.23). Solving this equation for moments gives Eq. (V.24), and for deflections, w, Eq. (V.54).

The interaction of the rigidly joined wall and floor (Fig. VII.7) brings about a bending moment, M_1, and a shearing force, Q_1, at the joint. Their values are found on the assumption that the wall and floor edges undergo the same angular deflections at the line of contact.

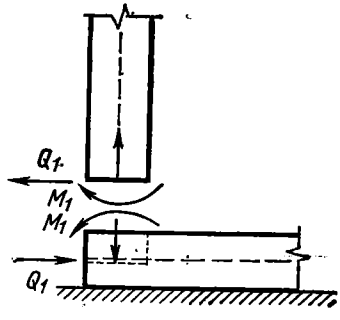


Fig. VII.7. To the design of the connection between a wall and a floor

The deflections of the wall edge are determined using the same formulas as for dome edges (see Table V.1). The floor edge deflections are found as those of a beam (a floor strip of unit width) lying on soil (see Ch. III).

When the wall and floor are rigidly connected to each other, the resultant equations for the circular forces, N_{cir}, and bending moments, M_x, in the wall at the level, x, from the floor, derived with allowance for the moment, M_1, and shearing force, Q_1, have the following form

$$N_{cir} = N^0_{cir} - p_0 R\,[\exp(-\varphi)\cos\varphi + \exp(-\varphi)\sin\varphi\,(1 - s/l)] \tag{VII.3}$$

$$M_x = 0.5 p_0 s^2\,[(1 - s/l)\exp(-\varphi)\cos\varphi - \exp(-\varphi)\sin\varphi] \tag{VII.4}$$

where N^0_{cir} is the circular force calculated at a given wall level, using Eq. (VII.2), p_0 is the hydrostatic pressure at the bottom of the wall, s is the elastic characteristic of the wall,

$$s = 0.76\sqrt{Rh} \tag{VII.5}$$

h is the wall thickness, and $\varphi = x/s$ is the non-dimensional coordinate.

At $x = 0$, $\varphi = x/s = 0$, $\exp(-\varphi) = 1$, $\sin\varphi = 0$ and $\cos\varphi = 1$, a maximum moment arises, equal to

$$M_{max} = 0.5 p_0 s^2\,(1 - s/l) \tag{VII.6}$$

Likely diagrams for N_{cir} and M_x are shown in Fig. VII.8*a*.

When a precast cylindrical wall is hinged to the floor (see Fig. VII.4*c*), the radial deflection of the wall gives rise to a frictional force round its edge

$$Q_{fr} = Nf \tag{VII.7}$$

where N is the normal force applied at the edge of the wall, due to its self-weight and the weight of the adjoining part of the roof plus

the backfill over its top; and f is the coefficient of friction of the wall against the floor, taken as 0.5.

At the level x above the floor, the circular tensile forces in the wall are given by

$$N_{cir} = N^0_{cir} - 2\,(R/s)\,Q_{fr} \exp(-\varphi) \cos\varphi \qquad \text{(VII.8)}$$

The maximum moment

$$M_{x,max} = Q_{fr} s \exp(-\varphi) \sin\varphi \qquad \text{(VII.9)}$$

is applied at the distance

$$x_1 = 0.6\sqrt{Rh} \qquad \text{(VII.10)}$$

from the floor.

Figure VII.8*b* shows the diagrams for N_{cir} and M_x for a movable connection between the wall and the floor.

Buried tanks are designed for the internal hydrostatic pressure that is likely to exist during tests and repair in the absence of back-

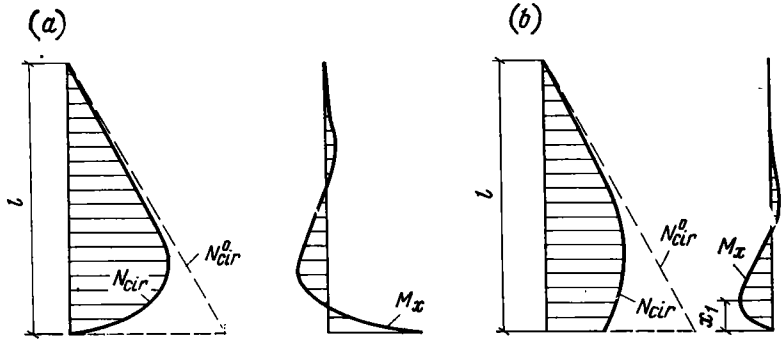


Fig. VII.8. To the design of the wall of a circular tank; circular force and bending moment diagrams

(*a*) rigid joint between the wall and the floor; (*b*) movable joint between the wall and the floor

fill, and for the lateral earth pressure to be exerted on an evacuated tank. To resist bending moments, the area adjoining the floor is additionally reinforced with vertical steel placed according to the diagrams of Fig. VII.8.

The lateral earth pressure on the walls may be determined as prescribed in Sec. VII.7.

The cross-sectional area of the wall hoop reinforcement is determined as for a member in axial tension. This is done individually for each zone 1 m high, starting from the floor and using the maximum

circular force in that zone, by the following formula

$$F_s = N_{cir}/R_s \qquad \text{(VII.11)}$$

The analysis of prestressed tank walls for incipient cracking is carried out as prescribed in Ch. VII in Vol. 1. Tank walls should satisfy the requirements of the first category, so they should be analysed in terms of the design forces.

The cross-sectional area of the vertical reinforcement in the walls is determined as for a slab in bending, separately for the internal hydrostatic pressure and for the earth pressure. The design amount of reinforcement is disposed at the bottom where the concrete cover is 1.5 cm. In the upper part of the wall, the reinforcement is installed for structural reasons.

No special requirements are to be met in the analysis and design of the roof and columns. Under normal conditions (in the absence of backwater), the weight of the floor and liquid over it is balanced by the earth thrust so that no bending of the floor occurs. Local bending moments arise in the floor only where it joins the walls and the column footings. Such areas are additionally reinforced, whereas elsewhere the reinforcement is distributed to suit structural requirements. As a rule, the floor is cast in-situ.

VII.3. RECTANGULAR TANKS

1. Construction Types

As already noted, rectangular tanks are preferable when the tank capacity is from 6 to 20 thousand cubic metres. They are also built in smaller capacities, if the tank is to be located in a limited space, say, as a part of other structure.

In the USSR, rectangular water tanks are standardized as regards size and capacity (Table VII.2).

TABLE VII.2. Capacity and Size of Standardized Rectangular Water Tanks

Capacity, m^3	100	250	500	1 000	2 000	3 000	6 000	10 000	20 000
Plan dimensions, m	6×6	6×12	12×12	12×18	18×24	24×30	36×36	48×48	66×66
Height, m	3.6	3.6	3.6	4.8	4.8	4.8	4.8	4.8	4.8

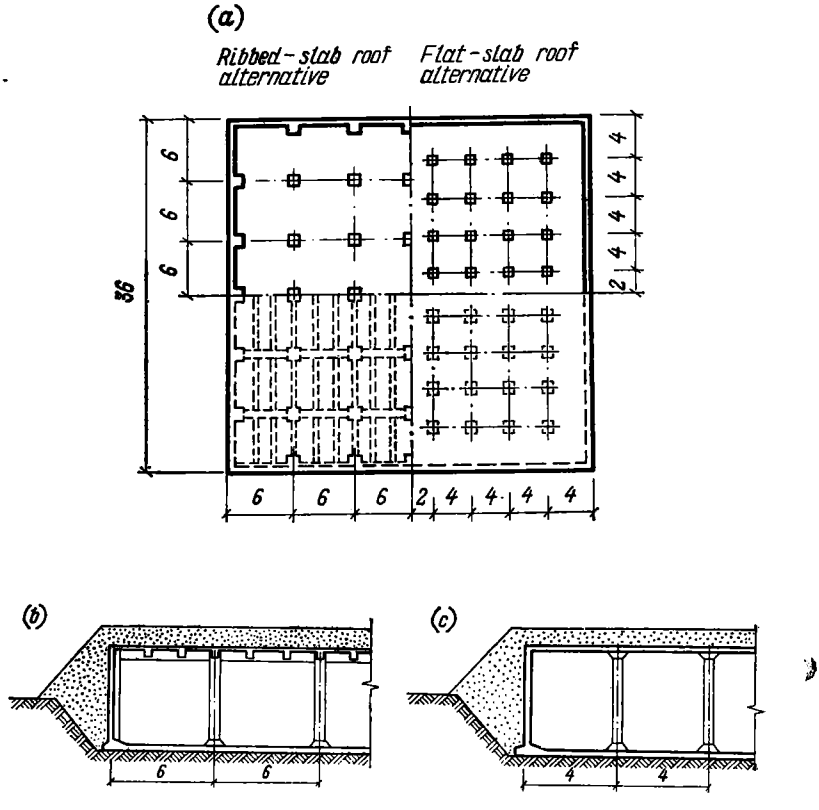


Fig. VII.9. In-situ rectangular tank
(a) plan; (b) section of a tank using a beam and girder roof; (c) section of a tank using a flat-slab roof

The roofs of rectangular tanks are usually made of flat slabs installed on columns, and the floors are also flat or sloping from the periphery to the centre, in order to increase the tank capacity.

Figure VII.9 illustrates the construction types of in-situ r.c. tanks: one with a beam and girder roof and a column grid size of 6 m square, and the other with a flat-slab roof and a column grid size of 4 m square. Walls up to 4 m high are made plain, higher walls have ribs.

Figure VII.10 illustrates the construction types of precast tanks: one with a slab and girder roof and a column grid size of 6 m square, and the other with a slab roof and a column grid size of 4 m square. In the first alternative, the roof is made of type-design girders and ribbed slabs 6 × 1.5 m in plan used in industrial building floors;

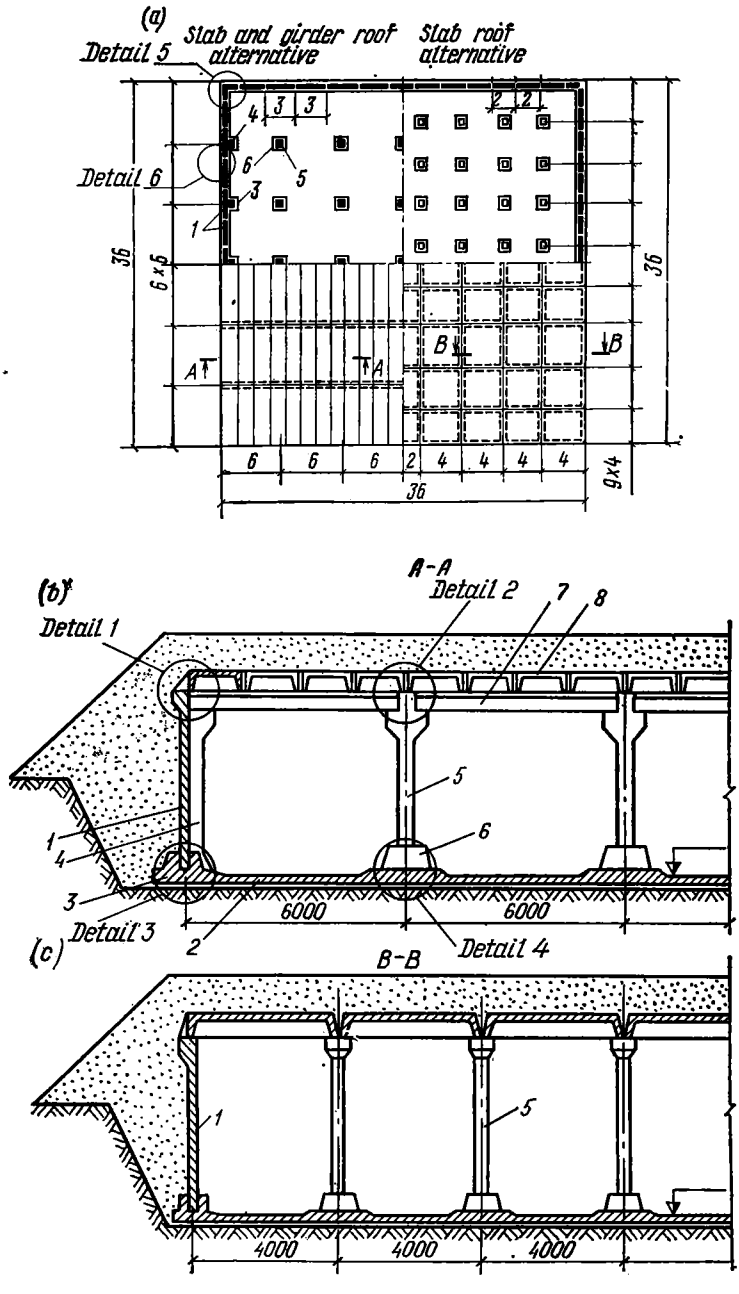


Fig. VII.10. Precast rectangular tank

(*a*) plan; (*b*) section of a tank using a slab and girder roof; (*c*) section of a tank using a slab roof; *1*—wall panels; *2*—in-situ floor; *3*—outer column footing (boss in the floor); *4*—outer column; *5*—intermediate column; *6*—footing block; *7*—roof beam; *8*—slab

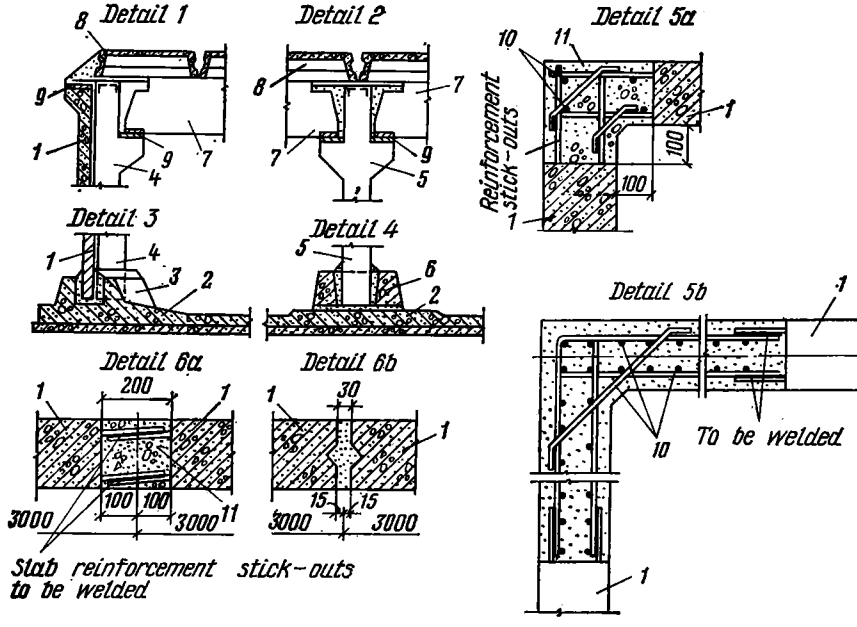


Fig. VII.11. Details of a precast rectangular tank
1 through *8*—same as in Fig. VII.10; *9*—embedded parts; *10*—additional reinforcement in in-situ portion; *11*—in-situ concrete in walls

in the second alternative, the roof consists of slabs (having ribs around the periphery) supported at the corners directly on columns.

The wall panels for any tank are chosen to be of the same type and size. In the tank of Fig. VII.10, the height of a wall panel is 4.8 m, its nominal width is 3 m, and its thickness is 200 mm. The wall panels are installed in a longitudinal slot in the floor, secured in the design position, and concreted. The vertical joint gaps may be rectangular and 200 mm wide (for welded reinforcement stick-outs), or of key construction 30 mm wide (without welded reinforcement). Since the joints of the former type must allow for the bending that the wall experiences in the horizontal direction between the pilasters, they should be installed where the bending moments are small (see Fig. VII.10).

The corners of the walls are cast in-situ, and their dimensions depend on the plan layout of the wall panels.

Precast square-section columns are installed in footing sockets, and the gaps are concreted. The floors are cast in-situ. Figure VII.11 illustrates the details of rectangular tanks.

Extended tanks have movement joints installed at every 54 m (Fig. VII.12).

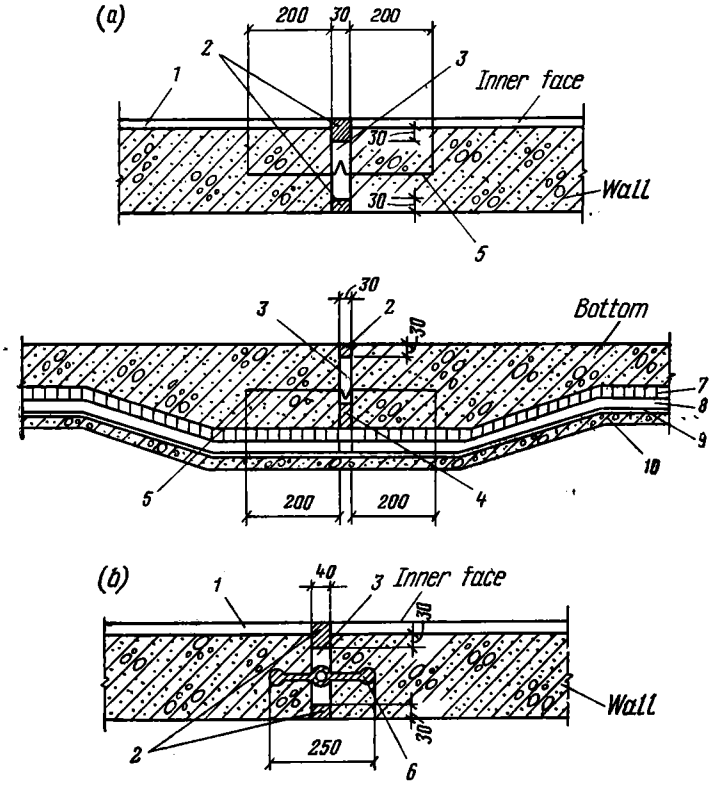


Fig. VII.12. Details of movement joints

(*a*) with steel compensators; (*b*) with three-cam rubber key; *1*—shotcrete plaster; *2*—asbestos-cement grout; *3*—asbestos strand soaked in asphalt; *4*—asphalt slabs; *5*—stainless steel (or ordinary steel) sheet compensator 1 to 2 mm thick; *6*—three-cam rubber key; *7*—layer of grout; *8*—layer of sand; *9*—ruberoid; *10*—layer of concrete

2. Design Procedure

The tank walls are designed to resist unilateral water pressure in the absence of backfill, and also unilateral earth pressure when the tank is empty. The earth pressure is taken as prescribed in Sec. VII.7.

An in-situ r.c. ribless wall and a precast wall with key-type joints (see Fig. VII.11, Detail *6b*) in which the horizontal reinforcement is not welded are designed using a beam model whether they have or have no ribs (or pilasters) (Fig. VII.13*a*). The span l_1 is taken equal to the distance from the top side of the floor slot to the roof.

For design purposes, we select a loaded vertical stip 1 m wide. We assume that the wall is rigidly clamped in the floor and hinged

at roof level (Fig. VII.13*b*). Figure VII.13*c* shows a diagram for the bending moments acting in a vertical direction. The moments at the support, M_{sup}, and in the span, M_{sp}, are determined by the formulas given in a course on the strength of materials.

In an in-situ or precast rib-reinforced wall, where all of the reinforcement is welded at the joints (see Fig. VII.11, Detail *6a*), each

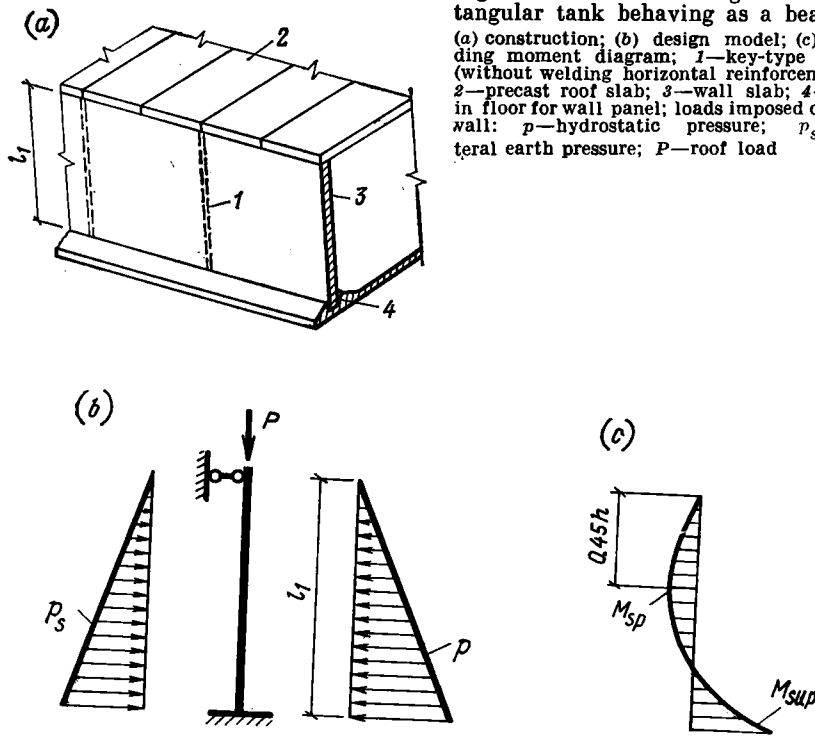


Fig. VII.13. To the design of a rectangular tank behaving as a beam
(*a*) construction; (*b*) design model; (*c*) bending moment diagram; *1*—key-type joint (without welding horizontal reinforcement); *2*—precast roof slab; *3*—wall slab; *4*—slot in floor for wall panel; loads imposed on the wall: p—hydrostatic pressure; p_s—lateral earth pressure; P—roof load

part of the wall between the ribs is designed as a slab spanning in two directions (Fig. VII.14) if $l_2/l_1 \leqslant 2$ (at $l_2 > l_1$). At the sides of the ribs and the floor, the slab is deemed to be rigidly clamped, and at roof level, hinged. In the case of precast roofs, the hinge is formed owing to moment-free restraints between the prefab roof slabs and wall panels; in the case of in-situ roofs, it is formed because the roof rests on a slab having a low flexural rigidity.

The maximum values of support and span moments are looked up in appropriate reference sources.

The amount of reinforcement required is chosen to resist the maximum moments at the supports and in the span as in a singly rein-

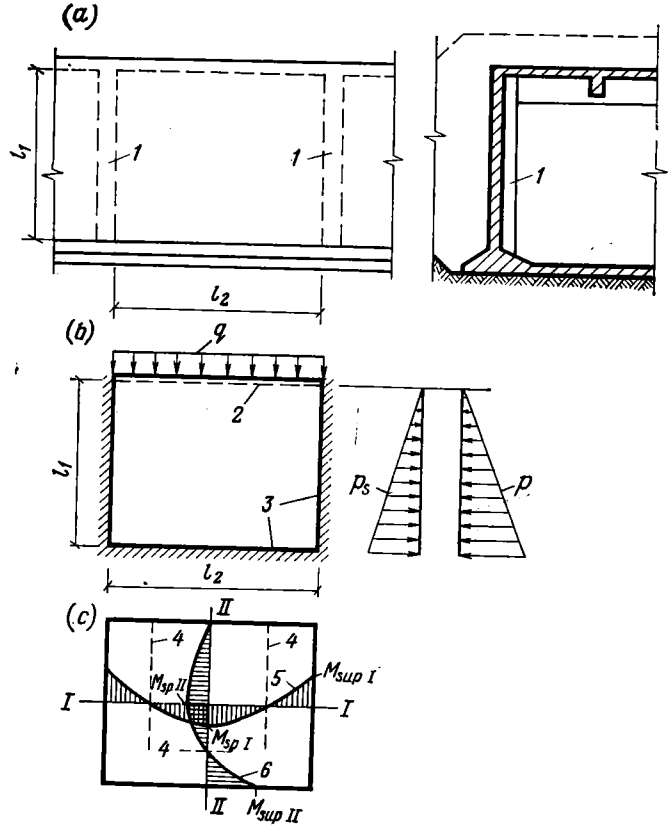


Fig. VII.14. To the design of a rectangular tank wall as a two-way slab
(*a*) construction; (*b*) design model; (*c*) bending moment diagrams; *1*—vertical ribs; *2*—hinged support; *3*—fixation; *4*—zero-moment lines; *5*—bending moment diagram along l_2; *6*—bending moment diagram along l_1; loads imposed on the wall: *p*-hydrostatic pressure; p_s—lateral earth pressure; *P* – roof load

forced rectangular-section slab subjected to bending. The normal forces induced in the wall by the roof load or lateral wall pressure are not taken into account because they have little effect on the final results. The amount of reinforcement is determined separately for the internal water pressure and the lateral earth pressure.

Individual reinforcing bars are combined into welded-bar mats which are installed near the interior and exterior wall surfaces with a minimum concrete cover. A likely reinforcement for a prefab wall panel is shown in Fig. VII.15.

In in-situ tanks, plain (ribless) walls are designed assuming an interaction with flat-slab roofs, and ribbed walls, assuming an interaction with ribbed roofs (see Fig. VII.9).

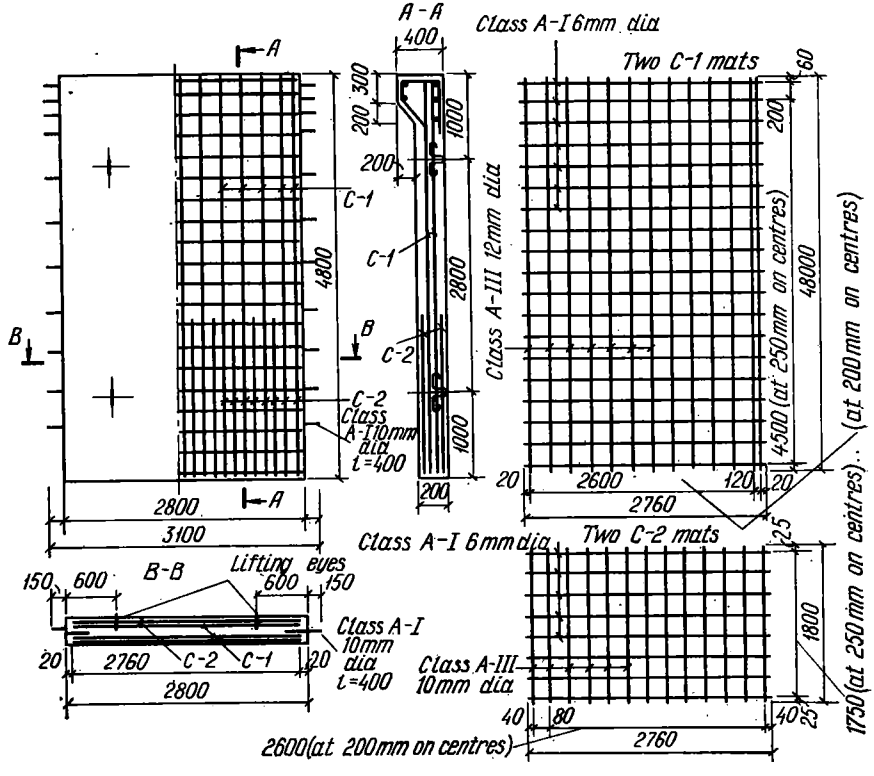


Fig. VII.15. Reinforcement of a rectangular tank wall panel

In addition to strength, the walls should be designed for crack opening ($a_{cr} \leqslant 0.2$ mm) as prescribed in Ch. VII in Vol. 1. In this case, all loads are regarded as sustained.

VII.4. WATER TOWERS

Water towers are widely used in the water supply systems of towns, villages and industrial buildings. Their function is to maintain the required water head in the lines and an uninterrupted water supply.

The main part of a water tower is a tank. The capacity of the tank is chosen to suit water consumption requirements and according to the capacity of the pumping station. The height of the tank above ground level depends on the design water head required.

The optimal location for a water tower and its height depend on the pattern of water consumption and relief. In capacity, water towers range between 15 m^3 and 3 000 m^3, and in height, between 6

and 50 m. Water towers may be of the shelter type (Fig. VII.16*a*) or the shelterless type (Fig. VII.16*b*). In the former type, the tank is enclosed in a shelter to keep the tank from freezing at sub-zero ambient temperatures and the low rate of water circulation, or to keep

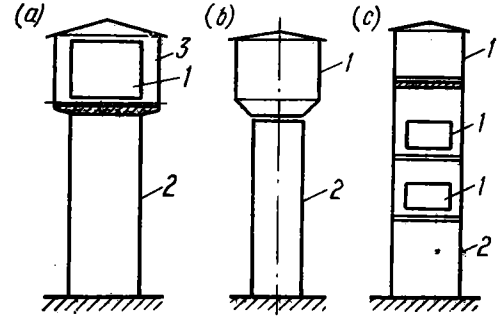


Fig. VII.16. Types of water towers
(*a*) shelter type; (*b*) shelterless type; (c) with several tanks; *1*—tank; *2*—support structures; *3*—shelter

the water from heating at high ambient temperatures. In the latter type, thermal insulation is applied directly to the tank walls.

Instead of one tank (Fig. VII.16*a* and *b*), a water tower may have several tanks (Fig. VII.16*c*), if water of different purity and temperature is required.

Existing evidence shows that, given the same height and construction type, the cost of water towers varies only insignificantly, with tank capacity. For example, if the effective tank capacity is increased by 30 to 40%, the cost of the tower will rise only by 3 to 6%. Similarly, given the same capacity and construction type, the cost of towers varies as gradually with height.

This facilitates the standardization of water tanks, so that a limited range of tanks and support types and sizes may be sufficient. On the basis of a technical and economic analysis the following type-design structures have been developed in the USSR: towers with a tank capacity of 25 and 50 m^3 for a support height of 9, 12, 15, 18, 21, 24 or 27 m, and towers with a tank capacity of 150, 250, 500 and 1 000 m^3 for a support height of 12, 18, 24, 30, 36 or 42 m.

Nowadays, the most popular tanks in the USSR are made of the shelterless or, at least, semi-shelter type. This simplifies construction, reduces the weight of support structures and cuts down the cost.

The tanks of water towers are made either of reinforced concrete or of steel. Among reinforced concrete tanks, the simplest structure consists essentially of a cylindrical wall and a flat non-bearing bottom

lying on a solid floor installed in the supporting structure (Fig. VII.17*a*).

Tanks made up of a cylindrical wall changing at the bottom into a truncated cone, and a load-bearing bottom are more economical of material and, therefore, cost by 30 to 40% less (Fig. VII.17*b*).

To ensure the required crack resistance, the walls of large reinforced concrete tanks ought to be prestressed. However, steel tensioning

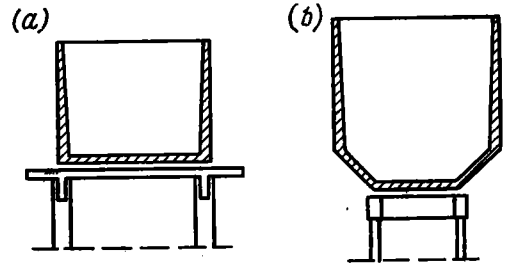


Fig. VII.17. Types of water tower tanks
(*a*) with cylindrical wall and non-bearing bottom; (*b*) with cylindrical wall changing into a truncated cone and a load-bearing bottom

in one structure at a time and at a great height is inconvenient and not always warranted. Because of this, tanks are often made of steel instead of reinforced concrete.

The supports of water towers are mostly made of reinforced concrete, but for small-capacity tanks (from 25 to 50 m^3) they may be made of metal or brick depending on the local conditions. In particular, for relatively high towers, metal supports may prove less expensive than reinforced concrete or masonry supports because they are more economical of material as their amount is decided by structural, rather than design considerations.

Masonry supports are used for low towers (from 9 to 12 m high) with small-capacity tanks (from 25 to 50 m^3), if it is more advantageous to use available brick than reinforced concrete which would have to be specially fabricated.

According to exposure conditions, reinforced concrete supports may be made as a solid monolithic cylindrical shell (Fig. VII.18), as a space frame (Fig. VII.19), or as an open-work lattice structure (Fig. VII.20). Other things being equal, precast r.c. open-work lattice supports are less expensive than the other types. Also, the cost of such supports is from two thirds to one half the cost of solid in-situ r.c. supports, and even less as compared with the cost of masonry supports. The cost of reinforced concrete and metal supports is about the same.

Figure VII.18 shows a water tower using a reinforced concrete support which is a monolithic shell cast in reusable travelling formwork. The shell thickness is chosen to be 150 mm; this is the minimum

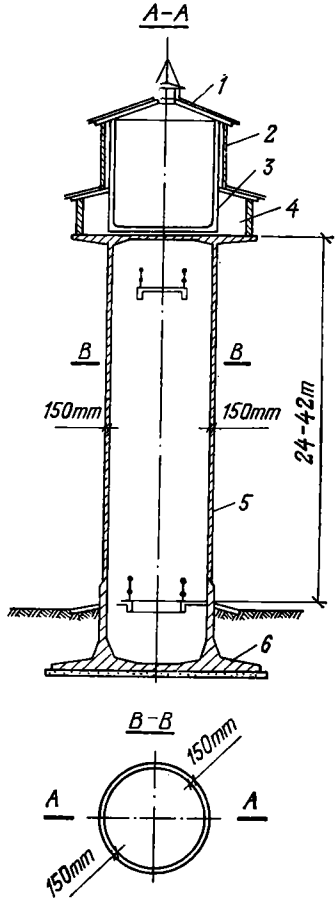


Fig. VII.18. Water tower with a solid in-situ r.c. support

1—tank roof; *2*—tank wall heat insulation; *3*—reinforced concrete tank; *4*—semi-shelter (heated gallery); *5*—r.c. support; *6*—r.c. footing

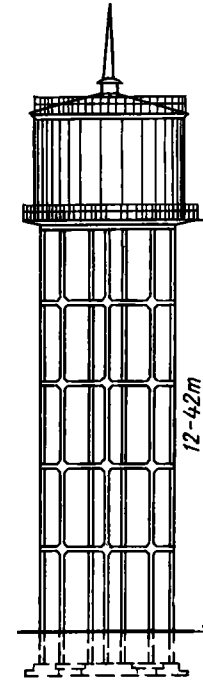


Fig. VII.19. Water tower with a frame r.c. support

value permitting formwork to be moved without causing tears in the concrete. This thickness is, however, about twice the value required by strength analysis, so the cost of the entire structure considerably rises. Such supports are attractive for towers at least 24 m high using

large-capacity tanks (at least 800 m³), because the difference between the shell thickness required for concrete-casting reasons and that required by design is now small. The shaft of the tower rests on an in-situ reinforced concrete footing having a circular rib running around the periphery of the shaft. The tower shown in the figure uses

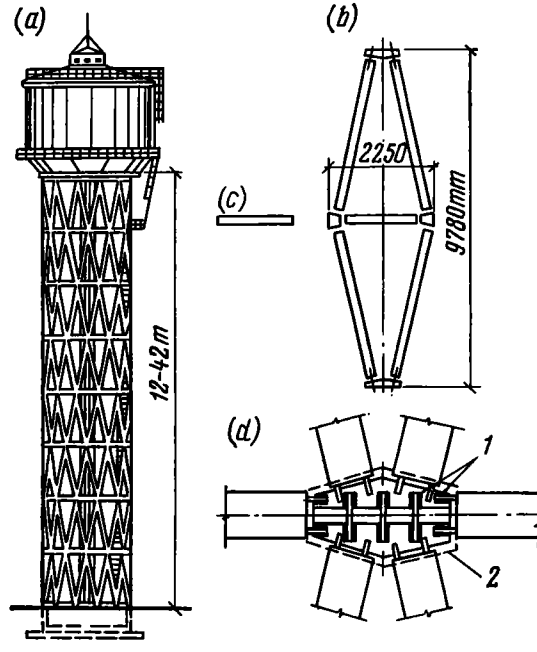


Fig. VII.20. Water tower with an open-work lattice r.c. support
(*a*) general view; (*b*) rhombic rod unit; (*c*) chord unit; (*d*) connection of support members during erection; *1*—to be welded during erection; *2*—in-situ concrete boundary

a reinforced concrete tank with a non-bearing bottom. A heated service gallery around the tank is covered by a semi-shelter, and the tank wall above the gallery is protected by heat insulation.

Water towers using frame r.c. supports (Fig. VII.19) are more economical of material and less expensive than those with solid in-situ r.c. supports. Frame supports require about 50% of the reinforced concrete necessary for in-situ supports. Frame supports may be assembled from prefabricated members. However, the joints between prefabricated members, especially important in this case, are slow to make and require careful alignment and quality control, which increases the cost of construction. For tanks with a capacity of not more than 200 m³, the supporting struts are only installed around the tank periphery. For tanks of a greater capacity with non-bearing

bottoms, they are also installed under the bottom. In the latter case, the tank construction does not depend on the support design.

Water towers using open-work lattice supports made from prefabricated reinforced concrete members (Fig. VII.20*a*) are a better (if not the best) choice. Such supports are assembled from rhombic rod units (Fig. VII.20*b*) and chord units (Fig. VII.20*c*). Each rhombic unit is preassembled from inclined struts and chords rectangular in cross section.

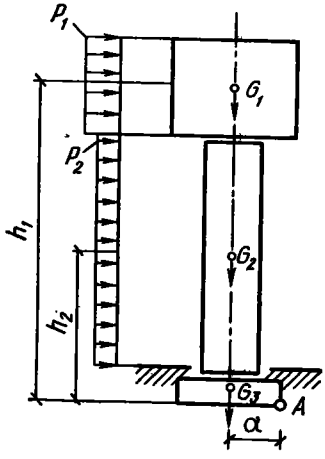


Fig. VII.21. To the design of a water tower

All prefab members have reinforcement stick-outs which are welded at the joints to steel plates during erection. The rhombic units are installed vertically, with their spices touching, and are connected to one another and also to the chord members by erection angles and fishplates. After the units have been properly installed and connected, the joints are concreted (Fig. VII.20*d*).

Such water towers are supported on a circular-strip in-situ r.c. footing. The buried part of the tower enclosing the necessary valves and control equipment is cast integral with the footing.

At the joints, the tank rests on the supporting structure disposed around its periphery.

In each particular case, the choice of the layout and construction type at the time of design is based on cost estimates of not only the tower proper, but also in relation to the cost of the entire project.

We have discussed the water tower types most frequently used in Soviet practice for industrial projects, villages, and small towns. In large cities, the designer has to meet more stringent architectural requirements and to reckon with the need for larger tanks. On the other hand, large cities offer better facilities for erection work. In the circumstances, each water tower may be designed on an individual basis.

The tanks, supports, footings and shelters of water towers should be designed as prescribed in relevant chapters of this text. For the supports and footings, the main loads (Fig. VII.21) are the weight of the water-filled tank, G_1, the dead weight of the support, G_2, the dead weight of the footing and backfill over its top, G_3, the lateral wind pressure applied to the shelter (tank), P_1, and to the support, P_2.

As a whole, the support is subjected to eccentrical compression due to G_1 and G_2 and to the bending moment induced by P_1 and P_2 which reaches its maximum at the footing. If the support is solid, it

is analysed as a single structure of large circular cross section. Frame and open-work lattice supports are analysed as space rod systems.

The size of the footing base is designed to carry the normal force and moment as prescribed in an appropriate standard, depending on the bearing capacity of the soil. As a first trial, the footing base size may be adopted according to the analysis of the tower as a whole for its overturning about the external side of the footing on the leeside (point A in Fig. VII.21). The overturning moment due to wind and the moment of stability due to the weight of the component parts of the structure are found by

$$M_{ov} = \Sigma P_i h_i \quad \text{and} \quad M_{st} = \Sigma G_i a \tag{VII.12}$$

Here, the right-hand sides are the sums of the moments due to the above forces taken about the point A. The wind load is multiplied by the overload factor, $n = 1.3$, and the dead weight of the structure (with the tank empty) is multiplied by the reduction factor, $n = 0.9$.

The coefficient of stability against overturning

$$k = M_{st}/M_{ov} \tag{VII.13}$$

should be at least 1.5.

VII.5. BUNKERS

Bunkers (or shallow bins) are storage containers for particulate materials. They may be prismatic or cylindrical, with the depth h

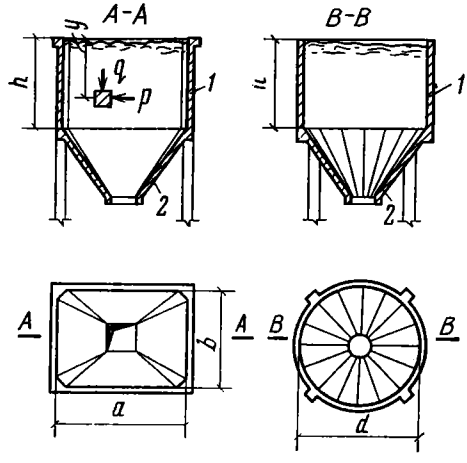


Fig. VII.22. Single in-situ r.c. bunkers
(a) prismatic; (b) cylindrical; 1—wall; 2—hopper

and plan dimensions chosen such that $h \leqslant 1.5a$ (where $a > b$) or $h \leqslant 1.5d$ (Fig. VII.22).

The friction of stored materials against the bunker walls during discharge is insignificant and is, therefore, discarded in design.

Bunker bottoms are usually made in the form of hoppers with an angle exceeding the angle of repose of the particulate material by 5 to 10° in order to ensure the complete discharge of the stored material.

Where different particulate materials are to be stored at one location, use is made of grouped bunkers (Fig. VII.23*a*). If it is neces-

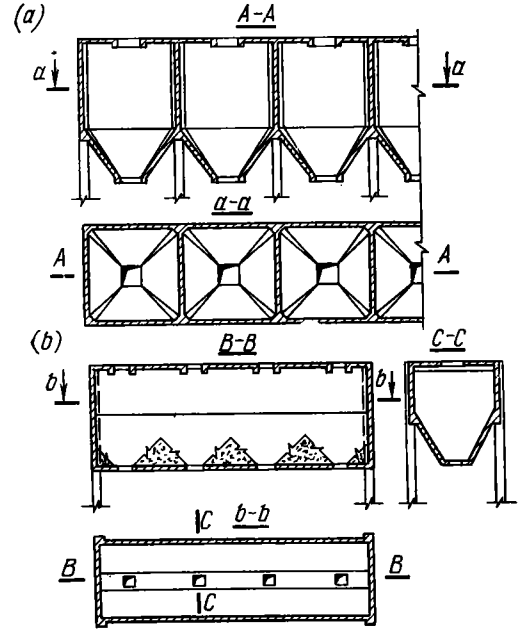


Fig. VII.23. In-situ bunkers
(*a*) grouped; (*b*) trough-type

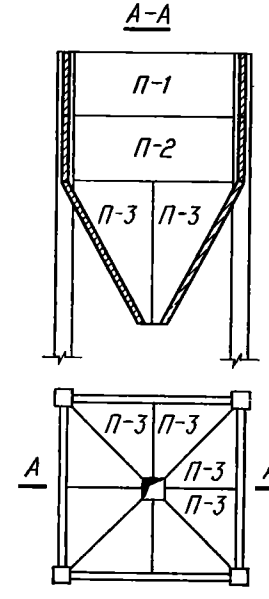


Fig. VII.24. Bunker assembled from prefab members

sary to store large quantities of a material, use is made of trough-type bunkers with several filling and discharge openings (Fig. VII.23*b*).

The filling openings of bunkers are covered by metal grates and the discharge openings have gates. The bunker walls are lined against wear by metal sheets, iron or diabase slabs.

Reinforced concrete bunkers may be cast in-situ or assembled from prefabricated members. The construction types for in-situ r.c. bunkers are shown in Figs. VII.22 and 23.

Precast bunkers are of a standard geometry. Their vertical walls are assembled from rectangular slabs (plain or ribbed), and the hopper walls are assembled from triangular or trapezoidal slabs (Fig. VII.24). During erection, all prefab members are connected by welding together their embedded parts. In some cases, the bottom parts of, or entire hoppers are made of metal.

The basic vertical static pressure of the particulate material, q^b, at a depth y below the surface is $q^b = \gamma y$, where γ is the weight per unit volume of the stored material (Fig. VII.22*a*). The basic horizontal static pressure at the same depth is found by

$$p^b = q^b \operatorname{tg}^2 (45° - \varphi/2) \qquad \text{(VII.14)}$$

where φ is the angle of repose of the stored material, ranging from 30 to 45° depending on the stored material.

The design pressures, q and p, are obtained by multiplying q^b and p^b by the overload factor 1.3. When bunkers are loaded by high-capacity clamshells which drop their load from a certain height, the design pressures should be multiplied by the dynamic factor, $k = 1.4$; if they are filled by conveyers, the dynamic factor is omitted.

The normal and tangential pressures exerted on the inclined walls, p_n and p_t, are determined by the following formulas (Fig. VII.25*a*)

$$p_n = q \cos^2 \alpha + p \sin^2 \alpha \qquad \text{(VII.15)}$$

$$p_t = (q - p) \sin \alpha \cos \alpha \qquad \text{(VII.16)}$$

To them are added the components due to the dead weight, g, of the inclined wall (multiplied by the overload factor 1.1)

$$g_n = g \cos \alpha \text{ and } g_t = g \sin \alpha \qquad \text{(VII.17)}$$

Bunkers are complex space structures which are rather difficult to design even in the elastic stage.

In this respect, trough-type bunkers (Fig. VII.23*b*) present a simpler problem; they may be designed as space folded systems (using V.Z. Vlasov's method).

A single rectangular bunker may fail in any one of several ways, due to local bending in the walls, wall rupture as a result of the internal lateral thrust, hopper breakaway, or general bending with rupture along normal or inclined sections (Fig. VII.26). The bunker should be properly designed to ensure sufficient strength against any likely failure.

Under a load normal to their surface, the walls experience local bending. For local bending, each wall is designed individually. The interaction between the neighbouring walls is taken into account approximately, assuming that the walls are clamped along the contact line. In addition to bending, the bunker is subjected to tension: the vertical walls undergo tension in a horizontal direction, and inclined walls, in both horizontal and vertical directions.

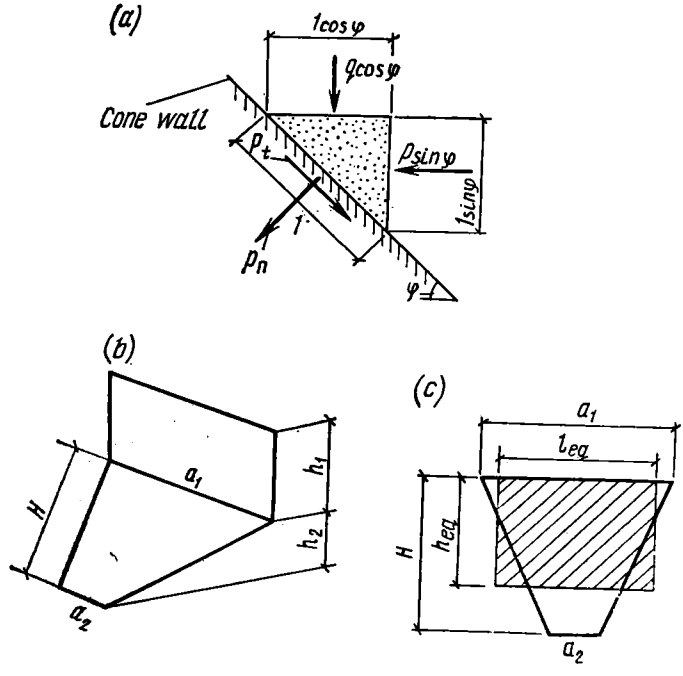


Fig. VII.25. To the design of bunkers

(*a*) to determining pressure on inclined hopper wall; (*b*) dimensions of bunker wall; (*c*) to determining equivalent dimensions of trapezoidal bunker wall

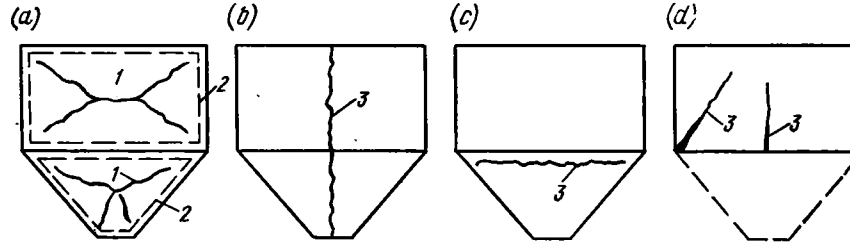


Fig. VII.26. Bunker failure

(*a*) due to bending of the walls at right angles to their plane; (*b*) due to internal lateral thrust; (*c*) hopper breakaway; (*d*) due to general bending with rupture along normal or inclined sections; *1*—outside cracks due to bending of wall at right angles to its plane; *2*—inside cracks due to wall bending at right angles to its plane; *3*—cracks due to in-plane forces

Trapezoidal slabs (Fig. VII.25*b*) are approximately designed for an average uniformly distributed load with a magnitude of

$$p_n^{av} = (1/3)\,\gamma h_2 [\cos^2\alpha + \sin^2\alpha\,\mathrm{tg}^2\,(45° - \varphi/2)] \times [1 + a_2/(a_1 + a_2) + 3h_1/h_2] \qquad \text{(VII.18)}$$

In this case, the trapezoidal shape is replaced by a rectangle having the following design side dimensions (Fig. VII.25*c*)

$$\left.\begin{aligned} l_{des} &= (2/3)\,(a_1 + 2a_2)\,a_1/(a_1 + a_2) \\ h_{des} &= H - a_1\,(a_1 - a_2)/6(a_1 + a_2) \end{aligned}\right\} \qquad \text{(VII.19)}$$

The moments are looked up in reference tables, assuming that the slabs span in two directions and are fixed at three sides.

In the slabs, the tension zones due to the bending moment s are formed near the external side of the bunker in the span, and near the internal side between the ribs. This tends to induce cracks as shown in Fig. VII.26*a*.

The tensile forces are given by

$$N_a = 0.5 p_n^{av} b' \sin\alpha \quad \text{and} \quad N_b = 0.5 p_n^{av} a' \sin\alpha \qquad \text{(VII.20)}$$

where p_n^{av} is the average normal pressure exerted on the wall, a' and b' are the plan dimensions of the bunker at the level of the slab strip in question, and α is the angle of inclination of the hopper wall (for a vertical wall, $\alpha = 90°$).

The slab reinforcement is chosen from analysis of rectangular sections for eccentrical tension.

The amount of wall reinforcement required can be reduced, if the walls are designed for bending at right angles to their plane in the state of ultimate equilibrium (similarly to the manner described in Sec. II.4) on the assumption that linear plastic hinges are formed in the walls at the cracks as shown in Fig. VII.26*a* and that the internal moments are essentially distributed. Since the cracks in this case are rather wide, the above method should be used when the stored material has no corrosive effect on the reinforcing steel.

A check for strength under the action of the horizontal forces, N_a and N_b, (Fig. VII.26*b*) should be carried out individually for the vertical and inclined walls on the assumption that the tensile forces are taken up solely by the horizontal reinforcement.

The hopper is checked for breakaway strength (Fig. VII.26*c*) at the upper base where the tensile forces, N, acting along the slope of the hopper reach their maximum. For the angle, α, these forces (due to the weight of the stored material, G_1, and self-weight of the hopper, G_2) are

$$N = (G_1 + G_2)/2\,(a + b) \sin\alpha \qquad \text{(VII.21)}$$

They are transferred only to the slope reinforcement which is used to connect the hopper to the vertical bunker walls.

The bending strength analysis of the bunker as a whole (Fig. VII.26*d*) is carried out in terms of bending moments at a normal section and in terms of shearing forces at an inclined section similarly to a reinforced concrete beam. In the normal-section analysis, account

should be taken of the horizontal reinforcement in the tension zone crossed by the crack. The service factor for the reinforcement is 0.8. In the inclined-section analysis, allowance is also included for the vertical reinforcement.

The hopper walls of an in-situ r.c. bunker are reinforced with square-mesh flat double fabric (tied or welded) (Fig. VII.27*a*, *b* and *d*). The ribs are additionally reinforced with external angular fabric, and angle brackets with separate bars placed on the inside

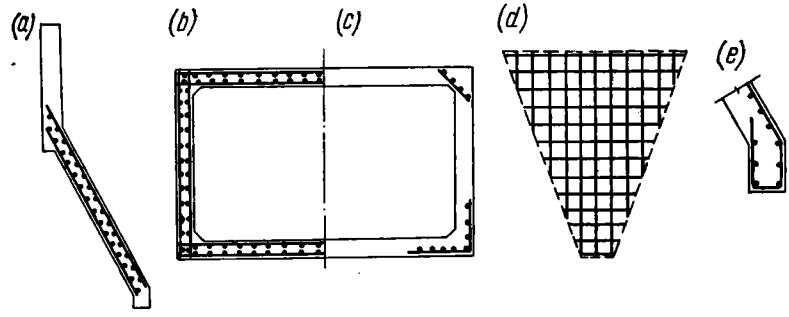


Fig. VII.27. Details of bunker hopper reinforcement
(*a*) section; (*b*) plan (main reinforcement fabric); (*c*) plan (additional corner reinforcement) (*d*) hopper welded-wire fabric; (*e*) reinforcement of discharge opening with welded-wire fabric

(Fig. VII.27*c*). The discharge openings are reinforced with bent fabric (Fig. VII.27*e*). The vertical walls of the bunker are reinforced in the usual manner.

The same principles of reinforcement hold generally for bunkers assembled from prefabricated members.

VII.6. SILOS

Silos (also called deep bins) are storing containers for particulate materials, for which the height, h, and the plan dimensions, a and b, (Fig. VII.22*a*) are chosen such that $h > 1.5a$ (where $a > b$). The friction of the stored material against the silo walls during the discharge is significant and is included in design calculations.

Silos, single or assembled in groups, are commonly built for industrial projects (silos for cement, coal, soda, etc.) and agricultural uses (grain elevators).

In a group of silos arranged under a common roof, there is a charging gallery at the top of the structure, and a discharge space at the bottom (Fig. VII.28).

Silos may be cylindrical or prismatic with four, six or eight faces. As regards the consumption of materials and the cost of erection,

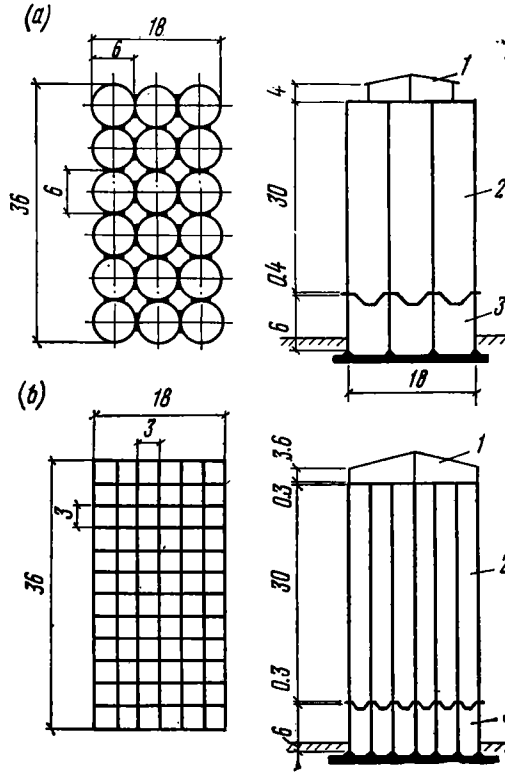


Fig. VII.28. In-situ silo groups
(*a*) with cylindrical silos; (*b*) with square silos; *1*—charging gallery; *2*—silo; *3*—discharge space

the optimal diameter for circular silos is 6 m, and the optimal side length for square-plan silos is from 3 to 4 m. These dimensions have been adopted in practice. If special storage conditions are to be satisfied, silos may be larger in plan. In this case, use is usually made of circular silos with a diameter of 12 m and more.

In the USSR, silo construction types are standardized. The type designs provide for single, grouped, one-row and two-row silos with a diameter of 6 m, and overall height of 15 to 25.8 m, and a storage capacity of 250 to 3 000 m^3, and also silos with a diameter of 12 m, an overall height of 24.6 to 42.6 m, and a storage capacity of 1 700 to 12 000 m^3.

There are three standard types of grain elevators with plan dimensions 36 × 24 m, 36 × 18 m and 24 × 18 m. The side of a silo group may be longer, but it should not exceed 48 m for circular silos and 42 m for square silos. Otherwise, movement joints would be required.

The standard height for silos is 30 m; on high bearing capacity soils (say, rock) it may be extended to 42 m or even more.

In circular silo groups, the interstices between the individual silos are also used to store particulate materials.

Silos may be cast in-situ or assembled from prefabricated members.

Figure VII.28 shows the layout of typical circular and square silo groups. Adjacent circular silos are combined into a single unit for a length of 1.5 to 2 m.

The minimum thickness of in-situ silo walls that permits the slip-form to be moved without causing tears in the concrete is given in Table VII.3.

TABLE VII.3. Minimum Thickness of In-Situ Silo Walls

Silo	Wall thickness, mm	
	exterior walls	interior walls
Circular, 6-m diameter	180	160
Circular, 12-m diameter	240	—
Square	160	150

Precast silos are arranged similarly to in-situ r.c. silos. Figure VII.29 illustrates the plan layout of a square precast silo group. The grouped silos are assembled in tiers from three standard member types, namely a closed-frame space block, an L-shaped block, and a flat block (Fig. VII.29*a* and *b*). The nominal height of each tier is 1.2 m. The prefabricated members may be plain 100 mm thick, or ribbed 60 mm thick. The individual silos are combined into a group by galvanized bolts (Fig. VII.29*c*).

Precast small-diameter (3 m) circular (cylindrical) silos may be erected from rings; such silos, however, are seldom used. Precast silos with a nominal diameter of 6 m are arranged as shown in Fig. VII.30*a*. Each tier comprises four elements connected by bolts (Fig. VII.30*b*). Prefabricated members may be plain (with a thickness of 100 mm) or ribbed (with a thickness of 60 mm and a rib height of 150 mm).

Figure VII.31 shows a likely arrangement for an octagonal silo consisting of two standard member types, namely a closed-frame space block and flat ribbed slabs. The prefabricated members are connected by bolts. This structure has not gained acceptance.

Figure VII.32 shows a precast large-diameter (12 m) circular silo assembled from fluted shell panels with a nominal width of 1.54 m and a nominal height of 3 m. The panels have end-face ribs with external slots intended to accommodate the prestressed hoop reinforce-

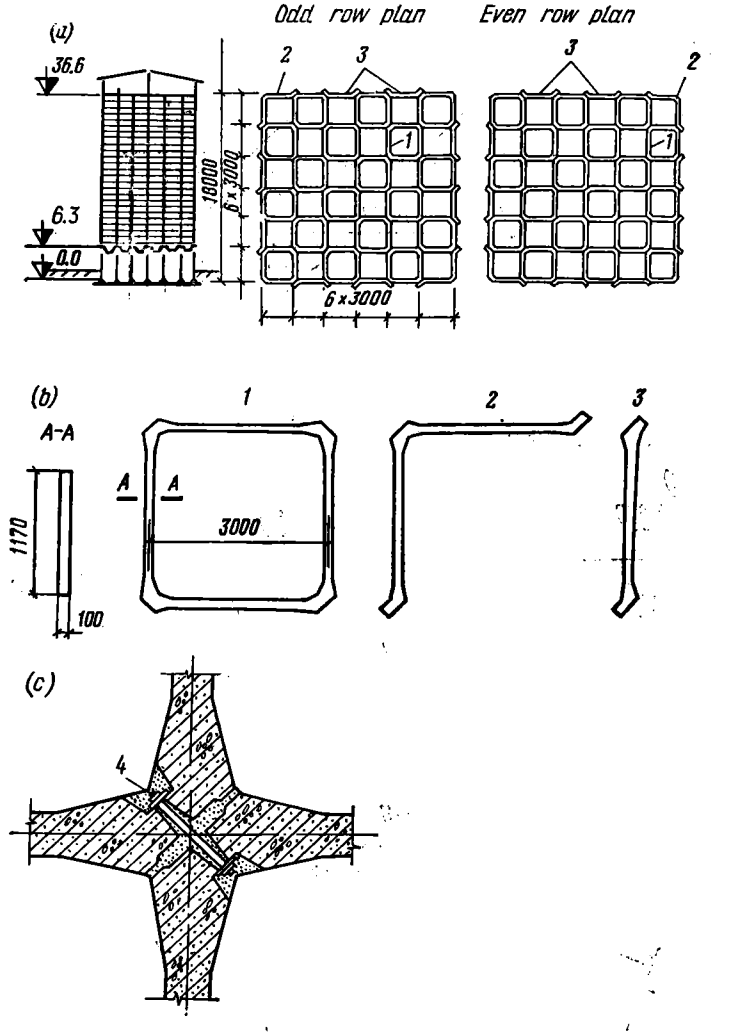


Fig. VII.29. Precast silo groups using square silos
(*a*) sections and plans; (*b*) prefab members; (*c*) connection of prefab members at internal corners; *1* through *3*—prefab members; *4*—bolt

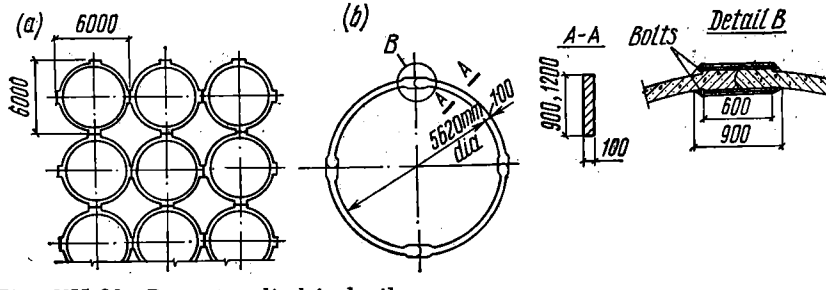


Fig. VII.30. Precast cylindrical silo groups
(*a*) plan detail; (*b*) tier

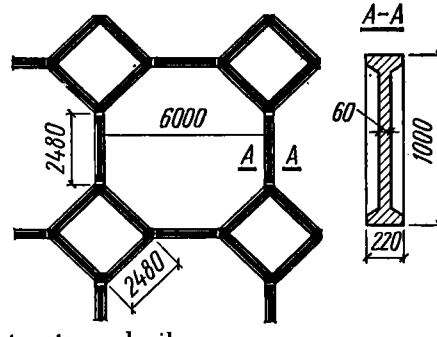


Fig. VII.31. Precast octagonal silo

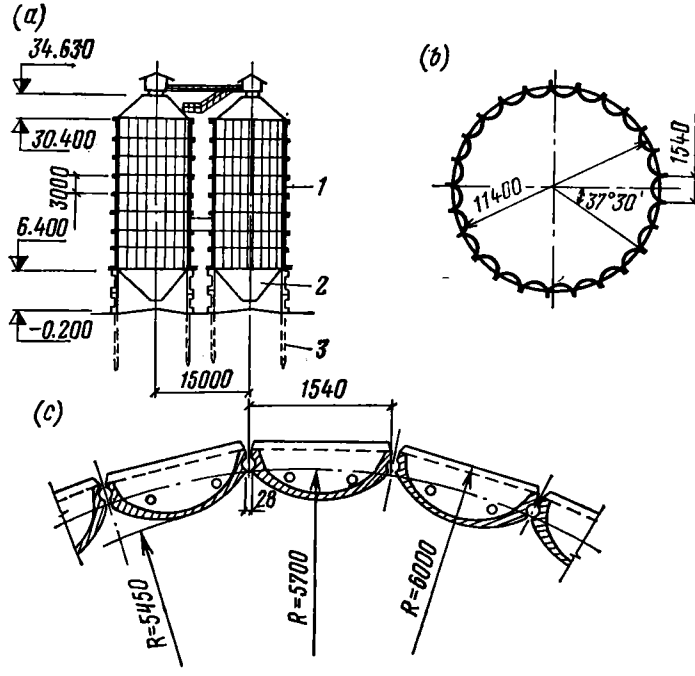


Fig. VII.32. Precast circular silo 12 m in diameter and using fluted walls (*a*) section; (*b*) plan; (*c*) plan detail; *1*—shell panels; *2*—metal hoppers; *3*—reinforced concrete piles

ment of the silo. This reinforcement is prestressed during the preassembly of the tier blocks into larger members on a special bed where the blocks are tensioned from the inside by compressed air. After tensioning, the reinforcement is shotcreted.

The walls of in-situ circular silos are usually carried as far as the footing slab. In the basement, the walls are strengthened by pilasters

which also support hoppers (Fig. VII.33*a*). Use is also made of flat bottoms backfilled on the top and supported by columns of their own (see Fig. VII.28*c*).

In the basement, the precast circular silos and hoppers (which may also be assembled from prefabricated members) are supported on U-type frames (Fig. VII.33*b*). Square silos are supported in a similar manner.

The basic vertical, q^b, and horizontal, p^b, pressure exerted by the stored material at a depth y from the surface is determined by the

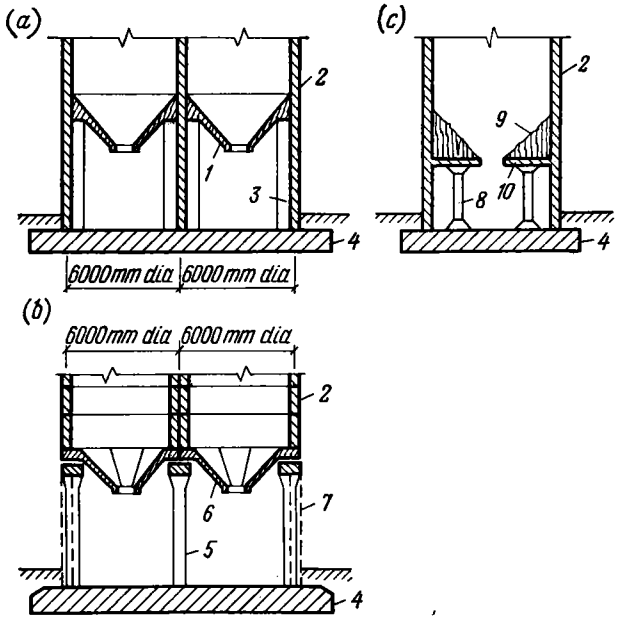


Fig. VII.33. Supports for cylindrical silos
(*a*) in-situ silos with in-situ hoppers; (*b*) precast silos with precast hoppers; (*c*) in-situ with flat bottom; *1*—in-situ hopper; *2*—silo walls; *3*—pilasters; *4*—footing slab; *5*—U-shaped frame; *6*—precast hopper; *7*—filler wall; *8*—columns; *9*—backfill; *10*—flat bottom

following formulas which have been derived theoretically and proved experimentally (Fig. VII.34*a* and *b*)

$$p^b = a\,(\gamma F/fu)\,\{1 - \exp[-(kfy/F)\,y]\} \qquad \text{(VII.22)}$$

$$q^b = p^b/k \qquad \text{(VII.23)}$$

$$k = \operatorname{tg}^2(45° - \varphi/22) \qquad \text{(VII.24)}$$

where F and u are the cross-sectional area and perimeter of the silo, respectively; γ is the weight per unit volume of the stored material; f is the coefficient of friction between the stored material and the

vertical walls, ranging from 0.4 to 0.8 depending on the material; a is the coefficient taking care of the dynamic effect of the material being charged, and also of some other effects overlooked in the theoretical approach; in the design of the horizontal reinforcement in the bottom part of the walls for 2/3 of their height it is assumed that

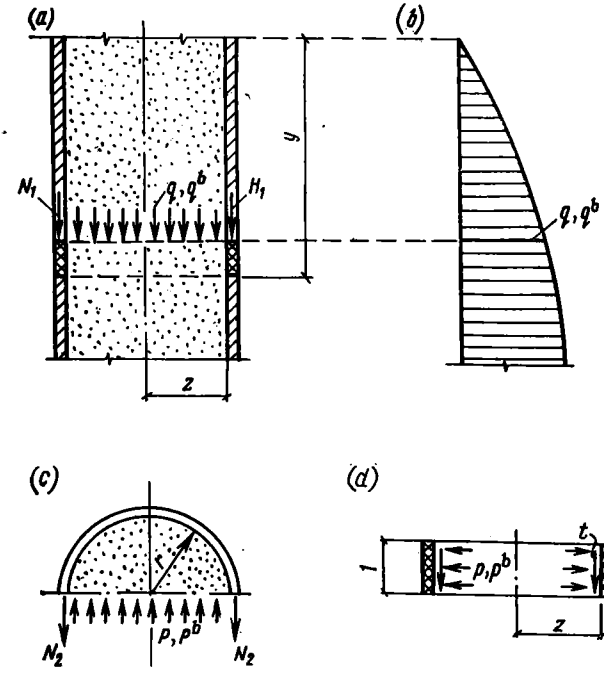


Fig. VII.34. To the design of cylindrical silo walls
(a) vertical section; (b) normal pressure in stored material; (c) to the design a circular force; (d) internal pressure exerted by stored material on ring member

$a = 2$, and in the design of the bottoms and hoppers it is taken that $a = 1.5$; in all other cases, it is assumed that $a = 1$.

The vertical pressure transmitted to the silo wall due to the friction between the stored material and the wall is $t = fp^b$.

The design load due to the stored material is determined by

$$q = q^b\ (n/m) \quad \text{and} \quad p = p^b\ (n/m) \tag{VII.25}$$

where n is the overload factor, and m is the service factor for the structure.

The overload factor for particulate materials is $n = 1.3$. In the design of the bottom zone of the silo walls, the columns below the

silo and the footing slabs subjected to compression, the design load due to the weight of the stored material is multiplied by 0.9.

In designing the horizontal reinforcement of the bottom part of the wall (at 2/3 of its height) for the interior circular silos arranged as in-line groups and also for the exterior and interior rectangular silos with a side length of up to 4 m, the service factor is taken as $m = 2$. For flat bottoms without backfilling and hoppers, $m = 1.3$, and for flat bottoms with backfilling having a thickness of 1.5 m and more, $m = 2$. In all other cases, $m = 1$.

The wall of a circular silo is in tension due to a horizontal force under the action of the force (Fig. VII.34*c*)

$$N_2 = pr \qquad \text{(VII.26)}$$

The wall of a silo of any shape in plan is compressed in a vertical direction by the force per metre run (Fig. VII.34*a*)

$$N_1 = (F/u)\, n\, (\gamma y - q) \qquad \text{(VII.27)}$$

The normal pressure along the hopper slope is found by Eq. (VII.15) as in bunkers.

The cross-sectional area of the horizontal reinforcement in circular silos per unit wall height is given by

$$F_s = N_2/R_s$$

The strength of the silo walls as compression members is checked in the most loaded regions (near the hopper or at the footing slab), in terms of the force, N_1 (per metre of the perimeter of the silo horizontal section), with allowance for the vertical load due to all the structures situated above.

A square in-situ r.c. silo is analysed at each tier as a closed frame subjected to the internal pressure, q (Fig. VII.35*a*). The wall experiences tension due to the force, $ql/2$, and bending due to the moment, $ql^2/12$, at the corners and the moment, $ql^2/24$, in the span (Fig. VII.35*b* through *d*).

The concrete used for silos should be at least of M-200 brand.

The walls of in-situ silos are usually cast in sliding formwork supported by jack frames. Because of this, they are reinforced in a horizontal direction with relatively short separate bars spaced 100 to 200 mm apart; the concrete cover should be at least 20 mm thick. The bars are joined at random with a lap splice $60d + 200$ mm long. In small-diameter silos, the walls are not prestressed and the reinforcing steel is class A-II deformed bars.

The walls of circular silos less than 6 m in diameter can be reinforced with single horizontal steel only (Fig. VII.36*a*). The exterior walls, however, should be doubly reinforced for at least 2/3 of their height (Fig. VII.36*b*) to carry bending moments arising when the rows of silos are filled in a staggered pattern.

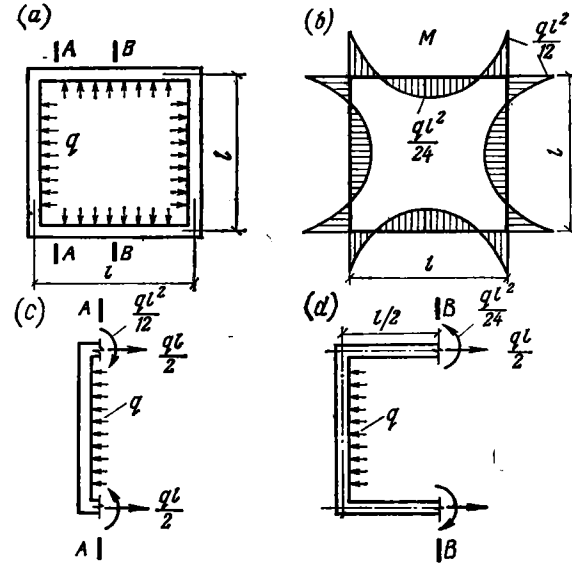


Fig. VII.35. To the design of square silo walls
(*a*) design model; (*b*) bending moment diagram; (*c*) moments and forces in walls at section *A-A*; (*d*) same, at *B-B*

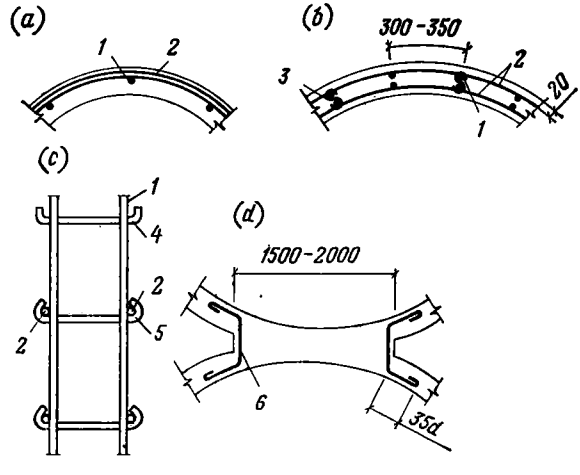


Fig. VII.36. Details of reinforcement of in-situ cylindrical silo walls
(*a*) single reinforcement; (*b*) double reinforcement; (c) vertical tied-bar mat; (*d*) additional reinforcement near adjacent silos; *1*—vertical bars; *2*—load-bearing hoop reinforcement; *3*—connecting studs; *4* and *5*—stirrup before and after placing horizontal bars; *6*—additional bars

The vertical bars are taken 10 mm in diameter and spaced 300 to 350 mm apart in the exterior silo walls, and 400 to 500 mm apart in the interior walls. The total area of the vertical bars should be at least 0.4% of the concrete area. Some vertical bars are installed as tied-bar mats (Fig. VII.36*c*) spaced 1 to 1.5 m apart to secure the horizontal reinforcement in the design position during concreting. The vertical bars are joined at random with a lap splice 35*d* long.

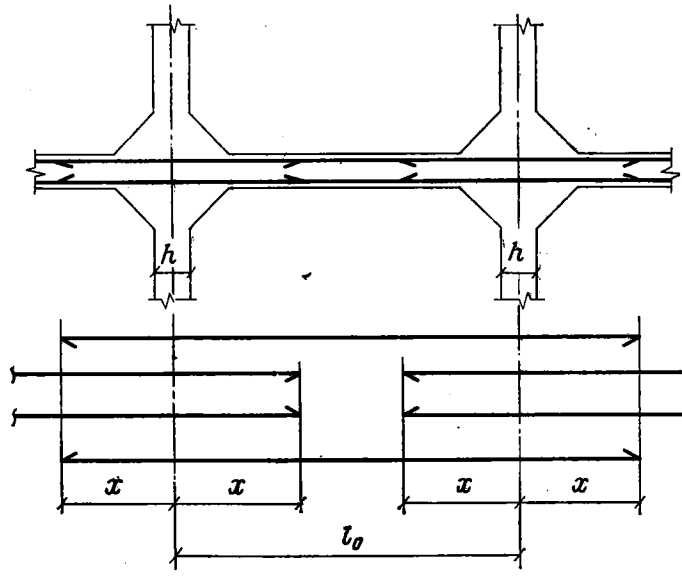


Fig. VII.37. Reinforcement of in-situ square silo walls

At the intersections, the vertical and horizontal bars are tied with binding wire. In the case of double reinforcement (Fig. VII.36*c*), both mats are connected by transverse stirrups at least 3 mm in diameter.

Where adjacent silos are interconnected, the entering corners are additionally reinforced with steel bars (Fig. VII.36*d*) of the same diameter and spacing as the main hoop bars.

The walls of square in-situ silos are doubly reinforced (Fig. VII.37) because the pressure may be exerted on the intermediate walls from either side and because the horizontal reinforcement in the corners should resist bending moments which are twice the moments in the span (see Fig. VII.35).

Precast silos are reinforced in a similar manner. Prefab members cast off-site can be reinforced with high-strength wire which cuts down the steel consumption.

Incipient cracking and crack-opening analysis for silo walls is carried out as prescribed for members in tension.

Experience has shown that if in-situ silo walls are made of M-200 concrete and reinforced with class A-II nonprestressed deformed bars with a percentage of reinforcement of not over 0.7%, the crack width will not exceed the allowable value (which is 0.2 mm for basic loads).

VII.7. RETAINING WALLS

Retaining walls made of reinforced concrete are more economical than masonry and concrete gravity walls. They are usually assembled

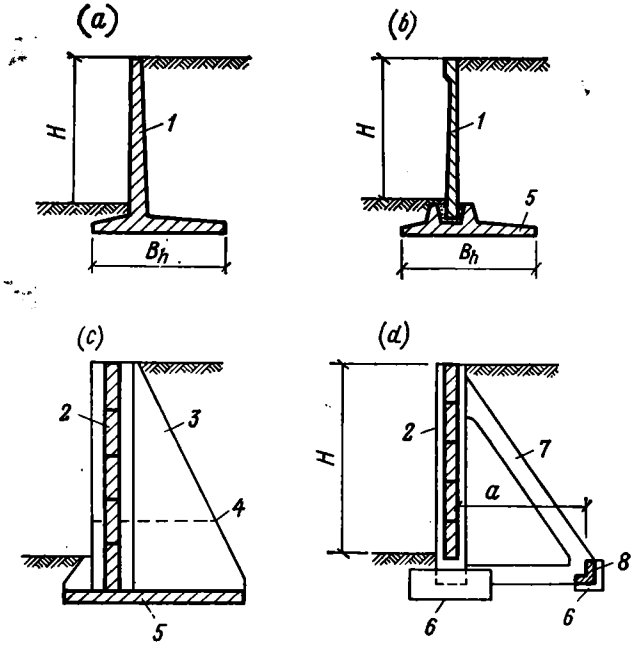


Fig. VII.38. Construction types of precast retaining walls
(*a*) single-member cantilever tee wall; (*b*) double-member cantilever tee wall; (*c*) counterfort wall; (*d*) tieback wall; *1*—prefab members; *2*—wall slabs; *3*—prefab counterfort; *4*—joint between prefab counterfort members; *5*—heel slab; *6*—frame supports; *7*—frames; *8*—anchor beam

from prefabricated members. The basic types of retaining walls are the cantilever tee wall, the counterfort wall and the tieback wall (Fig. VII.38).

Cantilever tee walls are used when the overall height of the retaining wall does not exceed 4.5 m. For higher walls, the counterfort or

tieback types are more economical. Cantilever tee walls may be manufactured as single blocks 2 to 3 m long (Fig. VII.38*a*). In the USSR, there have been developed type-design precast cantilever tee walls assembled from two prefab members, namely a wall (face) slab (also called a stem) and a heel slab (Fig. VII.38*b*). The standard walls are intended for a backfill height H of 1.2, 1.8, 2.4, 3 and 3.6 m. The nominal length of the wall slabs is 3 m, and that of the heel slabs is 3 and 1.5 m. The heel base width of a heel slab B is 2.2, 2.5, 3.1 and 3.7 m. To increase its stability against sliding, the heel base may be installed at an angle of 7°.

Structures of other types (Fig. VII.38*c* and *d*) are made up of prefab wall slabs installed in the slots of counterforts or frames. The counterforts are assembled from two or three component parts. They are laid at a spacing of two or three metres on precast heel slabs and connected to them by welding together embedded steel parts.

The frames of tieback retaining walls are supported on isolated footings and spaced 4 to 5 m apart. The anchor beam is intended to retain the entire structure against sliding under the action of the horizontal earth pressure. The distance, a, (see Fig. VII.38*c*) is taken as $0.3H$ to $0.6H$ if the angle of repose of the soil is from 30 to 45°, where H is the height of the retained soil.

In practice, use is also made of other types of retaining walls, namely those having an anchoring key below the heel slab or with a heel base sloping in the reverse direction to improve the stability against sliding in a horizontal direction, walls with wall slabs having intermediate relieving platforms made from the inside and disposed over the wall height, walls with ribbed surfaces to reduce the amount of concrete, etc. Sometimes, use is made of crib walls assembled from small reinforced concrete stringers, filled with rock. They require less materials than other retaining walls, but are more expensive to erect.

According to equations given in a course on the strength of materials, the earth pressure depends on the unit weight of the soil, γ, the angle of repose of the soil, φ, the angle of inclination of the retaining wall back face, and the angle of inclination of the backfill over the retaining wall. In the simplest case when the back face is vertical and the earth surface above the wall is horizontal, the resultant of the lateral earth pressure (that is, its basic value) per metre of the wall length (Fig. VII.39) is found by

$$E = 0.5\gamma H^2 \operatorname{tg}^2 (45° - \varphi/2) \qquad \text{(VII.28)}$$

The earth pressure is assumed to be linearly distributed over the wall height. So its magnitude at the bottom is $2E/H_0$, and the resultant is assumed to be applied at a distance $H/3$ from the bottom.

Under normal conditions, the unit weight of the soil, γ, ranges between 1.6 and 1.9 t/m^3, and the angle of repose, between 30 and

45°. The overload factor for the lateral earth pressure is taken as 1.2.

The uniformly distributed surcharge, p_{sur}, applied at the top of the backfill, taken with the overload factor 1.3 applied, is referred

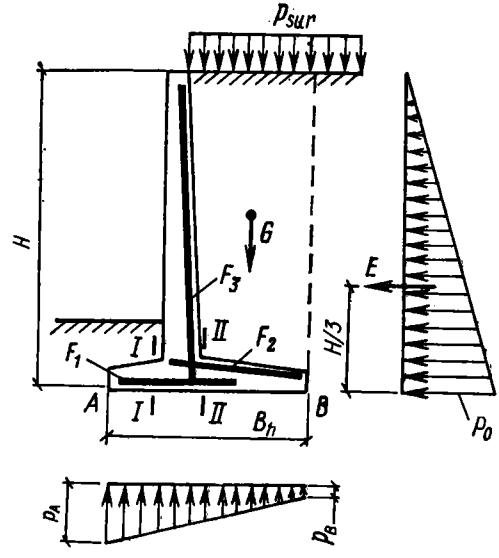


Fig. VII.39. To the design of a cantilever tee wall

to the weight of the soil layer with a height of $h_0 = p_{sur}/\gamma$ and is allowed for when determining the resultant of the pressure exerted on the wall by the following equation

$$E = 0.5\gamma H (H + 2h_0) \operatorname{tg}^2 (45° - \varphi/2) \qquad \text{(VII.29)}$$

As a first trial, the width of the heel slab, B, and its boom are chosen so that the maximum edge pressure exerted on the soil beneath the footing, given by

$$\left.\begin{matrix} p_A \\ p_B \end{matrix}\right\} = \Sigma G/F \pm \Sigma M/W \qquad \text{(VII.30)}$$

does not exceed $1.2\,R_0$, provided the condition $p_{av} = F/N \leqslant R_0$ is met and the stability of the wall against overturning and sliding is guaranteed, that is

$$M_{st}/M_{ov} \geqslant 1.5 \quad \text{and} \quad \Sigma\, Gf/E \geqslant 1.2 \qquad \text{(VII.31)}$$

Here, M is the moment due to all design forces acting on the wall about the centroid of the heel slab; F and W are the area and moment

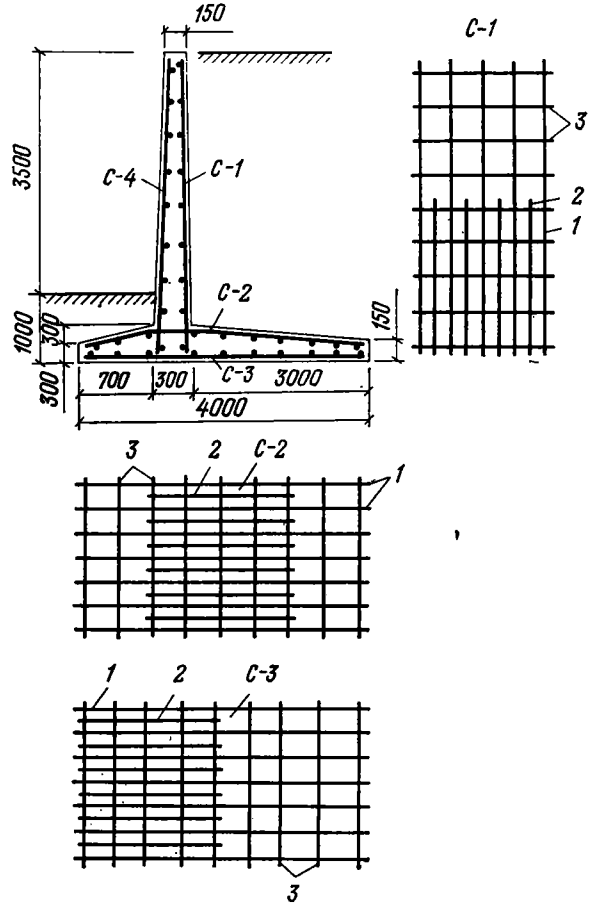


Fig. VII.40. Reinforcement of a cantilever tee wall
1—through load-bearing bars; *2*—additional load-bearing bars; *3*—erection bars

of resistance of the heel slab, respectively; R_0 is the assumed design pressure on the soil; M_{ov} is the overturning moment due to the earth pressure about the toe (point A in Fig. VII.39); M_{st} is the moment of stability due to the vertical loads (the weight of the wall and backfill over the heel slab) about the same point; ΣG is the sum of the vertical loads; and f is the coefficient of friction between concrete and soil ranging between 0.3 and 0.6, depending on the soil and its condition.

It is advisable that the pressure exerted on the soil at the heel (point B in Fig. VII.39) should approximately be equal to zero.

The final dimensions of the heel slab are found by design in terms of the soil bearing capacity and soil deformation as prescribed in a relevant standard.

The face and back projections of the heel slab are designed for bending as cantilevers fixed at sections *I-I* and *II-II*, respectively. The face cantilever is subjected to the soil pressure from beneath, and the back cantilever is additionally subjected to the pressure of the backfill over its top. The design amount of reinforcement, F_1 and F_2, is distributed near the bottom and top of the heel slab, respectively (see Fig. VII.39).

For the pressure, E, the wall itself is designed as a bending cantilever fixed at the heel slab. The design amount of reinforcement, F_3, is disposed near the back surface of the wall.

Figure VII.40 shows a likely reinforcement for a cantilever retaining wall. The load-bearing bars are combined into bar mats by erection reinforcement. To save steel, some load-bearing bars are placed only where maximum moments are likely to arise. The bar mat C-4 is installed to suit structural requirements.

The precast wall panels of Fig. VII.38*b* and *c* are designed for the lateral earth pressure as slabs acting as beams with a span taken between adjacent counterforts (frames). The counterfort is designed as a cantilever fixed at the heel slab. The joints between the prefabricated members are designed to resist the moments and other forces which are transmitted through them.

VII.8. UNDERGROUND CONDUITS AND DEEPLY BURIED STRUCTURES

Within the premises of industrial and civil projects, underground conduits are mostly installed at shallow depths. They are intended for heat supply lines, gas mains, steam lines, oil lines, water supply and sewage disposal systems, compressed air lines, and electric cables.

The internal height of low conduits is less than 1 200 mm, that of medium-height conduits is from 1 400 to 1 600 mm, and that of high conduits (also called passageway conduits, tunnels, or headers) is at least 1 800 mm. Tunnels are installed with a longitudinal slope to facilitate removal of accidental water, and are equipped with lighting, ventilation, signalization, fire-fighting means and other facilities.

The roofs of buried conduits should be at least 0.7 m below the surface (or 0.5 m if they run below paved road). Along their length, conduits have movement joints where they adjoin servicing chambers and compensating niches, and where the soil conditions undergo sudden changes. Movement joints should be spaced not more than 50 m apart.

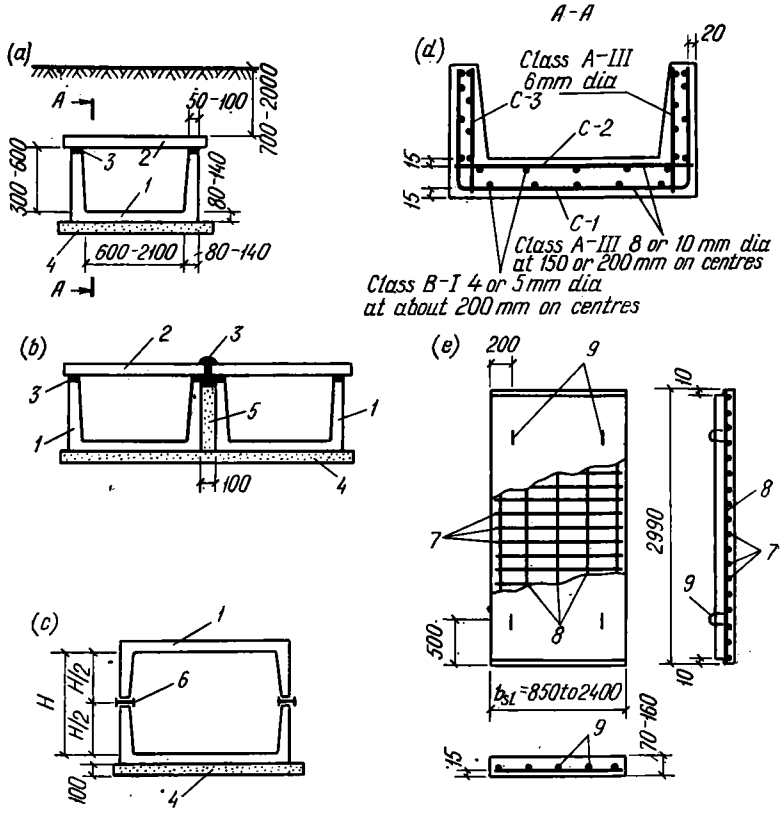


Fig. VII.41. Low conduits assembled from U-members
(*a*) single-duct conduit; (*b*) double-duct conduit; (*c*) single-duct conduit assembled from two U-members; (*d*) reinforcement of U-member; (*e*) reinforcement of prefab slab; *1*—U-member; *2*—roof slab; *3*—cement grout; *4*—compacted sand; *5*—sand-filled gap between U-members; *6*—steel spacer; *7*—load-bearing bars; *8*—erection bars; *9*—slab lifting eyes

As compared with masonry or plain concrete structures, reinforced concrete conduits require less materials and are less expensive. Also, the use of prefabricated reinforced concrete members cuts down the labour consumption and time of construction.

Figure VII.41 shows Soviet-made type-design low conduits assembled from prefabricated low-U-shaped members and deck slabs.

To reduce the number of standard U-member sizes, the conduits of Fig. VII.41*a* and *b* are made 300, 450 and 600 mm high and 600, 900, 1 200, 1 500 and 2 100 mm wide. The nominal length of precast conduits is 3 000 mm (the structural length, that is, the nominal length minus the gaps, is 2 970 mm).

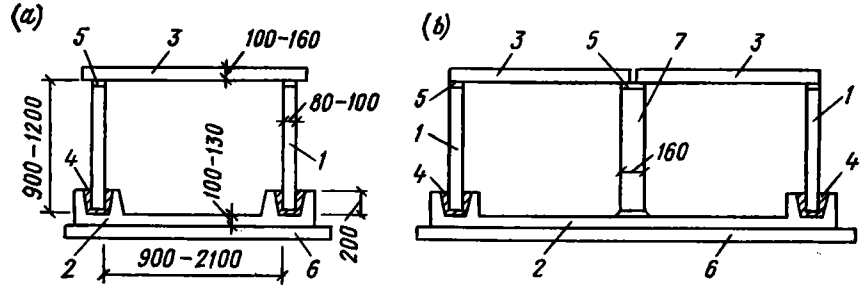


Fig. VII.42. Low conduits assembled from flat slabs
(*a*) single-duct conduit; (*b*) double-duct conduit; *1*—wall slab; *2*—bottom slab; *3*—roof slab; *4*—concreted slot; *5*—cement grout; *6*—compacted sand; *7*—wall partition

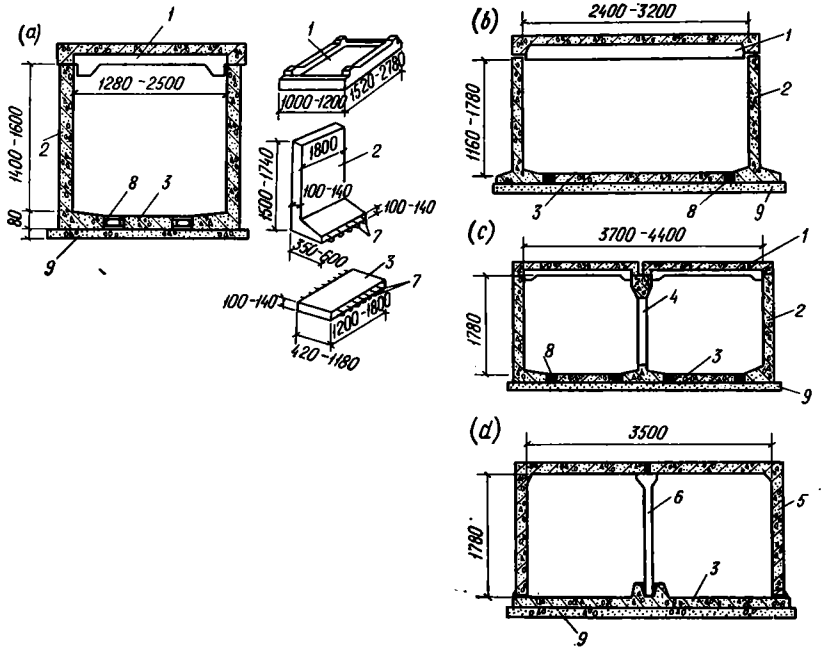


Fig. VII.43. Medium-height conduits
(*a*) single-duct conduit up to 2.5 m wide; (*b*) single-duct conduit 2.4 to 3.2 m wide; (*c*) and (*d*) double-duct conduits; *1*—roof slabs; *2*—wall blocks; *3*—bottom slabs; *4*—wall partition; *5*—L-shaped blocks; *6*—intermediate struts; *7*—loop reinforcement stick-outs; *8*—in-situ bottom portions; *9*—compacted sand and broken stone

Figure VII.42 shows Soviet-made type-design low conduits assembled from flat slabs. They are 900 and 1 200 mm high and 900, 1 200, 1 500 and 2 100 mm wide. The nominal length of wall and roof slabs and also that of the bottom slabs in single-duct conduits is 3 000 mm;

in double-duct conduits, the nominal length of bottom slabs is half that value. The wall slabs are rigidly embedded into the bottom slab slots.

Standard medium-height conduits are designed along the lines illustrated in Fig. VII.42*a* and *b*. In practice, use is also made of other construction types (Fig. VII.43) with ribbed roof slabs, wall blocks combined with support slabs, and L-shaped blocks (Fig. VII.43*d*).

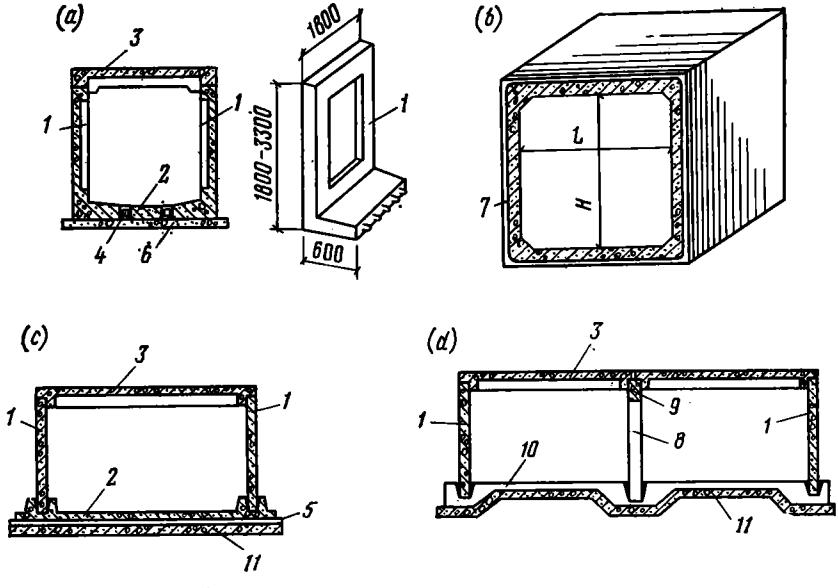


Fig. VII.44. Tunnels

(*a*) small-size tunnels assembled from three standard members; (*b*) small-size tunnels using one standard member (closed frame); (*c*) single-duct large-size tunnel; (*d*) double-duct large-size tunnel; *1*—wall blocks; *2*—bottom slab; *3*—roof slab; *4*—in-situ bottom portion; *5*—levelling layer; *6*—concrete layer; *7*—prefab closed-frame block; *8*—column; *9*—beam; *10*—in-situ bottom; *11*—concrete layer

Here, the bottom slabs are connected to the support slabs of the wall blocks by castellated joints (known in Soviet practice as Perederiy's joints) constituted by loop reinforcement stick-outs locked by tie bars (Fig. VII.43*a*). Double-duct conduits are divided along their length by open-work partitions or struts.

The clear width of tunnels (passageway conduits) (Fig. VII.44) is 1 500, 1 800, 2 100, 2 400, 3 000, 3 600 and 4 200 mm, and the clear height is 2 100, 2 400 and 3 000 mm. The construction types of standardized precast tunnels are shown in Fig. VII.44*c* and *d*.

The prefabricated members of conduits are made of M-300 concrete and reinforced with welded-bar mats or welded-wire fabric using class A-III bars and class B-I plain wire.

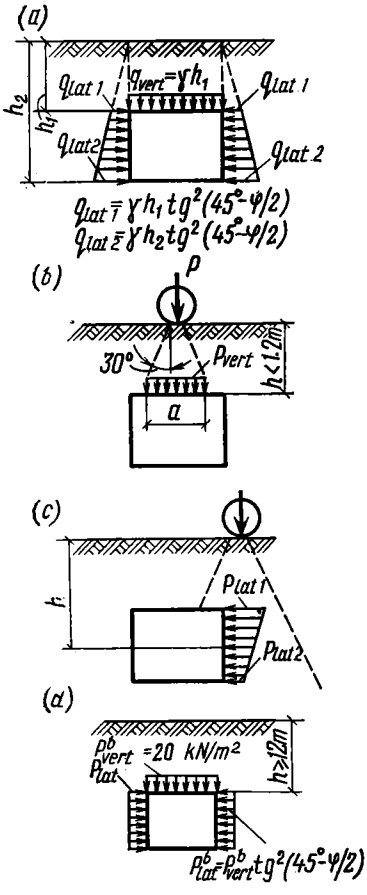


Fig. VII.45. Loads imposed on buried conduits

(*a*) vertical and lateral earth pressure; (*b*) vertical pressure due to surface live load (at $h < 1.2$ m); (*c*) lateral pressure under same conditions; (*d*) vertical and lateral pressure due to traffic (at $h \geqslant 1.2$ m)

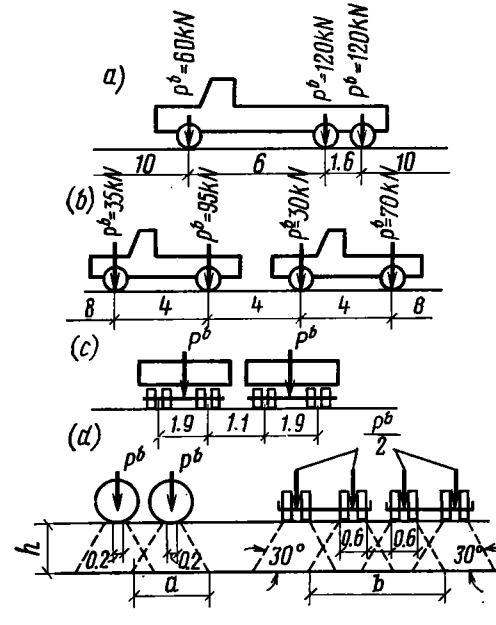


Fig. VII.46. Truck load

(*a*) axle position and load per axle for $H = 30$ load; (*b*) same, for $H = 10$ load; (*c*) distance between truck wheels in transverse direction; (*d*) propagation of pressure from tire contact area

Shallow-depth conduits are laid in trenches. In this case, the design vertical earth pressure, q_{vert}, and the design lateral pressure, q_{lat}, exerted on the roof and walls (Fig. VII.45) are determined by

$$q_{vert} = n\gamma h \quad \text{and} \quad q_{lat} = q_{vert}\,\mathrm{tg}^2\,(45° - \varphi^b/2) \qquad \text{(VII.32)}$$

where h is the distance from the earth surface in metres, γ is the load due to the unit volume of the soil taken as 16 to 20 kN/m³ depending

on the kind and moisture content of the soil, n is the overload factor taken as 1.2, and φ^b is the basic angle of internal friction of the soil, ranging between 25 and 45°.

Conduits should be designed with allowance for wheel loads applied at the surface. For mains services, the wheel load is $H = 30$ (two-lane traffic of three-axle trucks with a maximum load per axle $P^b = 120$ kN, Fig. VII.46*a*), for other systems, the load is $H = 10$ (one-lane traffic of two-axle trucks with a maximum load per axle $P^b = 95$ kN, Fig. VII.46*b*). The overload factor for the wheel load is 1.4. Thus, the design wheel load is $P = 1.4\ P^b$.

The distance between the truck wheels in the transverse direction is taken as shown in Fig. VII.46*c*. The length of tire contact area is 0.2 m in the longitudinal direction and 0.6 m in the transverse direction (Fig. VII.46*d*).

The concentrated load propagates at 30° to the vertical in the soil, and at 45° to the vertical in the road pavement.

The vertical pressure at the depth $h < 1.2$ m from the surface is given by $p_{vert} = P/ab$, where a and b are the sides of the pressure area at the depth h (Figs. VII.45*b* and VII.46*d*). When $h \geqslant 1.2$ m (Fig. VII.45*d*), the design vertical pressure (multiplied by the overload factor 1.4) is $p_{vert} = 28$ kN/m^2. The lateral earth pressure is found in either case by the second formula of Eqs. (VII.32), using the diagrams shown in Fig. VII.45*c* and *d*.

In buried structures, all of the vertical load due to the roof and walls is balanced by the soil reaction which is deemed to be uniformly distributed over the bottom base.

The roof slabs for conduits are analysed as a single-span beam hinged at the ends. In single-duct conduits, such as shown in Figs. VII.41*a*, VII.42*a*, VII.43*a* and VII.44*a* and *c*, the walls rigidly connected to the bottom are analysed as a U-shaped frame with a spreader (Fig. VII.47*a*). When the roof is removed (during construction or repair), the frame is analysed without the top spreader (Fig. VII.47*b*). Other conduits are analysed similarly, with allowance for their constructional features.

Sometimes, industrial and civil projects use deeply buried structures which are built without open trenches. Small buried structures may be of rectangular, circular, oval or similar cross section. In this case, the lining is made of simple reinforced concrete members.

For spans of 10 m or more, buried structures may be of arch construction, with short vertical walls and a flat or overturned arch bottom (Fig. VII.48). Such structures are generally cast in-situ.

Deeply buried structures are subjected only to the earth load within the so-called pressure vault, rather than to the load due to the weight of the entire earth layer. Above the pressure vault, there is a relieving vault which resists the load imposed by the weight of the remaining part of the soil.

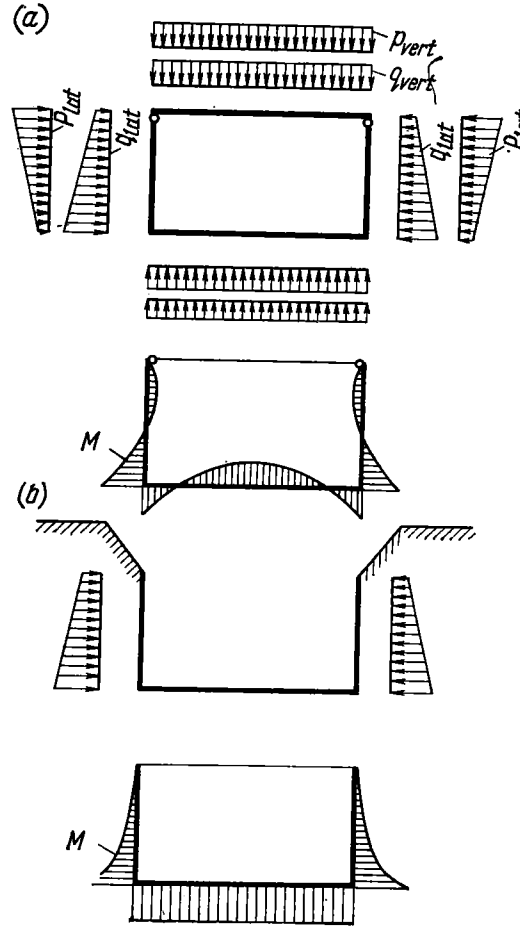


Fig. VII.47. To the design of conduits
(a) overturned U-shaped frame with spreader; (b) same, without spreader

According to the investigations carried out by M.M. Protodyakonov of the Soviet Union, the maximum slope of the pit sides should not exceed an angle of $(45° - \varphi/2)$ with the vertical (where φ is the angle of internal friction of the soil), and the height of the pressure vault should be

$$h_1 = a_1/f_{st} \tag{VII.33}$$

where (see Fig. VII.48)

$$a_1 = a + (f_1 + h_y) \operatorname{tg} (45° - \varphi/2) \tag{VII.34}$$

and f_{st} is the strength coefficient. According to Protodyakonov it ranges between 0.8 and 4 for soft soils (such as clayey soils, sand, conglomerates, low-strength slates, sandstone, and the like).

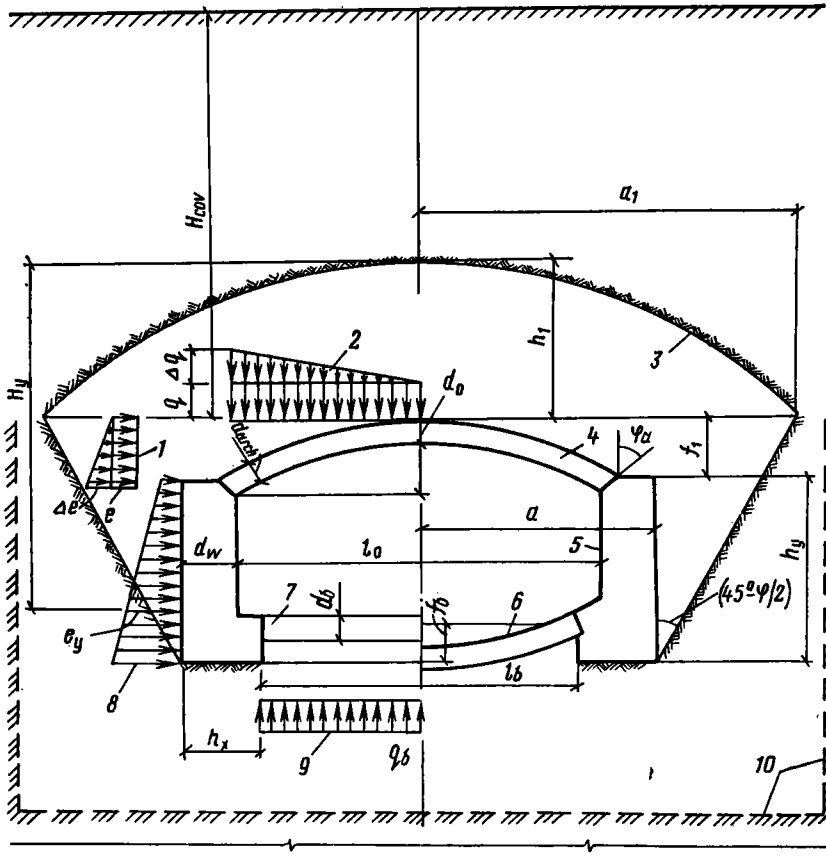


Fig. VII.48. Loading system for an in-situ deeply buried conduit

1—lateral load imposed on arch; *2*—vertical load imposed on arch; *3*—pressure vault boundary; *4*—elastic vault; *5*—rigid walls; *6*—overturned arch bottom; *7*—flat bottom on compacted sand and broken stone layer; *8*—lateral load imposed on wall; *9*—load due to soil upward thrust; *10*—boundary of soil which is deformed actively

A structure is said to be deeply buried when $H_{cov} \geqslant 5a_1$ (where H_{cov} is the soil cover above the structure).

Being subjected to the vertical earth pressure, a buried structure transfers it to the surrounding soil via the external side surfaces and the wall bottoms, thereby interacting with the most compressed surrounding soil.

Figure VII.48 shows the design model proposed by S.S. Davydov for deeply buried structures. In the model, the arch is subjected to the vertical downward loads, q and Δq, and the lateral loads, e and Δe, the walls are subjected to the lateral load, e_y, and the bottom to the vertical upward load, q_b.

The load imposed on the arch is found by

$$\left.\begin{aligned} q &= h_1\gamma_s + d_0\gamma_a \\ \Delta q &= (h_1 + f_1)\,\gamma_s + d_n/\cos\varphi_n\gamma_a - q \\ e &= h_1\gamma_s \operatorname{tg}^2(45^\circ - \varphi/2) \\ \Delta e &= f_1\gamma_s \operatorname{tg}^2(45^\circ - \varphi/2) \end{aligned}\right\} \qquad \text{(VII.35)}$$

and that applied to the walls is

$$e_y = \mu_0 H_y \gamma_s/(1 - \mu_0) \qquad \text{(VII.36)}$$

where μ_0 is Poisson's ratio equal to 0.3 for sand and 0.4 for clay.

The upward pressure, q_b, is determined using the methods of soil mechanics. If it is balanced by the weight of the bottom and compact-

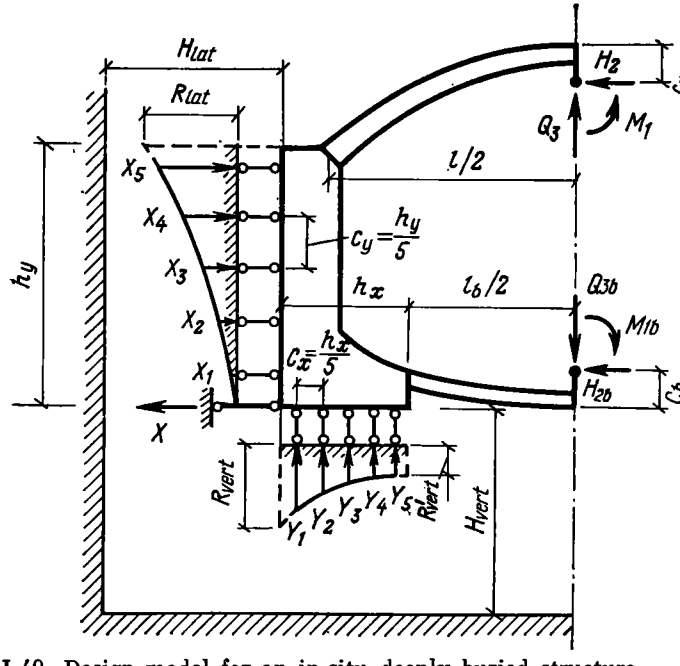


Fig. VII.49. Design model for an in-situ deeply buried structure

ed sand and gravel beneath the bottom with a factor of safety from 1.3 to 1.5, the bottom may be made horizontal. Otherwise, it is given the shape of an overturned arch.

In the design model, the interaction between the walls and the surrounding soil is taken care of by introducing concentrated constraints, $X_1, \ldots, X_5$ at the side and $Y_1, \ldots, Y_5$ at the bottom (Fig. VII.49). At the bottom, there is also friction between the structure and the soil, which is allowed for by introducing the horizontal constraint, X, at the bottom level.

For a symmetric structure loaded symmetrically, the basic system involves 17 unknowns, namely the moments and forces, M_1, M_{1b}, H_2 and H_{2b}; ten soil thrust forces, X_i and Y_i; the force of friction at the bottom, X; and two deflections at the point of fixation (the external wall corner at the bottom level), that is, the angular deflection, φ_0, and the vertical settlement, y_0. Here, we should take into account that $Q_3 = Q_{3b} = 0$.

The unknown forces, X_i and Y_i, and the angular deflection, φ_0, are determined subject to the condition that the soil and the wall (which is part of the buried structure, so that its own deformation may be neglected) deform together, and also to the three conditions of equilibrium of the system. The system may be made statically determinate in advance (in which case it is advisable to apply the unknowns M_1, H_2, M_{1b} and H_{2b} at the elastic centres of the arches). The deflection, y_0, is symmetrical in this system and has no effect on the state of stress.

The diagrams for the thrust of the soil acting in the elastic stage are shown in Fig. VII.49. If at some points the earth pressures, R_{lat} and R_{vert}, reach the plastic state, this should be allowed for by proper adjustments in repeated calculations.

As a first trial, the dimensions of the structure (Fig. VII.48) may be taken as

$$\left.\begin{aligned} d_0 &= 0.06\sqrt{l_0/f_0}\,(1+\sqrt{l_0/f_{st}})\,\sqrt{150/R} \\ d_B &= 0.5d_0 \text{ to } 0.8d_0 \\ d_a &= 1.25d_0 \text{ to } 1.5d_0 \\ d_w &= d_a \text{ to } 2d_a \\ h_x &= d_w \text{ to } 1.5d_w \\ f_b &= l_b/10 \text{ to } l_b/5 \end{aligned}\right\} \qquad \text{(VII.37)}$$

Here, R is the concrete brand number, and f_0 is the rise of the elastic vault; the lower values correspond to a harder soil and the higher values to a softer soil.

The axis of the top arch layer is assumed to follow a quadratic parabola. If necessary, it may be adjusted to match the pressure curve.

Once the unknowns are determined, the cross-sectional dimensions and the amount of reinforcement of the arch, walls and bottom are found according to the general rules.

CHAPTER VIII

REINFORCED CONCRETE STRUCTURES INTENDED FOR SPECIAL CONDITIONS

VIII.1. DESIGN OF EARTHQUAKE-RESISTANT BUILDINGS

1. Construction Features

Earthquakes are caused by volcanic activity, radioactive decay and rupture of deep earth layers accompanied by the displacement of the earth's crust. For an overwhelming majority of buildings located in seismic regions (that is, regions where earthquakes are likely to occur), horizontal displacements of the upper layers of the earth's crust are most disastrous. If, however, the epicenter of an earthquake is near the areas of construction, vertical displacements are also dangerous.

Earthquake-resistant buildings (to be built in seismic regions) should be designed in compliance with relevant standards and specifications.

The violence of earthquakes is evaluated using a standard scale which is a combination of an intensity and an instrumental magnitude scale. When the intensity of an earthquake is 6 or less, no additional strengthening of a structure is necessary, although the quality of construction should meet more stringent requirements. If the intensity of an earthquake is expected to be from 7 to 9, the structure should be designed specifically for earthquake resistance. Earthquakes of intensity 10 cause so strong seismic forces that construction in such regions would be economically unwarranted.

The problem of designing economical earthquake-resistant structures rests heavily on the determination or reliable quantitative estimates of expected earthquake intensities corresponding to particular regions. The subdivision of an area into regions each associated with a known or assigned seismic probability or risk serves as a useful basis for the implementation of code provisions on earthquake-resistant design. Appropriate seismic zoning maps are given in relevant national building codes. In such a map, each area is divided into

zones of approximately equal seismic probability or risk specified for average soil conditions, namely sands and clayey soils and a low ground water level. Each zone is further subdivided into seismic regions to take into account the actual soil conditions and help to determine the seismic risk of a particular construction site. Under favourable soil conditions, say, rock or superhard soil, the seismicity of a site should be reduced by one grade of intensity. In contrast, for clays and loams in a plastic state and also sands and sand loams with a high ground water level, the seismicity should be increased by one grade.

In an earthquake-resistant building, the vertical load-bearing elements (such as frames and shear walls) should be arranged so that the masses and rigidities of the component members are symmetric and uniformly distributed. It should be remembered that the structural measures improving the space rigidity of a building also contribute to its earthquake resistance. For this purpose, it is advisable to use transverse and longitudinal shear walls connected by floors.

Experience with reinforced concrete structures assembled from prefabricated members has shown that they stand earthquake shocks well. This is why such structures have found wide application in seismic regions. The joints between the component members should properly be concreted to resist seismic forces.

In plan layout, a building should be a clear-cut rectangle without protruding annexes and corners. If a complex plan layout is inevitable, provision should be made for anti-seismic joints installed to divide the building into rectangular blocks. Anti-seismic joints are usually combined with movement joints. To increase the earthquake resistance of a building, the footings within one block should be carried to the same depth. If the soil is soft, two-way strip footings or mat foundations should be used. In the case of soils with a high bearing capacity, isolated column footings connected at the top by beams running in two directions will do. For multi-storey buildings, it is advisable to provide basements and to rest them on pile foundations.

To be economical and meet the requirements of industrialized practice, buildings intended for seismic regions should also be designed so as to allow for the use of standardized component members, unified dimensions and construction types, and to facilitate the manufacture and erection of both precast and in-situ structures.

The optimal construction type (that is, one having the best economic and technical features) of an earthquake-resistant building is a shear wall-frame structure with regularly disposed shear walls. Experiments have shown that, although the total seismic load imposed on a shear wall-frame building increases due to the shear walls and a higher lateral rigidity, the seismic load shared by the frames is smaller than it is in the more flexible frame structures. Another im-

portant factor is the changed shape of the Q_f diagram for a frame-shear wall structure, owing to which the bending moments in the frame struts due to the lateral load remain almost the same over a considerable height, and, as a consequence, make it possible to use standard component members (see Ch. VI).

Under the action of earthquake shocks, the joints of reinforced concrete frames are in a complex state of stress, which is why a special attention should be given to their design. Present evidence shows

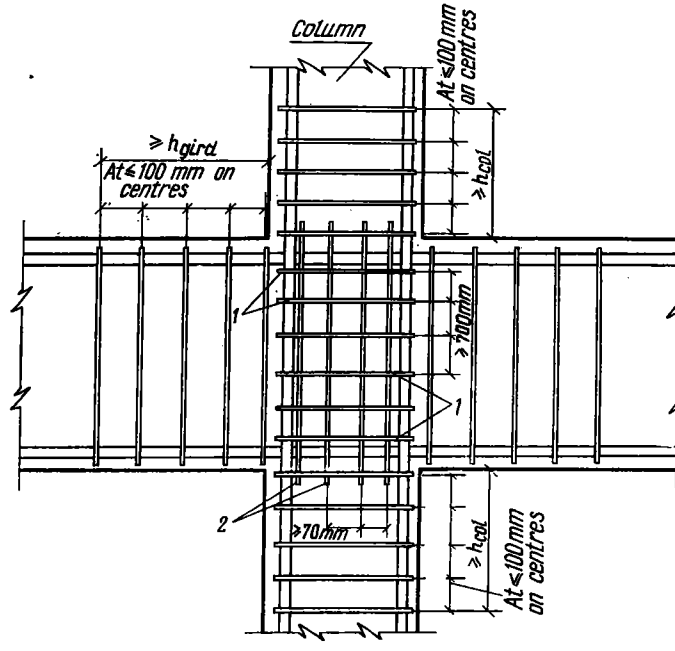


Fig. VIII.1. Reinforcement of an in-situ frame joint and the ends of girders and struts with transverse fabric
1—additional stirrups; *2*—additional vertical bars placed around the stirrup periphery

that a frame joint should be reinforced with additional stirrups and bars 8 to 10 mm in diameter and spaced 70 to 100 mm apart, and also with strengthened transverse steel (placed at the adjoining portions of the girders and columns) with a spacing half the distance required by design, but not more than 100 mm (Fig. VIII.1).

Under seismic loads, the plastic strain in the tensile reinforcement of a joint increases the strength resistance of a skeleton-type building.

The joints between precast girders and columns should preferably be made without embedded parts, by welding together reinforcement stick-outs and concreting (Fig. VIII.2). In such joints, the faces should be grooved to form concrete keys, and the transverse bars

of the girders and columns should be spaced close together. Within the joint, the column is additionally reinforced with stirrups and bars as described above.

Precast floors are assembled from slabs connected to one another and to the frame skeleton members by welding together embedded

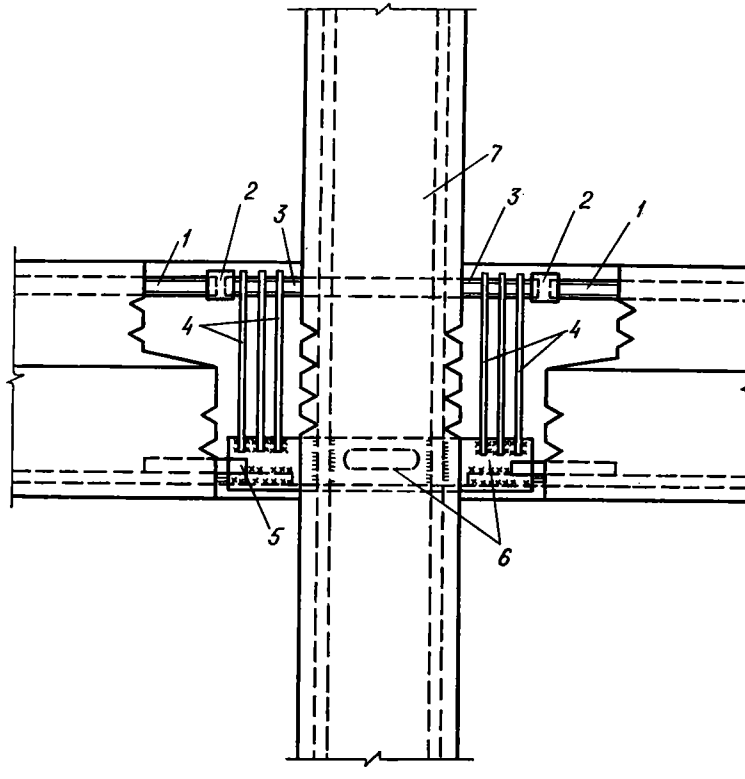


Fig. VIII.2. Reinforcement of a precast frame joint

1—girder longitudinal reinforcement stick-out; *2*—welded joint; *3*—column longitudinal reinforcement stick-out; *4*—girder transverse bars; *5*—strengthened reinforcement stick-out; *6*—support table with a hole for concreting, made of steel angles; *7*—column

parts and concreting the joints and shear keys. For this purpose, slots and grooved surfaces are provided in the floor slabs, which contributes to the shear strength of the floor.

Wall panels are rigidly connected to the skeleton and floors. The masonry or block filler walls are joined with the skeleton by reinforcing bars 6 mm in diameter, placed in the bonds at 50 cm on centres. This reinforcement is attached to the column stick-outs and carried into the masonry for at least 70 cm on either side.

If a wall has large window and door openings, horizontal anti-seismic reinforced concrete belts should be installed at the top of these openings. Physically, the anti-seismic belts are horizontal frames transferring the seismic load to the skeleton columns.

The protruding cantilever parts of the building, such as peaks, eaves and balconies, should rigidly be connected to the skeleton, and their number and size should be limited.

2. Basics of Earthquake-Resistant Design

The earthquake load imposed on a building is determined according to the period and mode of free vibrations of the building, its weight and earthquake intensity. The seismic vibrations of the soil and building footing are deemed to be sinusoidal and damped.

Seismic forces may propagate in any direction in space. For design purposes, however, they are assumed to be directed horizontally along the transverse and longitudinal axes of a building.

In earthquake-resistant design, the design loads should be multiplied by the following combination factors:

for dead loads 0.9
for long-time loads 0.8
for short-time and snow loads 0.5

Here, the loads due to wind and dynamic effect of the equipment, inertial forces of masses on rigid suspension and temperature climatic effects are not taken into consideration.

As a rule, seismic forces are assumed to be applied at floor levels, because it is here that the mass of each storey is deemed to be concentrated.

The design seismic force at the ith harmonic of free lateral vibrations for each kth storey (tier) is

$$S_{ik} = Q_k k_s \beta_i \eta_{ik} \qquad \text{(VIII.1)}$$

where S_{ik} is the lateral seismic force at the ith harmonic of free vibrations at the kth storey (tier) level, Q_k is the load per storey (tier)

TABLE VIII.1. Seismicity Factor

Design intensity grade	7	8	9
Seismicity factor	0.025	0.05	0.1

Note. For buildings 9 or more storeys high, the seismicity factor is multiplied by 1.4. For buildings 5 to 9 storeys high, it is multiplied by a coefficient taken from 1 to 1.4 as determined by linear interpolation.

including the weight of the floor, columns and walls and the live load multiplied by the combination factor, k_s is the seismicity factor depending on the earthquake intensity grade and given by Table VIII.1, $\beta_i = 1/T_i$ is the dynamic factor depending on the free vibration period T_i and taken to be not less than 0.8 and not more than 3, and η_{ik} is the vibration mode coefficient determined in terms of the ith mode of free lateral vibrations and dependent on the magnitude and position of the storey (tier) loads:

$$\eta_{ik} = X_{ik} \sum_{j=1}^{n} Q_j X_{ij} / \sum_{j=1}^{n} Q_j X_{ij}^2 \qquad \text{(VIII.2)}$$

Here, X_{ik} and X_{ij} are the ordinates for the ith mode of free vibrations in the kth storey (tier) and the remaining jth storeys (tiers), respectively.

The seismic forces calculated by Eq. (VIII.1) are assumed to be statically applied to the building.

For regular buildings in which the rigidity and weight insignificantly vary with height, seismic forces may be determined taking into account only the fundamental frequency of vibrations. Vibrations of the higher harmonics are taken into account only for buildings where the rigidity and weight considerably vary with height.

At the fundamental frequency of vibrations, the seismic load is replaced by an equivalent triangular load (based on the moment at the bottom), and the problem is solved using Eq. (VI.26). At the higher harmonics, the problem is solved in a similar manner. The design formulas for practical calculations are given in relevant manuals.

The design forces, N_d, (a bending moment, shearing force or longitudinal force) in the section in question, induced by the seismic load including that due to the higher harmonics, are found statistically as

$$N_d = \sqrt{N_{\max}^2 + 0.5 \sum_{i=1}^{n} N_i^2} \qquad \text{(VIII.3)}$$

where $N_{\max}$ is the maximum force in the section in question, determined by the diagrams of the forces induced by the seismic load due to the various modes of free vibrations; N_i is the force in the same section, determined by other force diagrams (other than $N_{\max}$). Here, all the forces N_i, except $N_{\max}$, are added together. In the design of reinforced concrete members, the additional service factor for earthquakes is assumed as for special short-time loads.

VIII.2. CONSTRUCTION FEATURES OF BUILDINGS ERECTED ON PERMAFROST

When choosing a construction type for a building to be erected in Arctic regions, it should be kept in mind that it will rest on permafrost. The design of a structure depends on the type and properties of the soil, the purpose of the building, its temperature conditions, and the period of construction. Under the circumstances, one can either take measures to retain the permafrost or allow for differential settlement of the soil if the permafrost is allowed to thaw.

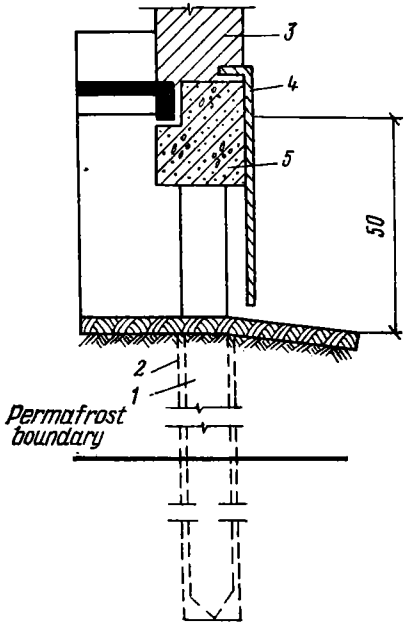


Fig. VIII.3. Detail of a building erected on reinforced concrete piles frozen into the permafrost
1—pile; *2*—hole; *3*—building wall; *4*—base slab; *5*—reinforced concrete pile-cap

Experience has shown that satisfactory results can be achieved by the use of buildings erected on reinforced concrete piles driven and frozen into leader holes bored in the permafrost kept intact (Fig.VIII.3). In hard permafrost, the hole diameter exceeds the pile size, and in plastic permafrost, it is smaller than the pile size. The piles are combined by a reinforced concrete strip pile-cap. To keep the permafrost intact, ventilated cellars are made under the floor.

If a building is erected without piles on a soil which is likely to settle when it thaws, two-way strip footings are installed. In this case, it is recommended to divide the building into several blocks about 20 to 30 m long, and install double transverse walls at the expansion joints.

In either method of construction, it is good practice to erect bearing-wall-type buildings as they are more rigid.

VIII.3. REINFORCED CONCRETE STRUCTURES FOR HIGH SERVICE TEMPERATURES

1. Behaviour of Concrete and Reinforcement on Heating

Some parts of an r.c. industrial building (such as casting, electric furnace and some other "hot" departments), may be exposed to su-

stained service temperatures above 50°C. Structures intended for such service conditions should be designed in compliance with appropriate standards and specifications.

At sustained service temperatures not exceeding 200°C, use is made of ordinary reinforced concrete with portland (or portland blast-furnace) cement as the binding agent; for temperatures higher than 200°C, special heat-resistant concrete should be used. The concrete brands for ordinary reinforced concrete are M-200 through M-600, for prestressed concrete, M-300 and higher. This text discusses reinforced concrete structures made of ordinary heavy reinforced concrete.

Structures may be exposed to heat (a) for a short time, when a structure is allowed just to reach the design temperature, and (b) for a long time, when a structure is allowed to remain at the design temperature for a long time.

Regular exposure to heat disturbs the inner structure of the concrete and, as a consequence, its strength decreases. In design calculations, it should be taken into account that the compressive strength of concrete subjected to elevated temperature decreases by 15% to 20%, and the tensile strength by 30% to 50%.

To take into account the reduction in the compressive and tensile strength due to heating in terms of the limit states of the first and second groups, the design strength of heated concrete is multiplied by the service factors, $m_{c,t}$ and $m_{ten,t}$ respectively:

$$\left.\begin{aligned} R_{pr,t} &= m_{c,t}R_{pr};\ R_{ten,t} = m_{ten,t}R_{ten} \\ R_{pr,t\,II} &= m_{c,t}R_{pr\,II};\ R_{ten,t\,II} = m_{ten,t}R_{ten\,II} \end{aligned}\right\} \qquad \text{(VIII.4)}$$

The tangent modulus of elasticity of concrete also decreases on heating. It is taken as

$$E_{c,t} = E_c\beta_c \qquad \text{(VIII.4}a\text{)}$$

Table VIII.2 gives the factors, $m_{c,t}$, $m_{ten,t}$ and β_c, as functions of temperature.

The thermal shrinkage of concrete (supplementary shrinkage on heating) is an irreversible strain and varies with the form of exposure to heat and temperature

$$\varepsilon_{c,sh} = \alpha_{sh}t \qquad \text{(VIII.5)}$$

The thermal shrinkage factor for concrete is as follows:
for short-time exposure to heat up to 100°C

$$\alpha_{sh} = 1.5 \times 10^{-6}$$

for short-time exposure to heat up to 200°C

$$\alpha_{sh} = 1.1 \times 10^{-6}$$

TABLE VIII.2. Service Factors and Reduction Factor for Tangent Modulus of Elasticity of Concrete Under Conditions of Regular Exposure to Elevated Service Temperatures

Exposure to heat	Factor	Temperature, °C			
		50	60	100	200
Short-time	$m_{c,t}$	1	0.85	0.85	0.8
	$m_{ten,t}$	1	0.7	0.7	0.6
Sustained	$m_{c,t}$	1	0.85	0.85	0.8
	$m_{ten,t}$	1	0.7	0.7	0.5
Short-time and sustained	β_c	1	0.9	0.8	0.6

for sustained exposure to heat at up to 100°C

$\alpha_{sh} = 4.5 \times 10^{-6}$

for sustained exposure to heat at up to 200°C

$\alpha_{sh} = 2.6 \times 10^{-6}$

The thermal expansion of concrete is a reversible strain depending only on temperature

$$\varepsilon_{c,t} = \alpha_{c,t} t \qquad \text{(VIII.6)}$$

For heating up to 100°C, the coefficient of linear thermal expansion for concrete is $\alpha_{c,t} = 11.5 \times 10^{-6}$, and, for heating up to 200°C, it is $\alpha_{c,t} = 10.6 \times 10^{-6}$.

The total thermal strain of concrete on heating (Fig. VIII.4) is

$$\varepsilon_{c,ten} = \varepsilon_{c,t} - \varepsilon_{c,sh} = (\alpha_{c,t} - \alpha_{c,sh})\, t = \alpha_{c,ten}\, t \qquad \text{(VIII.7)}$$

The coefficient of total thermal strain of concrete is as follows:

—for short-time exposure to heat up to 100°C

$\alpha_{c,ten} = 10 \times 10^{-6}$

—for short-time exposure to heat up to 200°C

$\alpha_{c,ten} = 9.5 \times 10^{-6}$

The basic density of heavy concrete dried to constant weight is taken as 2 300 kg/m³, and the basic average density of reinforced concrete, 2 400 kg/m³.

In cases where regular exposure to service temperatures up to 200°C is expected, r.c. structures use ordinary reinforcing steel which, however, should meet additional requirements. The maximum safe temperature for reinforcing steel is 450°C for nonprestressed bars, 250°C for prestressed bars, and 400°C for class B-I and Bp-I wire. In order to preserve the effect of cold-work hardening, the maximum safe temperature for class B-II and Bp-II wire and also reinforcing strands should not exceed 150°C.

When prestressed reinforced concrete structures are subjected to temperature cycling, the maximum safe temperature for the prestressed reinforcement should be set 50°C below the specified value. Temperature cycling refers to a condition when a structure is exposed to a temperature repeatedly varying within over 30% of its design value with a period of 3 hours to 15 days.

On heating, the strength of reinforcing steel decreases by 5% to 15%. The design strength is multiplied by the service factor, $m_{s,t}$, which takes care of the reduction in strength due to short-time and sustained exposure to heat (see Table VIII.3)

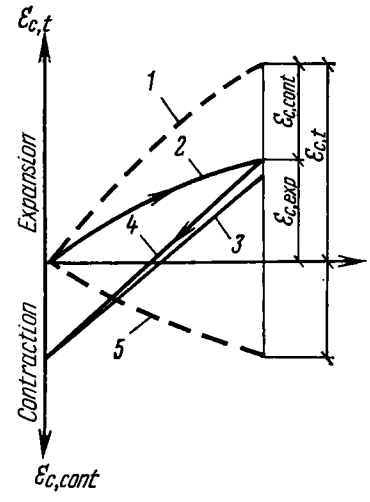


Fig. VIII.4. Thermal strain diagram for concrete
1—thermal expansion $\varepsilon_{c,t}$; *2*—total thermal strain $\varepsilon_{c,ten}$; *3*—strain due to temperature cycling; *4*—strain due to cooling after exposure to design temperature; *5*—thermal shrinkage $\varepsilon_{c,sh}$

$$R_{s,t} = m_{s,t} R_s \quad \text{and} \quad R_{s,t11} = m_{s,t} R_{s11}$$

TABLE VIII.3. Service Factor and Coefficient of Thermal Expansion for Reinforcing Steel Regularly Exposed to Elevated Service Temperatures

Reinforcement	Exposure to heat	Factor	Temperature, °C		
			50	100	200
Class A-I, A-II bars, wire of all types, and class K-7 strands	Short-time	$m_{s,t}$	1	0.95	0.9
	Sustained	$m_{s,t}$	1	0.95	0.85
	Short-time and sustained	$\alpha_{s,t}$	11.5×10^{-6}	11.7×10^{-6}	12.5×10^{-6}
Class A-III, A-IV, At-IV, At-V and At-VI bars	Short-time	$m_{s,t}$	1	1	0.9
	Sustained	$m_{s,t}$	1	1	0.9
	Short-time and sustained	$\alpha_{s,t}$	11.5×10^{-6}	12×10^{-6}	13×10^{-6}

The modulus of elasticity of reinforcing steel exposed to heat is

$$E_{s,t} = E_s \beta_s \tag{VIII.8}$$

where $\beta_s = 1$ for heating up to 100°C, and $\beta_s = 0.96$ for heating up to 200°C.

The coefficient of thermal expansion for steel is looked up in Table VIII.3 according to the type of reinforcement and temperature. In cracked reinforced concrete members, the average coefficient of thermal expansion for the steel in the concrete is

$$\alpha_{s,t,av} = \alpha_{c,ten} + (\alpha_{s,t} - \alpha_{c,ten})\, k \tag{VIII.9}$$

where k is a coefficient depending on the percentage of longitudinal reinforcement; for statically determinate systems, it is given in Table VIII.4.

TABLE VIII.4. Coefficient k

Percentage of reinforcement	0.2	0.4	0.7	1	2
k	0.55	0.8	0.9	0.85	1

The total loss of prestress in reinforcement is the sum of the main losses at the normal temperature and additional losses at elevated temperature (such as thermal shrinkage and creep of the concrete, stress relaxation in the steel, different strain in the concrete and steel at elevated temperatures, and so on).

When the reinforcement is to be heated to 100°C, its diameter should not exceed 25 mm; at a temperature of up to 200°C, it should be not more than 20 mm.

The longitudinal nonprestressed steel should be anchored at least a distance l_{an} past the section in which it is assumed to be acting at full strength. The value of l_{an} is found by Eq. (I.20) (see Ch. I, Vol. 1), assuming the design strength for the steel and taking the strength of the concrete with allowance for heating

$$l_{an} = (m_{an} R_{s,t}/R_{pr,t} + \Delta\lambda_{an})\, d \tag{VIII.10}$$

or at least $l_{an} = \lambda_{an} d$, whichever is the greater.

At a temperature of 100 to 200°C, the concrete cover for the load-bearing reinforcement should be increased by 5 mm, and taken not less than $1.5d$, where d is the diameter of the load-bearing reinforcement.

Heating up to 200°C followed by cooling leaves the bond strength between the steel and the concrete practically unchanged.

2. Temperature-Induced Strain and Forces

In reinforced concrete structures, the temperature distribution at a steady flow of heat is determined using the methods of heat engineering. The temperature of the steel is deemed to be the same as that of the concrete.

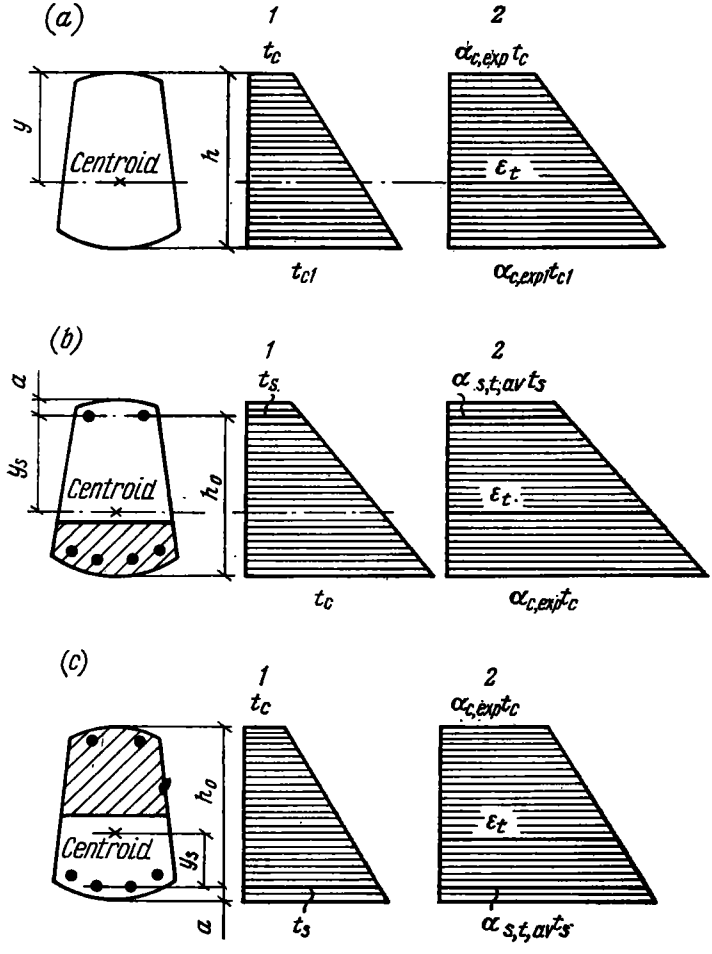


Fig. VIII.5. Design models for temperature and strain distribution due to nonuniform heating at a temperature linearly varying over the height of a section

(*a*) plain concrete; (*b*) reinforced concrete, with the tension zone near the cooler face; (*c*) reinforced concrete, with the tension zone near the hotter face; *1*—temperature distribution; *2*—elongation distribution

For temperature linearly distributed over the height of a section, the free thermal strain of the member, ε_t, is determined at the centroid of the section, and the curvature of the deflected axis, $1/\rho_t$, is taken as the slope of the strain diagram:

(a) in a crack-free reinforced concrete member (Fig. VIII.5*a*)

$$\varepsilon_t = [\alpha_{c,ten} t_c (h - y) + \alpha_{c,ten1} t_{c1} y] n_t / h \qquad \text{(VIII.12)}$$

$$1/\rho_t = (\alpha_{c,ten1} t_{c1} - \alpha_{c,ten} t_c) n_t / h \qquad \text{(VIII.13)}$$

(b) in a reinforced concrete member containing cracks in the tension zone located near the cooler face (Fig. VIII.5*b*)

$$\varepsilon_t = [\alpha_{s,t,av} t_s (h_0 - y_s) + \alpha_{c,ten} t_c y_s] n_t / h_0 \qquad \text{(VIII.14)}$$

$$1/\rho_t = (\alpha_{c,ten} t_c - \alpha_{s,t,av} t_s) n_t / h_0 \qquad \text{(VIII.15)}$$

(c) in a reinforced concrete member containing cracks in the tension zone located near the hotter face (Fig. VIII.5*c*), ε_t is found by Eq. (VIII.14) and the curvature of the deflected axis is given by

$$1/\rho_t = (\alpha_{s,t,av} t_s - \alpha_{c,ten} t_c) n_t / h_0 \qquad \text{(VIII.16)}$$

where t_c, t_{c1} and t_s are the temperatures of the concrete and reinforcing steel; n_t is the overheating factor taken as 1.1 for design in terms of the first group of limit states, and as 1 for design in terms of the second group of limit states; and y and y_s are the distances from the centroid of the section to the cooler face and to the point of application of the force in the tensile reinforcement, respectively.

The sag of the member due to nonuniform heating is

$$f_t = s_t (1/\rho_t) l^2 \qquad \text{(VIII.17)}$$

Here, the coefficient, s_t, depends on the support conditions of the member; for simply supported members, $s_t = 1/8$.

The thermal moment due to nonuniform heating in members fixed at the support, and also in closed frames of uniform cross section is

$$M_t = (1/\rho_t) B \qquad \text{(VIII.18)}$$

where B is the rigidity of the section.

3. Design with Allowance for Effect of Temperature

Structures intended for service at elevated temperatures are designed for the worst combinations of self-weight, external load and thermal forces with allowance for their duration. The design temperature is multiplied by the overheating factor, n_t.

In design in terms of the first and second groups of limit states, the design strength of the concrete and steel is multiplied by the service factors, $m_{c,t}$, $m_{ten,t}$ and $m_{s,t}$.

Statically determinate structures are designed for sustained exposure to the design temperature, and statically indeterminate structures are additionally designed for the short-time exposure which gives rise to maximum thermal forces. In the design of statically indeterminate structures, allowance should be made for the decrease in section rigidity due to sustained exposure to heat.

In strength design, the design compressive strength of the concrete is taken in accordance with the average temperature of the compression zone. For T-section members, it is taken according to the average temperature of the flange overhangs.

In incipient cracking analysis, the tensile strength of the concrete is determined for the concrete temperature at the tensile steel level, and the geometry of the transformed section is determined with allowance for the effect of elevated temperatures.

The width of normal cracks arising due to the self-weight, external load and temperature is determined using the formulas given in Ch. VII, Vol. 1. The width with allowance for additional crack opening due to the difference between the coefficient of thermal expansion of the steel in the concrete and the coefficient of total thermal strain of the concrete is found by

$$a_{cr} = 20\,(3.5 - 100\mu)\,k\eta\,\sqrt[3]{d}[c_{\Delta}\sigma_s/E_{s,t} + (\alpha_{s,t,av} - \alpha_{c,ten})\,t_s] \qquad \text{(VIII.19)}$$

In crack-free and cracked portions, the curvature of the deflected axis of reinforced concrete members is determined according to the moduli of elasticity of concrete and steel, $E_{c,t}$ and $E_{s,t}$, and the design strength of the materials at a given temperature. The total sag is the sum of the sag under load, f_M, and the deflection due to heating, f_t. If f_t reduces the total sag of a member, it may be dropped.

VIII.4. REINFORCED CONCRETE STRUCTURES FOR LOW SUB-ZERO TEMPERATURES

1. Requirements for Reinforcing Steel and Concrete

When designing reinforced concrete structures intended for sustained exposure to low sub-zero temperatures, it is necessary to take into account the cold brittleness of reinforcing bars, which varies with the class of reinforcement, steel grade and steel-making process. Appropriate notes must be included in the specifications.

At a temperature of —40°C and below and under static and dynamic loads, use may be made of bar reinforcement of class A-I, grade ВСт.3сп2; class A-II, grade 18Г2С, and 10ГТ; class A-III, grade

18Г2С and 25Г2С (the latter grade only in tied-bar mats); and also class A-IV, grade 20ХГ2Ц, and class A-V bars (without welded joints). At low sub-zero temperatures, heat-treated bars are used only under static loads. No special requirements are placed on ordinary, high-strength, plain and deformed wire and wire strands.

For a temperature of —30 to —40°C and static loads, embedded steel parts should be made of grade ВСт3пс6 and ВСт3пс5 carbon steel, and for dynamic and repeated loads, grade ВСт3сп5 carbon steel.

At low sub-zero temperatures, use should be made of no-voids concretes in which the space between the aggregate grains is fully occupied by hardened cement. These may be heavy concretes or porous-aggregate concretes. The design concrete brands should be specified not only in terms of compressive strength, but also in terms of frost resistance and water resistance. The brand numbers of concrete for joints should not be less than those of the members being joined.

2. Analysis and Design

If reinforced concrete structures intended for service at above-zero temperatures but exposed to low sub-zero temperatures (below —40°C) during construction use reinforcement which is allowed only for heated locations, the design should state temporary limits on external load. Under such conditions, the load-bearing capacity of structures during erection should be checked at the time of design for loads multiplied by the overload factor, $n = 1$, with the design strength of reinforcing steel multiplied by 0.7.

Under the circumstances, no-voids concretes should be used.

In the strength design of members and structures, the design axial compressive strength of concrete, R_{pr}, should be multiplied by the service factor, $m_{c,f-th}$, (Table VIII.5). This factor takes care of the reduction in concrete strength due to freeze-thaw cycles of water-saturated members.

TABLE VIII.5. Service Factor $m_{c,\,f-th}$ for Concrete Members and Structures Subjected to Freeze-Thaw Cycles

Service conditions	Design ambient temperature	Concrete	
		heavy	porous-aggregate
Freeze-thaw cycles in water-saturated state	Below —35°C	0.7	0.8
	Below —20°C (not below —35°C)	0.85	0.9
Freeze-thaw cycles under the conditions of sporadic water saturation (due to atmospheric precipitation)	Below —35°C	0.9	1
	—35°C and above	1	1

For a design temperature of below—40°C, contraction joints should be spaced closer together than under ordinary conditions, and the distance should be checked by design.

The design sub-zero temperature is defined as the average ambient temperature for the coldest five days of a winter month, depending on the region of construction and according to relevant standards.

VIII.5. REINFORCED CONCRETE STRUCTURES FOR CORROSIVE ENVIRONMENTS

1. Classification of Corrosive Environments

When designing reinforced concrete buildings and structures, it is necessary to remember that the environments may cause corrosion of the concrete or reinforcing steel. Accordingly, concrete, binding agents and anti-corrosion materials should be chosen to resist such effects, as prescribed in appropriate standards and specifications.

Corrosive environments may be gaseous, liquid or solid. According to the effect that they exert on a reinforced concrete structure, corrosive environments may be classified into low-, medium- and high-corrosive (destructive). According to the service conditions (such as ambient humidity, temperature, and so on), one and the same environment may have a different effect on a structure.

The effect of gases, acid fumes, water vapour and fog depends on their composition, humidity and temperature. The corrosiveness of gaseous environments increases with rising humidity.

The corrosiveness of liquids depends on the chemical composition, temperature, speed at which the liquid flows toward the surface of a structure, and also on the type of binding agent and the density of the concrete.

Solids (dust, etc.) may become corrosive in the presence of moisture and condensate as they dissolve salts and form liquid corrosive environments.

In increasing order of corrosiveness toward reinforced concrete, gases are divided according to their type and concentration into three groups, namely B, C and D (Table VIII.6). The corrosiveness of gases in the same group increases with rising relative humidity for heated locations or increasing level of humidity for unheated buildings and open-air structures (Table VIII.7).

A liquid environment becomes highly corrosive if it contains acids, nitrates, rhodanates, chlorites, and the like.

Liquid corrosive environments may cause corrosion of three types: I—leaching of soluble concrete components by soft water, mainly due to filtration under pressure, II—complete decomposition of concrete by highly soluble compounds produced by reactions

TABLE VIII.6. Classification of Corrosive Gases According to Their Type and Concentration

Group	Gas	Concentration, mg/m³	Group	Gas	Concentration, mg/m³
B	Ammonia	0.2		Hydrogen sulphide	11-200
	Sulphurous anhydride	0.5-1		Nitrogen oxides	5.1-25
	Hydrogen fluoride	0.02-5		Chlorine	1.1-5
	Hydrogen sulphide	0.01-10		Hydrogen chloride	5.1-10
	Nitrogen oxides	0.1-5			
	Carbon dioxide	$>1\,000$	D	Sulphurous anhydride	201-1 000
	Chlorine	0.1-1		Hydrogen fluoride	11-100
	Hydrogen chloride	0.05-5		Hydrogen sulphide	201-2 000
				Nitrogen oxides	26-100
C	Sulphurous anhydride	1.1-200		Chlorine	5.1-10
	Hydrogen fluoride	5.1-10		Hydrogen chloride	11-100

TABLE VIII.7. Corrosiveness of Gaseous Environments Toward Reinforced Concrete

Humidity		Group of corrosive gas	Corrosiveness toward reinforced concrete
heated locations, relative, %	unheated buildings and open-air structures, level of humidity		
$\leqslant 60$	Dry	Free from corrosive gases	Noncorrosive
		B	Noncorrosive
		C	Low
		D	Medium
61-75	Normal	Free from corrosive gases	Noncorrosive
		B	Low
		C	Medium
		D	High
>75	Humid	Free from corrosive gases	Low
		B	Medium
		C	High
		D	High

between the hardened cement and the liquid corrosive media and devoid of binding properties, and III—accumulation of poorly soluble crystallizing salts formed by chemical reactions in the concrete as the level of water containing a large amount of salts varies.

2. Requirements for Concrete and Reinforcing Steel

The aggregates and binding agents for concrete chosen according to a relevant standard should stand well the action of a given corrosive environment. In special cases, use may be made of acid-resistant soluble-glass concretes and polymer concretes.

For structures intended for service in corrosive environments, concretes using cement as the binding agent should have the necessary density which is stated as a brand number in terms of water resistance. Table VIII.8 gives the necessary data for heavy concretes.

TABLE VIII.8. Factors Characterizing Concrete Density

Concrete density	Designation	Factor characterizing concrete density		
		brand in terms of water resistance	water absorption, % by weight	water-cement ratio, max
Normal	H*	B-4	5.7-4.8	0.6
Increased	П	B-6	4.7-4.3	0.55
Superhigh	О	B-8	4.2 and lower	0.45

* Russian letters are used throughout. — *Translator's note.*

A long service life for the reinforcing steel in concrete can be ensured by the use of steels which are most resistant to a given corrosive environment, an appropriate dense concrete cover, protective coating over the reinforcement, and protective varnish or film coating applied to the concrete surface.

The choice of minimum concrete cover and concrete density depends on the corrosiveness of the environment, its type (gaseous or liquid), and the type of structure (see Table VIII.9).

TABLE VIII.9. Requirements for Reinforced Concrete Structures Intended for Corrosive Environments

Corrosiveness	Minimum concrete cover, mm			Concrete density of structures reinforced with		
	for gases*		for liquids	class A-I, A-II, A-III, A-IV, B-I bars and wire	class B-II, Bp-II, wire, strands	class A-V, Ат-IV, Ат-V, Ат-VI bars
	flanges of ribbed slabs, walls, beams	trusses, columns				
Low	15	20	25	H**	П	П
Medium	15	20	30	П	О	О
High	20	25	35	О	О	Not allowed

* For heat-treated bars and high-strength wire, the concrete cover should be not less than 25 mm.

** Russian letters are used throughout. — *Translator's note.*

Class A-V bars and thermally strengthened reinforcement of all classes cannot be used in prestressed structures intended for highly

corrosive gaseous and liquid environments. In structures made of porous-aggregate concretes, class B-II and Bp-II high-strength wire and class A-V, Ат-IV and higher bars can be used only with a special protective coating.

If the desired concrete density and concrete cover cannot be provided, it is recommended to use galvanized reinforcement.

Embedded parts and welded joints in reinforced concrete structures should be covered by dense concrete, or galvanized, aluminized, or given a coat of varnish or paint.

3. Anticorrosion Protection

Anticorrosion protection is chosen according to the corrosiveness and properties of the environment. In low-corrosive environments, use is made of varnishes and paints; in medium- and high-corrosive environments, of varnishes, paints, putties, pasted insulation and lining.

Varnishes or paints should form an impenetrable film 0.1 to 1 mm thick on the surface of a structure. The film thickness is chosen by design. Such coatings may be used with or without fillers and reinforcement. The materials are chosen according to the corrosiveness of the environments: (a) for low-corrosive environments, they are based on natural drying oil, polyester resins, and the like; (b) for medium-corrosive environments, on PVC and epoxy resins, thiokol, chlorinated rubber, etc.; (c) for high-corrosive environments, on epoxy resins, thiokol, chlorinated rubber and similar materials applied in several layers.

Putties should form on the surface of a structure an impenetrable layer 1 to 10 mm thick. They are based on organic binders, such as bituminous materials with fillers, polymer putties based on polyester and epoxy resins, and so on.

Pasted insulation can be bituminous or polymer roll insulation, polymer films and sheets. Such materials may or may not have a backing or reinforcement.

Lining is intended to protect the insulation against corrosive environments and mechanical injury. They include glass, acid-resistant tile, stone tile, and the like.

VIII.6. STRENGTHENING OF STRUCTURAL MEMBERS

1. Conditions Necessitating Strengthening

Structural members are strengthened to increase their load-bearing capacity and rigidity. A need for strengthening usually arises if a structure is to stand up to changed exposure conditions, to carry heavier equipment and increased live loads, and so on. The damade

caused to structural members during erection, in service, or by a fire may likewise call for strengthening. The decision on the feasibility and expediency of strengthening should be taken in each particular case, depending on the service requirements and the condition of the structure. Also, the cost of strengthening should be compared with the cost of an entirely new structure.

Structural members may be strengthened in one of two ways: (1) by changing their design, and (2) by building up the cross-sectional area and amount of reinforcement in the members. The former is mainly used to strengthen undamaged structures without an interruption in the production process. The latter applies to both damaged and undamaged structures.

2. Strengthening by Changing the Design

The load-bearing capacity of bending members (such as beams, girders, and the like) may significantly be increased by introducing posttensioned tie rods. The tie rods enhance the load-bearing capacity by changing the state of stress in the structure (it becomes eccentrically compressed). Each tie is composed of two or, sometimes, four rods which are tensioned pairwise by a bolt device. The tie rods may be horizontal, trussed, or both (Fig. VIII.6). How the tie rods are attached to the structure being strengthened is clear from the figure. The necessary prestress is achieved by bringing the tie rods slightly closer to one another. The tie rods are given a coat of an anticorrosion compound.

The new structure with the tie rods reliably connected to the supports behaves as a single entity. The load applied to the structure tensions the tie rods, thereby causing the member to act in compression and bending.

Columns are strengthened by adding prestressed broken-line spreaders on one or either side of the column (Fig. VIII.7). Each spreader piece comprises angles connected by welded straps. At the points of bending, the side legs of the angles are cut. The spreader pieces are prestressed by bracing them pairwise. In unilateral spreaders, the prestress is achieved by bringing them closer to the side surface of the column. On straightening, the spreaders take up some of the vertical load, thereby unloading the column. Unilateral spreaders are mainly used for strengthening eccentrically loaded columns to which the load is applied with large eccentricities.

3. Strengthening by Build-up

Slabs in in-situ ribbed floors and prefabricated floor slabs laid on reinforced concrete or steel beams are strengthened by casting a new in-situ slab over the existing concrete. After the layers of the old

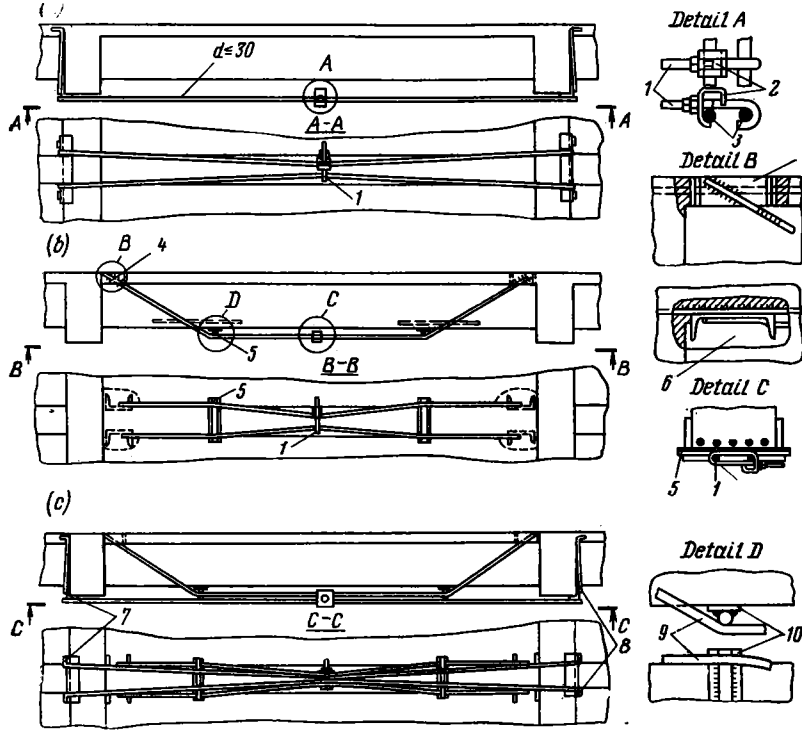


Fig. VIII.6. Strengthening of beams and girders by tie rods

1—strain pin; *2*—stop-washer; *3*—tie rods; *4*—channel support anchor; *5*—round-bar pads; *6*—slab hole to be stopped after anchor has been installed; *7*—angle stop; *8*—angle-stop anchors; *9*—tie rods; *10*—strip-steel pads

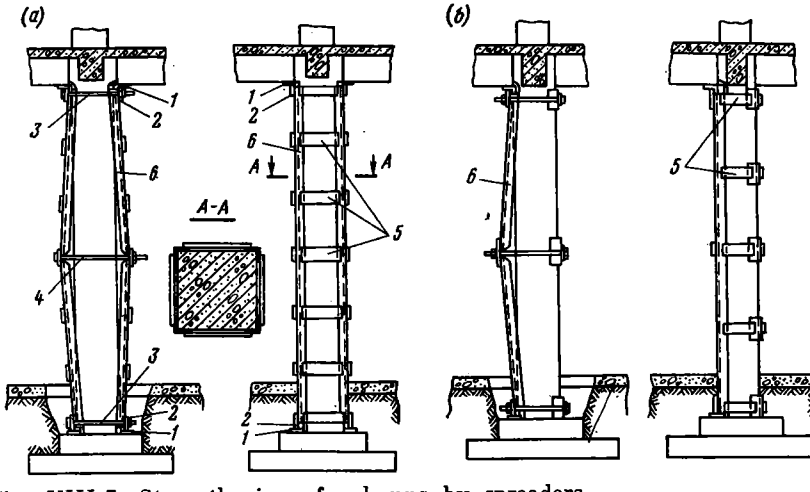


Fig. VIII.7. Strengthening of columns by spreaders

(*a*) columns in compression; (*b*) columns in eccentrical compression; *1*—angle stops; *2*—straps; *3*—coupling bolts; *4*—strain pin; *5*—straps to be welded after spreaders have been installed; *6*—spreaders

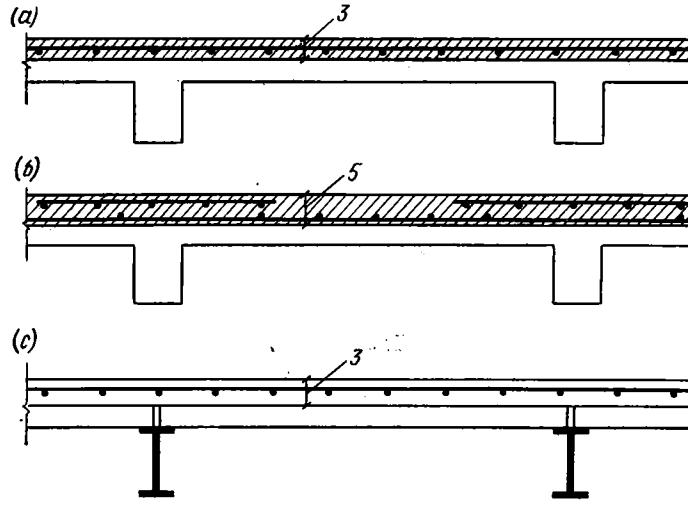


Fig. VIII.8. Reinforced concrete slabs strengthened by build-up

floor and the damaged top layer of the old concrete have been removed, reinforcement is placed and covered with a layer of concrete at least 3 cm deep (Fig. VIII.8*a*). The new slab thus formed is regarded as an in-situ slab. If the old slab has accumulated so much grease or dirt that an adequate bond between the old and new concrete cannot be secured, the new slab should be cast at least 5 cm thick and have reinforcement in the span and at the supports (Fig. VIII.8*b*). The effective load is shared by the old and new slabs in proportion to their rigidities. The slabs of precast floors are strengthened in a similar manner (Fig. VIII.8*c*).

In strengthening beams and girders, the concrete cover is removed, and additional longitudinal bars are welded to the bare reinforcement. Then, the reinforcement is grouted or shotcreted (Fig. VIII.9*a*). The load-bearing capacity of a beam may considerably be raised by increasing its cross-sectional area from beneath and placing additional reinforcement (Fig. VIII.9*b*). Additional bent-up bars and transverse stirrups are welded to the existing reinforcement.

Columns may also be strengthened by jackets reinforced spirally or with longitudinal bars and stirrups. The jacket should be at least 5 cm thick when concreted in formwork, or at least 3 cm thick if shotcreted (Fig. VIII.10). To strengthen the columns of multistorey buildings and to pass additional longitudinal reinforcement, floor slabs have holes, and the columns are locally widened where they intersect with the beams.

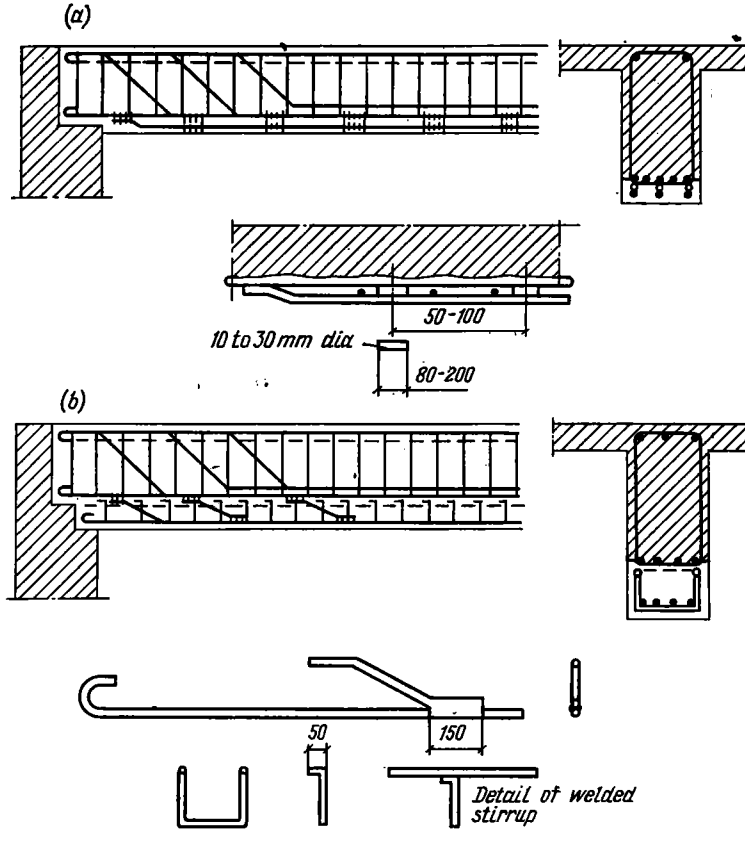


Fig. VIII.9. Detail of reinforcement in reinforced concrete beams strengthened by build-up

4. Construction Requirements

A reinforced concrete structure should be strengthened taking into account its stress-strain state under load. Where additional reinforcement is to be welded, the concrete cover should be removed so that half of the cross-sectional area of the existing longitudinal bars is exposed. It should be remembered that the removal of the concrete cover from the tensile face does not markedly impair the strength only when the existing reinforcement has no lap splices. The removal of the concrete cover from the compressive face always results in a temporary reduction of the strength.

The bond between the new and old concrete will sufficiently be reliable if the surface of the old concrete is properly prepared. For this purpose, it is necessary to remove from the surface all loose and

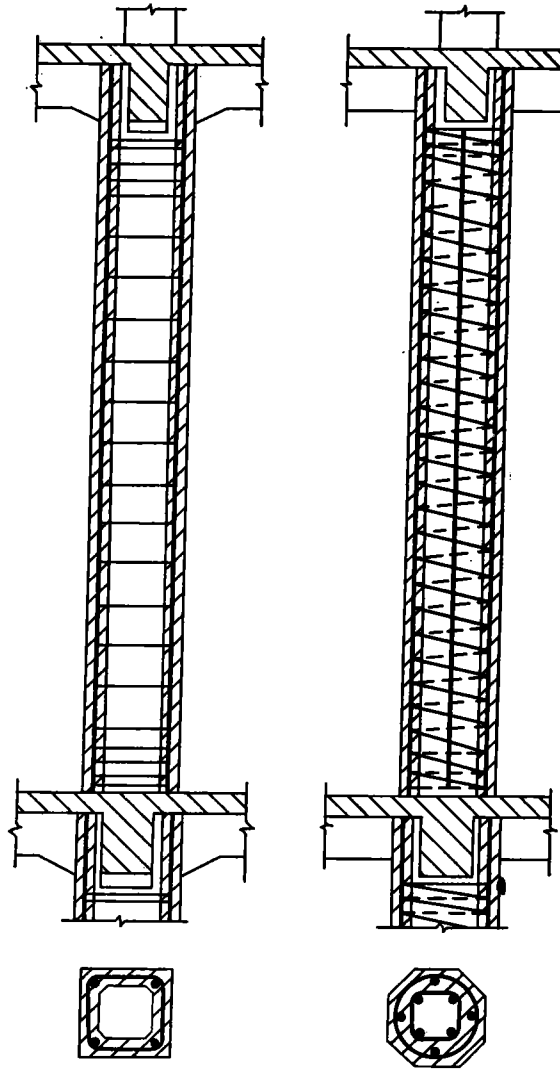


Fig. VIII.10. Reinforced concrete columns strengthened by build-up

damaged parts, dust, etc. Then, the surface should be roughened and wire-brushed. After the reinforcement has been installed, the surface is saturated with water under pressure, and maintained in a moist condition. Directly prior to concreting, water pools should be removed from the surface. During the first 7 days after concreting, the concrete should be moist cured by hosing.

APPENDIX I

Design Strength of Concrete in MPa

Strength	Concrete	Brand number in terms of compressive strength										
		150	200	250	300	350	400	450	500	600	700	800
Axial compressive (prism crushing), R_{pr}	Heavy	7	9	11	13.5	15.5	17.5	19.5	21.5	24.5	28	31
	Porous-aggregate	7	9	11	13.5	15.5	17.5	—	—	—	—	—
Axial tensile, R_{ten}	Heavy	0.63	0.75	0.88	1	1.1	1.2	1.28	1.35	1.45	1.55	1.65
	Porous-aggregate	0.63	0.75	0.88	1	1.1	1.2	—	—	—	—	—

Note. For concretes using alumina cement as binding agent, the design tensile strength, R_{ten}, should be multiplied by 0.7.

APPENDIX II

Service Factors for Heavy and Porous-Aggregate Concretes

Reason for introducing service factor	Designation	Value	Design strength to be multiplied by service factor
Duration of loading (in strength analysis):			
(a) for dead, long- and short-time live loads (except loads of relatively short duration, such as crane, wind, manufacture and transit loads), and also special loads due to deformation of settlement soils and permafrost for:			
naturally hardened and heat-cured concretes if structure is intended for exposure conditions favourable for strength build-up (ambient humidity over 75%, hardening under water)	m_{c1}	1	R_{pr}, R_{ten}
in other cases	m_{c1}	0.85	R_{pr}, R_{ten}
(b) for dead, long- and short-time live loads, and also special and emergency loads	m_{c1}	1.1	R_{pr}, R_{ten}
Multiply repeated loading (for fatigue design)	m_{c2}	see Ch. VIII (Vol. 1)	R_{pr}, R_{ten}
Freeze-thaw cycles	m_{c3}	0.7-1	R_{pr}
Prestress for members reinforced with:			
wire	m_{c4}	1.2	R_{pr}
bars	m_{c4}	1.2	R_{pr}
Concreting of members in vertical position (prefab slabs, in-situ columns, etc.), layer of concreting being more than 1.5 m deep	m_{c7}	0.85	R_{pr}
In-situ r.c. columns with longer section side less than 30 cm	m_{c8}	0.85	R_{pr}
Joints between prefab members narrower than 1/5 of smallest section dimension and narrower than 10 cm	m_{c9}	1.15	R_{pr}
Autoclaving	m_{c10}	0.85	R_{pr}

APPENDIX III

Basic Strength of Concrete (in MPa)

Strength	Concrete	Brand number in terms of compressive strength										
		150	200	250	300	350	400	450	500	600	700	800
Axial compressive (prism-crushing), R^b_{pr}	Heavy	8.5	11.5	14.5	17	20	22.5	25.5	28	34	39	45
	Porous-aggregate	8.5	11.5	14.5	17	20	22.5	—	—	—	—	—
Axial tensile, R^b_{ten}	Heavy	0.95	1.15	1.3	1.5	1.65	1.8	1.9	2	2.2	2.35	2.5
	Porous-aggregate	0.95	1.15	1.3	1.5	1.65	1.8	—	—	—	—	—

Strength	Concrete	Brand number in terms of tensile strength						
		P-10	P-15	P-20	P-25	P-30	P-35	P-40
Axial tensile, R^b_{ten}	Heavy	0.78	1.17	1.56	1.95	2.35	2.7	3.1
	Porous-aggregate	0.78	1.17	1.56	1.95	2.35	—	—

NOTE. For concretes using alumina cement as binding agent, the basic tensile strength, R^b_{ten}, should be multiplied by 0.7.

APPENDIX IV

Tangent Modulus of Elasticity $E_c \times 10^{-3}$ in MPa for Concrete in Compression and Tension

Concrete	Brand number in terms of compressive strength										
	150	200	250	300	350	400	450	500	600	700	800
Heavy, naturally hardened	21	24	26.5	29	31	33	34.5	36	38	39	40
Porous-aggregate with average density, t/m³:											
1.4	10.5	11.5	12.5	13.5	14.5	—	—	—	—	—	—
1.8	13.5	15	16.5	17.5	18.5	19	—	—	—	—	—
2.2	17	18.5	20	21.5	22.5	23.5	—	—	—	—	—

NOTE. The tabular values of the tangent modulus of elasticity for heavy concrete should be reduced by 10% in the case of heat curing, and by 25% in the case of autoclaving.

APPENDIX V.1

Basic and Design Strength and Modulus of Elasticity for Reinforcing Bars and Plain Wire

Reinforcement	Basic strength R_s^b, MPa	Design strength, MPa			Modulus of elasticity E_s MPa
		tensile steel		compressive steel	
		(a) longitudinal, (b) transverse and bent-up in diagonal-section bending design, R_s	transverse and bent-up in shear design, $R_{s,t}$	$R_{s,com}$	
Hot-rolled plain bars, class A-I	240	210	170	210	210 000
Hot-rolled deformed bars, class A-II	300	270	215	270	210 000

Appendix V.1 (continued)

Reinforcement	Basic strength R_s^b, MPa	Design strength, MPa			Modulus of elasticity E_s, MPa
		tensile steel		compressive steel	
		(a) longitudinal, (b) transverse and bent-up in diagonal-section bending design, R_s	transverse and bent-up in shear design, $R_{s,t}$	$R_{s,com}$	
Hot-rolled deformed bars, class A-III	400	340	240 (270)	340	200 000
Hot-rolled deformed bars, class A-IV and thermally hardened, class At-IV	600	500	400	400	200 000
Hot-rolled deformed bars, class A-V and thermally hardened, class At-V	800	640	510	400	190 000
Hot-rolled deformed bars, thermally hardened, class At-VI	1 000	800	640	400	190 000
Ordinary plain wire used in welded-wire fabric, class B-I	550	315	220 (190)	315	200 000
Ordinary deformed wire, class Bp-I:					
3 to 4 mm in dia	550	350	2 600 (280)	350	170 000
5 mm in dia	525	340	2 500 (270)	340	170 000

Note. The values of $R_{s,tr}$ in brackets are specified for tied-wire fabric and tied-bar mats.

APPENDIX V.2

Basic and Design Strength and Modulus of Elasticity for High-Strength Wire and Wire Strands

Reinforcement	Diameter, mm	Basic strength R_s^b, MPa	Design strength, MPa			Modulus of elasticity E_s, MPa
			tensile steel		compressive steel	
			(a) longitudinal, (b) transverse and bent-up in diagonal-section bending design, R_s	transverse and bent-up in shear design, $R_{s,\,tr}$	$R_{s,\,com}$	
High-strength plain wire, class B-II	3	1 900	1 230	980	400 for all types of reinforcement bonded to concrete	200 000
	4	1 800	1 160	930		
	5	1 700	1 100	880		
	6	1 600	1 030	830		
	7	1 500	970	710		
	8	1 400	900	720		
High-strength deformed wire, class Bp-II	3	1 800	1 160	930		200 000
	4	1 700	1 100	880		
	5	1 600	1 030	830		
	6	1 500	970	770		
	7	1 400	900	720		
	8	1 300	840	670		
Seven-wire strands, class K-7	4.5	1 900	1 230	980	–	180 000
	6	1 850	1 190	950		
	7.5	1 800	1 160	930		
	9	1 750	1 130	900		
	12	1 700	1 100	880		
	15	1 650	1 060	850		

APPENDIX VI

Design Areas and Weights of Hot-Rolled Deformed Bars and Ordinary and High-Strength Wire

Dia, mm	Design cross-sectional areas in cm², bars										Weight, kg/m	Hot-rolled deformed bars							Wire	
	1	2	3	4	5	6	7	8	9	10		A-II	A-III	A-IV	At-IV	A-V	At-V	At-VI	ordinary	high-strength
3	0.071	0.14	0.21	0.28	0.35	0.42	0.49	0.57	0.64	0.71	0.055 (0.051)	—	—	—	—	—	—	—	×	×
4	0.126	0.25	0.38	0.50	0.63	0.76	0.88	1.01	1.13	1.26	0.098 (0.090)	—	—	—	—	—	—	—	×	×
5	0.196	0.39	0.59	0.79	0.98	1.18	1.37	1.57	1.77	1.96	0.154 (0.139)	—	—	—	—	—	—	—	×	×
6	0.283	0.57	0.85	1.13	1.42	1.70	1.98	2.26	2.55	2.83	0.222	—	×	—	—	—	—	—	×	×
7	0.385	0.77	1.15	1.54	1.92	2.31	2.69	3.08	3.46	3.85	0.302	—	×	—	—	—	—	—	×	×
8	0.503	1.01	1.51	2.01	2.51	3.02	3.52	4.02	4.53	5.03	0.395	—	×	—	—	—	—	—	×	×
9	0.636	1.27	1.91	2.54	3.18	3.82	4.45	5.09	5.72	6.36	0.499	—	×	—	—	—	—	—	—	—
10	0.785	1.57	2.36	3.14	3.93	4.71	5.5	6.28	7.07	7.85	0.617	×	×	×	×	×	×	×	—	—
12	1.131	2.26	3.39	4.52	5.65	6.79	7.92	9.05	10.18	11.31	0.888	×	×	×	×	×	×	×	—	—
14	1.539	3.08	4.62	6.16	7.69	9.23	10.77	12.31	13.85	15.39	1.208	×	×	×	×	×	×	×	—	—
16	2.011	4.02	6.03	8.04	10.05	12.06	14.07	16.08	18.10	20.11	1.578	×	×	×	×	×	×	×	—	—
18	2.545	5.09	7.63	10.18	12.72	15.27	17.81	20.36	22.90	25.45	1.998	×	×	×	×	×	×	×	—	—
20	3.142	6.28	9.41	12.56	15.71	18.85	21.99	25.14	28.28	31.42	2.466	×	×	×	×	×	×	×	—	—
22	3.801	7.6	11.4	15.20	19.00	22.81	26.61	30.41	34.21	38.01	2.984	×	×	×	×	×	×	×	—	—
25	4.909	9.82	14.73	19.63	24.54	29.45	34.36	39.27	44.13	49.09	3.853	×	×	×	×	×	×	×	—	—
28	6.158	12.32	18.47	24.63	30.79	36.95	43.1	49.26	55.42	61.58	4.834	×	×	×	—	—	—	—	—	—
32	8.042	16.08	24.13	32.17	40.21	48.25	56.30	64.34	72.38	80.42	6.313	×	×	×	—	—	—	—	—	—
36	10.18	20.36	30.54	40.72	50.9	61.08	71.26	81.44	91.62	101.80	7.99	×	×	—	—	—	—	—	—	—
40	12.56	25.12	37.68	50.24	62.8	75.36	87.92	100.48	113.04	125.60	9.87	×	×	—	—	—	—	—	—	—

NOTES. 1. The "×" sign refers to rolled bars and wire.
2. The values in brackets are specified for class Bp-I wire.

APPENDIX VII

Types and Design Cross-Sectional Areas of Standard Welded-Wire Fabric

Type	Design cross-sectional areas (cm^2)											
	of all longitudinal wires for fabric width (B) mm											of transverse wires, per metre run of fabric
	900	1 100	1 300	1 400	1 500	1 700	2 300	2 500	2 700	2 900	3 500	
200/250/3/3	0.43	0.5	—	0.57	0.64	0.71	0.92	0.99	1.07	1.14	—	0.28
150/250/3/3	0.5	0.57	—	0.71	0.78	0.85	1.14	1.28	1.35	1.42	—	0.28
200/250/4/3	0.76	0.88	—	1.01	1.13	1.26	1.64	1.76	1.89	2.02	—	0.28
150/250/4/3	0.88	1.01	—	1.26	1.39	1.51	2.02	2.27	2.39	2.52	—	0.28
200/250/5/4	1.18	1.37	—	1.57	1.76	1.96	2.55	2.74	2.94	3.14	—	0.5
150/250/6/4	1.98	2.26	—	—	3.11	—	4.53	5.09	5.38	5.66	—	0.5
100/250/6/4	2.83	3.4	—	—	4.53	—	6.79	7.36	7.92	8.49	—	0.5
150/250/9/5	4.45	5.09	—	—	7	—	10.18	11.45	12.08	12.72	—	0.78
100/250/9/5	6.36	7.63	—	—	10.18	—	15.26	16.54	17.81	19.08	—	0.78
250/200/3/4	0.36	0.43	0.43	—	—	0.57	0.78	—	—	0.99	1.14	0.63
250/150/3/4	0.36	0.43	0.43	—	—	0.57	0.78	—	—	0.99	1.14	0.84
250/150/4/5	0.63	0·76	0.76	—	—	1.01	1.39	—	—	1.76	2.02	1.31
250/200/4/8	0.63	0.76	0.76	—	—	1.01	1.39	—	—	1,76	2.02	2.52

Type	Design cross-sectional areas (cm^2) of all longitudinal wires for fabric width B (mm)											Design cross-sectional areas (cm^2) of transverse wires, per metre run of fabric
	900	1 100	1 300	1 400	1 500	1 700	2 300	2 500	2 700	2 900	3 500	
250/150/5/9	0.98	1.18	1.18	—	—	1.57	2.16	—	—	2.74	3.14	4.24
200/200/3/3	—	0.5	0.57	0.57	0.64	0.71	0.92	0.99	1.07	1.14	1.35	0.36
150/150/3/3	—	0.57	0.71	0.71	0.78	0.85	1.14	1.28	1.35	1.42	1.7	0.47
100/100/3/3	—	0.85	0.99	1.07	1.14	1.28	1.7	1.85	1.99	2.13	2.56	0.71
200/200/5/5	—	1.37	1.57	1.57	1.76	1.96	2.55	2.74	2.94	3.14	3.72	0.98
100/100/5/5	—	2.35	2.74	2.94	3.14	3.53	4.7	5.1	5.49	5.88	7.06	1.96
150/150/7/7	—	3.08	3.85	3.85	4.24	4.62	6.16	6.93	7.32	7.7	9.24	2.57
100/100/7/7	—	4.62	5.39	5.78	6.16	6.93	9.24	10.01	10.78	11.55	13.86	3.85
200/200/8/8	—	—	—	—	—	—	6.54	7.04	—	—	—	2.52
200/200/9/9	—	—	—	—	—	—	8.27	8.9	—	—	—	3.18
150/150/9/9	—	—	—	—	—	—	10.18	11.45	—	—	—	4.24
100/100/8/8	—	—	—	—	—	—	12.07	13.08	—	—	—	5.03
100/100/9/9	—	—	—	—	—	—	15.26	16.54	—	—	—	6.36

APPENDIX VIII

Dimensions and Weight of Class K-7 Wire Strands

Nominal strand dia, mm	Wire dia, mm	Strand area, cm²	Theoretical weight of metre length of strand, N
4.5	1.5	0.127	1
6	2	0.227	1.73
7.5	2.5	0.354	2.79
9	3	0.51	4.02
12	4	0.906	7.14
15	5	1.416	11.16

NOTE. The nominal diameter of a class K-7 wire strand is three times the nominal wire diameter.

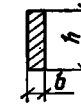
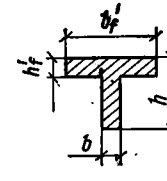
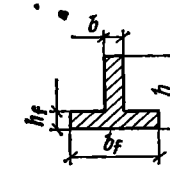
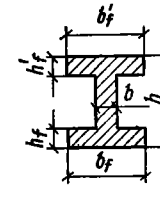
APPENDIX IX

Diameters and Minimum Spacings of Wires and Bars in Resistance Spot-Welded Wire Fabric and Bar Mats

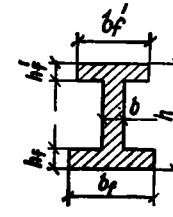
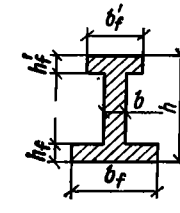
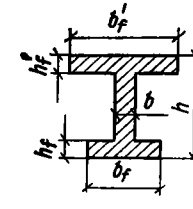
Diameter of wire or bar in one direction, mm	3	6	8	10	12	14	16	18	20	22	25	28	32	40
Minimum allowable diameter of wire or bar in the other direction, mm	3	3	3	3	4	5	5	6	6	8	8	10	10	12
Minimum allowable centre-to-centre distance between wires or bars in one direction, mm	50	50	75	75	75	75	75	100	100	100	150	150	150	200
Minimum allowable centre-to-centre distance between longitudinal bars placed in two rows, mm	—	30	30	30	40	40	40	40	50	50	50	60	70	80

APPENDIX X

Coefficient ν for Determining Elastic-Plastic Moment of Resistance $W_{cr}=\nu W_0$ of Tension Zone

Cross section	ν	Shape of cross section
Rectangular	1.75	
T-section with flange in compression zone	1.75	
T-section with flange in tension zone:		
(a) $b_f/b \leqslant 2$, irrespective of h_f/h	1.75	
(b) $b_f/b > 2$ and $h_f/h \geqslant 0.2$	1.75	
(c) $b_f/b > 2$ and $h_f/h < 0.2$	1.50	
Symmetric I-section:		
(a) $b'_f/b = b_f/b \leqslant 2$, irrespective of $h'_f/h = h_f/h$	1.75	
(b) $2 < b'_f/b = b_f/b \leqslant 6$, irrespective of $h'_f/h = h_f/h$	1.50	
(c) $b'_f/b = b_f/b > 6$ and $h'_f/h = h_f/h \geqslant 0.2$	1.50	
(d) $6 < b_f/b = b_f/b \leqslant 15$ and $h'_f/h = h_f/h < 0.2$	1.25	
(e) $b'_f/b = b_f/b > 15$ and $h'_f/h = h_f/h < 0.1$	1.10	

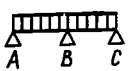
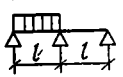
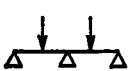
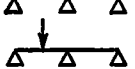
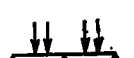
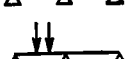
APPENDIX X (continued)

Cross section	v	Shape of cross section
Asymmetric I-section with $b_f'/b \leqslant 3$:		
(a) $b_f/b \leqslant 2$, irrespective of h_f/h	1.75	
(b) $2 < b_f/b \leqslant 6$, irrespective of h_f/h	1.50	
(c) $b_f/b > 6$ and $h_f/h > 0.1$	1.50	
Asymmetric I-section with $3 < b_f'/b < 8$:		
(a) $b_f/b \leqslant 4$, irrespective of h_f/h	1.50	
(b) $b_f/b > 4$ and $h_f/h \geqslant 0.2$	1.50	
(c) $b_f/b > 4$ and $h_f/h < 0.2$	1.25	
Asymmetric I-section with $b_f'/b \geqslant 8$:		
(a) $h_f/h > 0.3$	1.60	
(b) $h_f/h \leqslant 0.3$	1.25	

APPENDIX XI

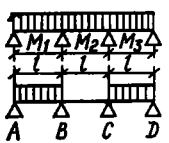
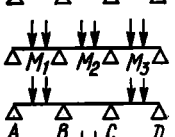
Bending Moments and Shearing Forces in Equal-Span Continuous Beams (for Uniformly Distributed and Concentrated Loads)

For a uniformly distributed load $M=(\alpha g+\beta p)\,l^2$ and $Q=(\gamma g+\delta p)\,l$ — Two-Span Beams — For a concentrated load $M=\alpha G+\beta P$ and $Q=\gamma G+\delta P$

Loading system	Span moments		Support moments	Shearing forces			
	M_1	M_2	M_B	Q_A^l	Q_B^l	Q_B^r	Q_C
	0.07	0.07	—0.125	0.375	—0.625	0.625	—0.375
	0.096	—0.025	—0.063	0.437	—0.563	0.063	0.063
	0.156	0.156	—0.188	0.312	—0.688	0.688	—0.312
	0.203	—0.047	—0.094	0.406	—0.594	0.094	0.094
	0.222	0.222	—0.333	0.667	—1.334	1.334	—0.667
	0.278	—0.056	—0.167	0.833	—1.167	0.167	0.167
	0.266	0.266	—0.469	1.042	—1.958	1.958	—1.042
	0.383	—0.117	—0.234	1.266	—0.734	0.234	0.234

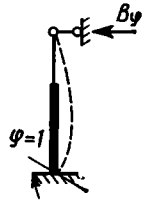
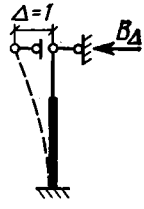
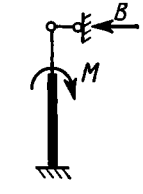
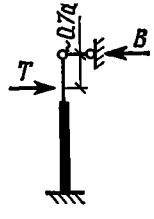
*APPENDIX XI (continued)

Three-Span Beams

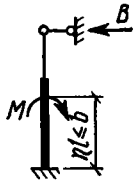
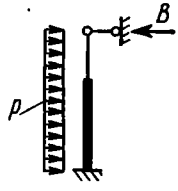
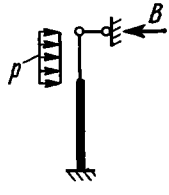
Loading system	Span moments		Support moments		Shearing forces					
	M_1	M_2	M_B	M_C	Q_A	Q_B^l	Q_B^r	Q_C^l	Q_C^r	Q_D
	0.08	0.025	—0.1	—0.1	0.4	—0.6	0.5	—0.5	0.6	—0.4
	0.101	—0.05	—0.05	—0.05	0.45	—0.55	0	0	0.55	—0.45
	—0.025	0.075	—0.05	—0.05	—0.05	—0.05	0.5	—0.5	0.05	0.05
	—	—	—0.117	—0.033	0.383	—0.617	0.583	—0.417	0.033	0.033
	—	—	—0.067	—0.017	0.433	—0.567	0.083	0.083	—0.017	—0.017
	0.244	0.067	—0.267	—0.267	0.733	—1.267	1	—1	1.267	—0.733
	0.289	—0.133	—0.133	—0.133	0.866	—1.133	0	0	1.133	—0.866
	—0.044	0.2	—0.133	—0.133	—0.133	—0.133	1	—1	0.133	0.133
	—	—	—0.311	—0.089	0.689	—1.311	1.222	—0.778	0.089	0.089
	—	—	—0.178	—0.044	0.822	—1.178	0.222	0.222	—0.044	—0.044

APPENDIX XII

A. Design Formulas for Two-Piece Columns (Fig. A)

Loading system	Support reaction B
	$B_\varphi = 3E_c I_b/l^2\,(1+k+k_1)$
	$B_\Delta = 3E_c I_b/l^3\,(1+k+k_1)$
	$B = 3M\,(1-\alpha^2)/2l\,(1+k+k_1)$
	$B = T\,(1-\alpha+k_1)/(1+k+k_1)$
	$B = 3M\,(1+k/\alpha)/2l\,(1+k+k_1)$

APPENDIX XII (continued)

Loading system	Support reaction B
	$B=3M\eta\,(2-\eta)/2l\,(1+k+k_1)$
	$B=3pl\,[1+\alpha k+1.33\,(1+\alpha)\,k_1]/8\,(1+k+k_1)$
	$B=pl\,[3\,(1+\alpha k)-(3+\alpha)(1-\alpha)^3+k_1]/8\,(1+k+k_1)$

Here, $\alpha=a/l$, $k=\alpha^3\,(I_b/I_t-1)$, $k_1=(1-a)^3\,I_b/8n^2I$, $I_b=Fc^2/2$, F is the cross-sectional area of one column piece, and n is the number of panels in a two-piece column. The other notation is explained in Fig. A.

B. Design Formulas for Single-Piece Columns (Fig. B)

When finding the reaction B for solid single-piece columns, k_1 in the above formulas for open-work two-piece columns should be taken as zero, and I_b should be determined as for a solid section.

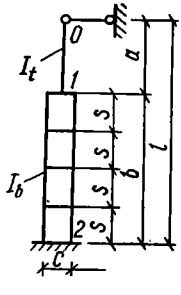


Fig. A

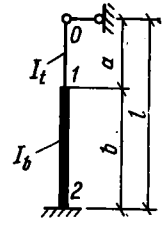


Fig. B

APPENDIX XIII

Tables for Designing Multi-Storey Frames

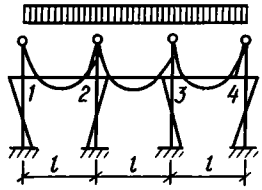
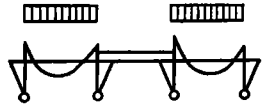
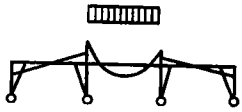
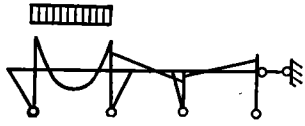
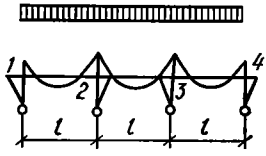
The support moments in the girders of a three-bay frame are found by $M=(\alpha g+\beta p)\, l^2$. The coefficients, α and β, depend on the combination of dead load, g, and live load, p, on the ratio of the rigidities per metre run of the struts entering a frame joint from below, i_b, the rigidities of the struts joining from above, i_t, and the rigidities per metre run of the girders, i_g:

$$i_b = B_b/l_b;\ i_t = B_t/l_t \text{ and } i_g = B_g/l$$

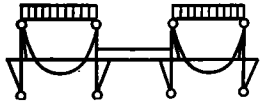
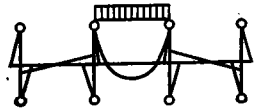
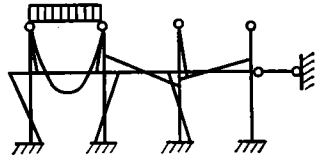
where l_b and l_t are the total lengths of the bottom and top parts of the struts (the height of the multi-storey frame), respectively.

The span moments are determined using the support moments.

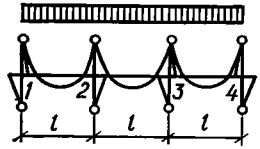
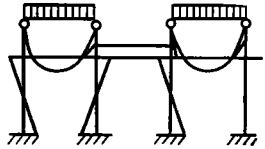
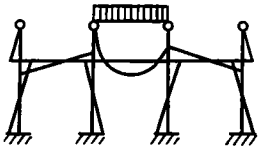
Top-Storey Frame

Loading system and moment diagram	i_b/i_g	Support moments			
		M_{12}	M_{21}	M_{23}	M_{32}
	0.25	0.025	0.099	0.093	—
	0.5	0.038	0.098	0.089	—
	1	0.052	0.095	0.086	—
	1.5	0.059	0.093	0.085	—
	0.25	0.032	0.059	0.034	—
	0.5	0.047	0.064	0.026	—
	1	0.061	0.069	0.017	—
	1.5	0.067	0.072	0.013	—
	0.25	0.007	0.04	0.059	—
	0.5	0.009	0.034	0.064	—
	1	0.009	0.026	0.069	—
	1.5	0.008	0.021	0.072	—
	0.25	0.03	0.069	0.049	0.015
	0.5	0.045	0.071	0.039	0.013
	1	0.059	0.073	0.028	0.011
	1.5	0.066	0.075	0.022	0.009
	0.5	0.038	0.098	0.089	—
	1	0.052	0.095	0.086	—
	2	0.063	0.092	0.085	—
	3	0.068	0.089	0.084	—

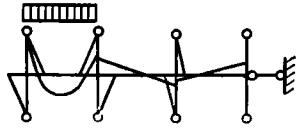
APPENDIX XIII (continued)

Loading system and moment diagram	i_b/i_g	Support moments			
		M_{12}	M_{21}	M_{23}	M_{32}
	0.5	0.047	0.064	0.026	—
	1	0.060	0.069	0.017	—
	2	0.770	0.074	0.011	—
	3	0.074	0.076	0.008	—
	0.5	0.009	0.034	0.064	—
	1	0.008	0.026	0.069	—
	2	0.007	0.018	0.074	—
	3	0.006	0.013	0.076	—
	0.5	0.045	0.07	0.039	0.013
	1	0.059	0.073	0.028	0.011
	2	0.069	0.075	0.019	0.008
	3	0.074	0.077	0.014	0.006

Ground-Floor Frame

Loading system and moment diagram	$\frac{i_b+1.5i_t}{i_g}$	Support moments			
		M_{12}	M_{21}	M_{23}	M_{32}
	0.5	0.030	0.099	0.092	—
	1	0.044	0.097	0.088	—
	2	0.057	0.094	0.086	—
	3	0.063	0.092	0.085	—
	5	0.069	0.090	0.084	—
	0.5	0.038	0.061	0.031	—
	1	0.053	0.066	0.022	—
	2	0.065	0.071	0.014	—
	3	0.070	0.074	0.011	—
	5	0.076	0.077	0.007	—
	0.5	0.008	0.038	0.061	—
	1	0.009	0.031	0.066	—
	2	0.008	0.023	0.071	—
	3	0.007	0.018	0.074	—
	5	0.007	0.013	0.077	—

APPENDIX XIII (continued)

Loading system and moment diagram	$\frac{i_b+1.5i_t}{i_g}$	Support moments			
		M_{12}	M_{21}	M_{23}	M_{32}
	0.5	0.036	0.07	0.045	0.014
	1	0.051	0.072	0.034	0.012
	2	0.064	0.074	0.024	0.010
	3	0.07	0.076	0.018	0.007
	5	0.075	0.078	0.013	0.006

NOTE. For each loading system, the bending moments in the struts are found by the difference between the girder support moments at the joint, distributed in proportion to the rigidities per metre run of the struts. The rigidity per metre run of the ground-floor strut is taken as $i_b/1.5$.

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